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Mathematical discussion in classrooms as a technologically-supported activity fostering participation and inclusion

Chiara Giberti[1] · **Ferdinando Arzarello**[2] · **Silvia Beltramino**[3] · **Giorgio Bolondi**[4]

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Abstract

Whole-class mathematical discussion in a problem-solving activity is recognized as a powerful pedagogical activity but also a challenge for teachers who must consider several difficulties that learners might face, particularly in terms of an overload of Working Memory and Executive Functions. This study investigates how the use of a digital platform (Padlet) can support participatory and inclusive mathematical classroom discussion. We proposed a teaching experiment based on graphical tasks anticipating integral calculus to grade 13 students, and we examined how the use of the digital platform plays a role in the construction and interpretation of new mathematical objects emerging from the activity. The use of Instrumental Genesis and Double Instrumental Genesis frameworks allowed us to make the affordances of the tool emerge. As a result, we got evidence of how mathematical discussion may develop as a network of interactions, feedback, and connection of input and discuss examples of how active participation and inclusion are enhanced by the tool affordances. Indeed, the digital platform allowed easy interaction, with many ways to represent and express the ongoing evolution of personal and shared meanings and the possibility to manage the time of the activity. This fostered students' participation and students which did not participate in previous discussions were actively engaged in it.

Keywords Mathematical discussion · Digital technologies · Inclusion · Participation

1 Introduction

Mathematics education plays a crucial role in promoting the active, responsible, and conscious participation of students in our complex and multifaceted society. In this scenario, new technologies in education are fundamental, but incorporating digital tools into

✉ Chiara Giberti
chiara.giberti@unibg.it

1 Department of Human and Social Science, University of Bergamo, Bergamo, Italy

2 Department of Mathematics "Giuseppe Peano", University of Turin, Turin, Italy

3 Liceo Scientifico "Maria Curie" of Pinerolo, Turin, Italy

4 Faculty of Education, Free University of Bozen-Bolzano, Brixen, Italy

mathematics education is a "non-trivial issue" with far-reaching implications for all aspects of the learning process (Drijvers et al., 2013). This was clearly highlighted in 2020 during the COVID emergency period when the sudden need for digitalization led to the adoption of a transmissive approach to mathematics teaching and learning, putting aside mathematics activities based on a constructivist approach (Bakker & Wagner, 2020).

The need to reflect on how to introduce digital technologies in education has already been tackled by several researchers, also considering the effect of media technologies. For instance, Jenkins has reflected on media education and "participatory cultures," which he defines as "a culture with relatively low barriers to artistic expression and civic engagement, strong support for creating and sharing creations, and some type of informal mentorship" where "members also believe their contributions matter and feel some degree of social connection with one another," and their potential benefits on education (Jenkins, 2009, p. xi). Within this framework, Jenkins (2009) considers interactivity as a property of technology, while participation is considered as a feature of culture: new media allows interactivity that enables people to reach and share new content in new ways. However, despite this, the wide dissemination of technologies makes sense only if it is associated with the development of necessary cultural knowledge and attempts to "encourage youths to develop the skills, knowledge, ethical frameworks, and self-confidence needed to be full participants in contemporary culture" (Jenkins, 2009, p. 9).

Mathematical discussion is a crucial moment in classroom activities and a challenge for teachers, especially in view of enhancing student participation to co-construct new mathematical concepts. In this paper, we address this challenge of developing an inclusive and participatory mathematical classroom discussion, and our interest is to investigate how innovative technologies can help overcome this challenge. In particular, we present a research study focused on the introduction of a specific technological tool, a digital platform named Padlet,[1] where students have the opportunity to post their ideas and comment on their classmates' posts. On the one hand, we highlight how this tool helps the teacher in orchestrating (in the sense of Drijvers et al., 2013) the mathematical discussion emerging from the resolution of a mathematical problem. On the other hand, we consider Padlet as a digital media used for school purposes (Hunter & Hunter, 2018; Imm & Stylianou, 2012), which promotes the participation of all students in discussion in a constructive and generative way and thus leads us to consider the classroom as a tiny but significant participatory culture.

This research is an extension of a previous research study (Giberti et al., 2022a, 2022b) based on data collected from a grade 6 class of a small mountain village during the COVID emergency, and the research was based on a problem that required measure estimation. In this work, we expand on the previous results by considering a totally different context in terms of school background: we involved a class from the metropolitan area of Torino, a big city in Italy, and the experiment considered in this paper took place in 2023 when schools in Italy had returned to normal after the pandemic. We focus on grade 13 students, considering their stronger mastery of both technology and the specificities of mathematical discussion, and we proposed a mathematical problem as a preliminary introduction to integral calculus. This enables us to verify the potentialities of this approach based on Padlet to promote mathematical discussion in the classroom and allows us to generalize our results within a broader context.

[1] The version of Padlet used in this work was released in 2023; we used the free Padlet account. More information about Padlet can be found at https://padlet.com/.

2 Theoretical framework

2.1 Mathematical discussion

Communication, not only in terms of verbal communication but including all communicative actions, has a crucial role in mathematics education because "students' learning of mathematics in teaching processes is enclosed in language and communication" (Steinbring, 2015, p. 282). Mathematical discussion is defined by Bartolini Bussi as a "polyphony of articulated voices on a mathematical object" (Bartolini Bussi et al., 1995, p.16). The term "voice" refers to a form of speaking and thinking, which represents the perspective of an individual. Voices are not identified with interlocutors' verbal expression, for instance, a student might use the same voice of a classmate if he/she supports the same idea; furthermore, the same student might use multiple voices when he/she presents a new perspective or refers to a classmate's argument (Bartolini Bussi et al., 1995). Following this definition, the teacher guides and orchestrates the discussion: (i) introducing a particular discussion in the classroom activity and (ii) influencing the development of the discussion with his/her own interventions.

Bartolini Bussi and colleagues (1995) differentiate between different types of mathematical discussion: in this paper, we analyze a mathematical problem discussion and, in particular, a *balance discussion*, i.e., the process of informing, analyzing, and evaluating proposed individual solutions to a given problem. Usually, this discussion is introduced by the teacher a few days after the lesson in which the students tackled the problem. This time interval allows the teacher to collect and read their proposed solutions in order to plan the discussion and allows the students to distance themselves from their own work and thinking. In the discussion, the comparison of strategies is followed by moments aimed to make explicit (i) the solution processes, (ii) the identification of new learning outcomes, and (iii) the institutionalization of learning, i.e., the formulation of new mathematical concepts connected to previous learning.

The practice of comparing solutions and recognizing connections between them is crucial for enhancing relational thinking in mathematics and gaining a more in-depth understanding of mathematical concepts (Skemp, 1978). Richland and colleagues (2017) outline the difficulties students might encounter during a discussion based on comparison of solutions, introducing two main concepts:

- Working Memory (WM), described as "the cognitive resource that enables humans to hold information in mind and to manipulate that information without losing it" (Richland et al., 2017, p. 43); for instance, if a student describes a solution verbally during the discussion, others must actively retain the information in order to consider it later; otherwise, it will be forgotten.
- Executive Functions (EF) related to WM, are "general purpose control mechanisms that modulate the operation of various cognitive subprocesses and thereby regulate the dynamics of human cognition" (Miyake et al., 2000, p. 50). EF are necessary, in particular, in new, complex, and demanding tasks and are crucial for students to follow and participate in a mathematical discussion (Richland et al., 2017). The role of EF emerges, for instance, during the discussion, when a student changes his/her explanation mid-way and listeners must replace the initial version in their WM with the revised one.

These two concepts, belonging to the Cognitive Load Theory, have been already adopted to analyze interaction between individuals (Choi et al., 2014) also considering digital and online environments (Skulmowski & Xu, 2022). In our study, we will consider the three main EF proposed by Miyake and colleagues (2000): *shifting* (process of transitioning between various tasks, operations, or mental frameworks), *updating* (continuous monitoring and encoding of incoming information, facilitating the revision of items stored in WM by replacing outdated data with newer, more pertinent information), and *inhibition* (capacity to intentionally restrain dominant, automatic, or prepotent responses).

When the demands on WM and EF are too high, it can result in some students finding it difficult to understand the connections between solutions, thereby causing a rise in errors of distraction, and struggle with comparison of representations. This is because the cognitive load becomes too much for them to handle effectively (Richland et al., 2017). Thus, the orchestration of a mathematical balance discussion becomes a challenge for teachers who should be mindful of the diversity in their students' EF resources and make adjustments to provide appropriate support and time during the mathematical discussion. By doing so, they can facilitate the participation of all students, regardless of their varying levels of WM and EF.

Students' differences in WM and EF must be considered together with their social competencies that enable students' participation. Several levels of participation can be defined (Cohen & Lotan, 2014), and students can participate in the discussion by:

- Following others' interactions
- Reacting to others' solicitations
- Leading a discussion
- Putting forward new issues

Students' participation plays a key role in creating an inclusive learning environment (Demo et al., 2021; Santi et al., 2022) in which all students have the possibility to develop their mathematical competencies and are closely linked to individual social well-being in school activity (Booth & Ainscow, 2002). We will thus consider that a mathematical discussion is inclusive if the participation of students is fostered, if WM is lightened, and if EF resources are developed. Promoting a participatory and inclusive discussion is a challenge for teachers, and, in this work, we propose the use of digital technologies to facilitate teachers in achieving this goal.

2.2 Digital technologies to promote mathematical discussion

The use of digital technology in mathematics education has been widely explored in recent years, with various advancements achieved through the dedicated efforts of mathematics teachers and researchers (Borba, 2021), but it also presents its own set of challenges and merely having access to technology does not guarantee improvement in the quality of teaching and learning (Ball et al., 2018). Teachers have to rethink their pedagogical practices in light of increased pedagogical opportunities, and they have to manage the process of *instrumental genesis* (Trouche, 2004). This combines two interrelated processes: *instrumentalization*—the various functionalities of the artefact are transformed into actions for mathematics teaching/learning—and *instrumentation*—the progressive construction of cognitive schemes of instrumented actions by the agents who use the tool (Haspekian, 2014, p.247). In this paper, we investigate how such processes can develop when a technological tool like the Padlet becomes an instrument used within a mathematical teaching experiment (Kelly & Lesh, 2012). The general features of the tool require a specific instrumental approach: the fact that Padlet has not been designed to support mathematical discussion but for more general purposes pushes the teacher and the students to explore the variety of ways it can be used in this field. Haspekian introduced the notion of *double instrumental genesis* to describe these two joint processes analyzing how a spreadsheet is introduced as a teaching/learning tool in mathematics activities:

> Indeed, for students, the spreadsheet may become a mathematical instrument through an instrumental genesis. However, as a spreadsheet is not by definition a didactical tool to serve mathematics education, it also has to progressively become such an instrument during a professional genesis on the part of teachers. These are two different instruments, which both exist for the teacher (Haspekian, 2014, p. 247).

This happens also in our case for the Padlet. In particular, it becomes an instrument for mathematics education through a suitable professional instrumentation from the teacher and the researcher, as described below. This process consists in what Trouche (2004) calls *instrumental orchestration*, that is, "the teacher's intentional and systematic organization and use of the various artefacts available in a […] learning environment in a given mathematical task situation, in order to guide students' instrumental genesis" (Drijvers et al., 2013, p. 1350). Three elements are involved in the instrumental orchestration:

- The *didactical configuration* (the arrangement of the teaching setting and artifacts)
- The *exploitation mode* (decision about how the task is introduced and worked through, including the use of artifacts and time management)
- The *didactical performance* (teaching decisions made during the lesson)

These processes require that the designers of the teaching experiment and the students themselves suitably exploit possible new suitable affordances of the instrument during the double instrumentation process. The term "affordance" was introduced by Gibson (1986) and became the subject of intense discussion in the field of Information and Communication Technologies in education. Hammond (2010, p. 216) proposed the following definition of affordances: "the perception of a possibility of action […] provided by properties of, in this case, the computer plus software. […] they may, drawing on intuition and deduction from user accounts, be 'perceived directly', and perception of actions can precede internal mental ordering." We will adopt it in this study as a specific feature of the double instrumentation process.

As a final remark, we observe that from the side of students, the process of getting used to the fresh Padlet affordances has been supported by favorable concurrent aspects of its instrumentation in our teaching experiment. In fact, students have found at school the opportunity of using a transparent digital environment that supports them in handling social networking and acts, to which they, as *Post-Millennial Generation Z* members, are acquainted in their life outside school. In turn, this approach can create favorable conditions to support students' mathematical knowledge construction, as described in Engelbrecht and colleagues (2020, p. 830):

> sharing interaction spaces, such as those that facilitate asynchronous online discussion, creates opportunities for participants to reorganize their knowledge in the course of the social interaction. In this sense, the affordance of new media helps participants to communicate knowledge in multimodal ways generating different ways of discourse.

3 Research question

As stated by Jacinto and Carreira (2022), “the role and impact of digital tools in mathematical problem-solving processes remains an under-explored topic” (Jacinto & Carreira, 2022, p.2560). Our previous findings on this issue highlighted that the use of Padlet can support the role of the teacher in the orchestration of mathematical discussion and help all students in overcoming the obstacles due to excessive effort in terms of WM and EF, thus promoting a more inclusive mathematical discussion (Giberti et al., 2022a, b). In this paper, we extend our previous results by considering a higher school grade (grade 13 vs. grade 6) and starting from a different mathematical problem in which the mathematical elements are much more explicit and relevant, requiring structured argumentation and different representations and argumentations based on numerical data. In other words, the previous experiment revolved around a “real” situation that contained mathematizable elements (but in many cases developed on a plane that was not explicitly mathematical), whereas here the situation is already in context with mathematical elements (plots). Furthermore, in our previous experiment, the use of technology was “forced” by the emergency situation, while here it is embedded in the “ordinary” of class discourse. Thus, the implementation of the experiment in a strictly different context will enable us to confirm the potentialities and affordances of this instrument and generalize our results in terms of participation and inclusion of the students in the mathematical discussion in a problem-solving activity.

Thus, our research question is: In which way can Padlet technology support participatory and inclusive mathematical classroom discussion?

4 Methods

4.1 Participants

The class involved was from an Italian science-oriented high school (grade 13); it was composed of 15 females and 6 males, none of whom had special educational needs. The mathematics teacher, the third co-author of this contribution, is an experienced teacher who has always promoted discussion within the mathematics lessons, following a constructivist approach to mathematics teaching and learning as recommended in the Italian National Guidelines (MIUR, 2012). Information regarding students’ performances in mathematics and their participation in classroom mathematics activities were reported by the teacher both before and after the experiment. Most of the students were already used to actively participating in the discussions during mathematics lessons, although not everybody contributed in the same manner or with the same frequency. With regard to the mathematical content related to the proposed problem, students knew the meaning of derivatives, evaluating the growth of a function, reading a graph, calculating limits, and the meaning of the terms “primitive” and “derivative.” They had not yet studied integral calculus, although they had occasionally encountered some integrals in physics lessons. Before the experiment, the teacher identified three students (their nicknames were Stampella, Carota, and Cinzia) as students who usually have more difficulties in mathematics and in taking part to mathematical discussions. These students were identified on the basis of her previous long-term experience with the class (which she had taught since grade 9) and considering their usual level of participation in classroom discussions and their performances; we will refer to these students as "previously fragile" in mathematics with the aim of focusing on their engagement in the different steps of the activity proposed.

4.2 The platform

Padlet allows multiple input modes (text posts, images of hand-written comments or calculations, emojis such as thumbs-up and smiley face, audio uploads, and video uploads). In this and previous experiments, the Padlet was used to collect students' solutions to the same problem. Padlet allows students to post their ideas using a nickname, meaning that interactions within the Padlet remain completely anonymous. Furthermore, a specific Padlet feature allows the teacher to save the posts and make them visible to other students only when all the classmates have completed their work; in this way, each student can manage his/her time and not be influenced by what their peers have already published. The solutions shared within the Padlet were then read and commented on by the classmates. The teacher presented the platform's functions to the class before the experiment; only a few of the students had already used it, but in those cases, it was for different activities.

4.3 The task-related stimulus for the discussion: design and a-priori analysis

The mathematical task was developed by the teacher and the researchers specifically for the experiment. The research aim was to identify a problematic situation: (i) meaningful in relation to previous and further learning goals of the class, (ii) allowing students to find multiple and different solutions, and (iii) promoting a rich, fertile, and significant mathematical discussion. The whole team, composed of the teacher and researchers (all authors of this paper), then decided to focus on the problem of identifying the average of a continuous phenomenon, starting from its plot. The task was designed to invite students to compare the average of the temperatures measured by a digital thermometer in two different cities in the same time interval. Our aim, from a mathematical point of view, was to promote a discussion in which students were encouraged to reflect in terms of area under the curve and then promote a co-construction of the idea of integral as the average of a continuous phenomenon. The problem was then composed by three plots, included in one column of the Padlet each (Figs. 1 and 2), which represented similar situations of growing complexity. The different steps of the task were designed to foster participation in a multistage problem (Ayres & Sweller, 1990): they were given to students in three successive steps.

The a-priori analysis of the problem (Artigue, 2020) involved the teacher and researchers during two online meetings and highlighted possible strategies and related difficulties of each proposed situation. The collaboration between teachers and researchers as designers and participant observers is a peculiarity of Italian research in mathematics education (Arzarello & Bartolini Bussi, 1998). In Plot A, the temperature in city X was higher than that in city Y at any time; thus, we expected that most students would answer with a general argumentation without any calculation. Other strategies could be based on an attempt to calculate the average, for instance, by considering the difference between the highest and lowest values or considering the area under the two curves. In Plot B, the two curves intersect but, despite this, the fact that the temperature was higher in city X is evident even without any calculations. Also in this case, we supposed that students could adopt both qualitative (based on a "visual" comparison of areas or an estimate of the area between the two curves) and quantitative strategies (wrong or right attempts to calculate the average

Fig. 1 Screenshot of the Padlet before the activity

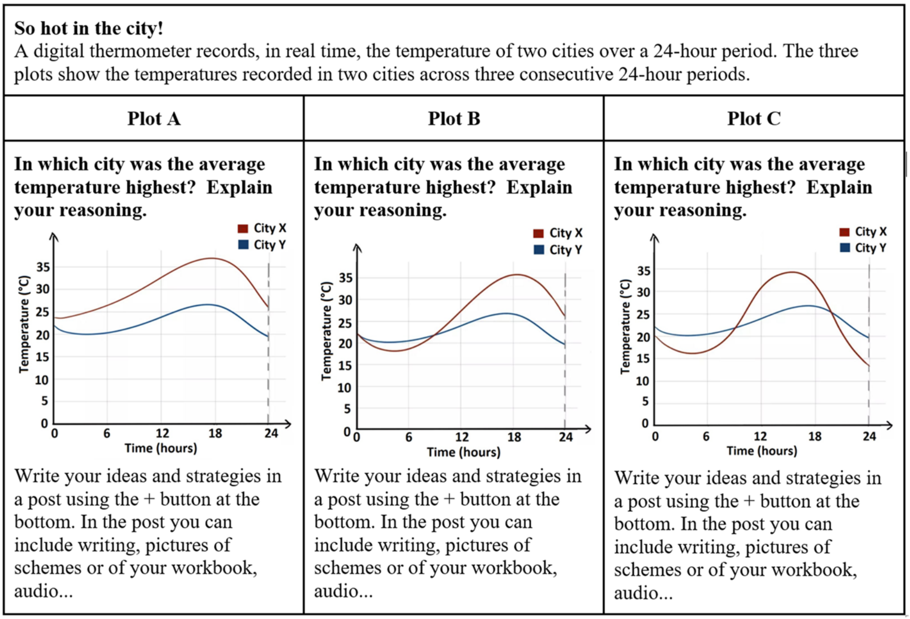

Fig. 2 Translation of the Padlet screenshot

by considering maximum and minimum values or areas). Finally, in Plot C, the difference between the two curves is not evident and it is necessary to base any answer on quantitative considerations. In this case, qualitative considerations based on the visual interpretation of the plot are not sufficient. At the same time, however, quantitative reflections considering minimum and maximum values are not sufficient either. We therefore expected that this last situation would reinforce the necessity for students to refine the quantitative strategies adopted and then reflect on how to compare the areas under the curves, before identifying new quantitative ways to calculate these areas.

4.4 Data collection

The experiment was developed in two classroom lessons of 2 h each. In the first lesson, the problem was introduced by the teacher, using Padlet. More precisely, the task was used by the teacher to enact the process described in Table 1. This can happen without the direct intervention of the teacher who, throughout the duration of the work on Padlet, does not intervene but simply observes the ideas already posted in the Padlet and manages the timing of the phases.

4.5 Data analysis

We analyzed Padlet data through qualitative methods based on a coloring technique (similar to the one used in Bolondi et al., 2023) to highlight the way students participated in the discussion. More precisely, we created a diagram (Fig. 3) that illustrated the interaction between posts and comments; in the diagram, the dark blue rectangles represent the posts, and the other rectangles represent the comments relative to each post; in Table 2, we report the type of comments considered. In relation to the process described in Table 1, the posts did not influence each other because students did not have the possibility to see others' posts until the next step began. In the comment phase, conversely, the comments were collected in time and all the comments were immediately visible to all students; thus, we have to consider that the sequence of comments is not casual, but they are related and influenced each other. The coding procedure of the posts and comments was performed by two of the researchers independently and then the very rare differences between the two diagrams were discussed with the other researchers to decide the correct coding (O'Connor & Joffe, 2020).

We refer to students' participation at the micro-level of classroom interaction (Demo & Veronesi, 2019) considering their intervention both in the digital platform and in the final discussion and the level of their participation as defined by Cohen and Lotan (2014).

Furthermore, in the analysis of the classroom discussion, we followed Richland and colleagues (2017), and we categorized the oral interventions of the teacher and students, as well as their activities involving the use of Padlet during the final discussion, to highlight episodes in which participation in the development of mathematical discussions might be hindered due to an excessive load of WM and EF. At the same time, we focused on possible strategies employed by students and the teacher to overcome those barriers to participation.

Regarding the analysis of classroom discussion (Step 8), in our research, two of the three elements of instrumental orchestration (didactical configuration and exploitation mode) were defined in advance (see Table 1) and agreed with the teacher. We then analyzed the findings that emerged from the didactical performance of the teacher during the classroom discussion and how this performance is supported by the fact that the mathematical discussion had already been launched on the Padlet. We searched also for unexpected elements of didactical performance, i.e., decisions made during the lesson in relation to new affordances of Padlet envisaged by the teacher.

Data from the final interview with the teacher were explored to deepen her perception on how the discussion developed both in Padlet and then in the classroom, also in comparison

Table 1 Nine-step pedagogical design structure of the teaching experiment with indication regarding the (desired) students' activities (Watson & Ohtani, 2015)

Steps	Lesson	Description	Teacher activity	Students' activity	Data collected
Step 0	Lesson 1	Introduction	The teacher introduces the Padlet to the students, explaining how to login and add posts or comments. The teacher explains the nine-step structure and asks students to explain their ideas and comment in Padlet using posts signed with their nicknames.	Students decide their nicknames and login to Padlet.	-
Step 1	Lesson 1	Hypotheses collection Plot A	The teacher shows only the first column in the Padlet (Plot A with the description of the situation and the request). The Padlet was set up by the teacher so as to enable her to moderate the posts and collect all students' answers before making them visible. In this and subsequent steps (steps 1 to 6), the teacher observes students working individually and reads the posts in the Padlet but does not intervene in any way.	Students work individually and post their hypotheses and solutions using the modality of their choice (written posts, pictures, audios, small videos, or links). Students see only their own input.	Students' interventions in Padlet.

Table 1 (continued)

Steps	Lesson	Description	Teacher activity	Students' activity	Data collected
Step 2	Lesson 1	Commenting on hypothesis Plot A	Once all the students have posted their solution in the first column, the teacher makes all the posts visible and asks students to read all their classmates' solutions and comment on them. Due to Padlet constraints, the comments were possible only in written form; an additional possibility was to allow students to use the "like/dislike" modality, but the teacher, in agreement with the researchers, decided to forgo this possibility in order to encourage students to argue their opinions.	Students read all their classmates' solutions and comment on them in written form using their nicknames.	Students' interventions in Padlet.
Step 3 and Step 4	Lesson 1	Hypotheses and comments collection Plot B	Repetition of steps 1 and 2 in relation to Plot B (second column).	Same as in steps 1 and 2.	Students' interventions in Padlet.
Step 5 and Step 6	Lesson 1	Hypotheses and comments collection Plot C	Repetition of steps 1 and 2 in relation to Plot C (third column).	Same as in steps 1 and 2.	Students' interventions in Padlet.

Table 1 (continued)

Steps	Lesson	Description	Teacher activity	Students' activity	Data collected
Step 7	Lesson 1	Final comments	The teacher asks students to reflect on the three situations, on their own and their classmates' solutions, and asks them to add a general comment in the fourth column.	Students retrace the three situations with the support of the Padlet in which all the solutions and comments are now collected. The students post their general comments in the fourth column. Posts could refer to how the activity worked in general or they could explain their reflections on the comparison between their own and their classmates' hypothesis in the three situations.	Students' interventions in Padlet.
Step 8	Lesson 2	Classroom discussion	The teacher displays the Padlet on the interactive whiteboard; this enables the teacher and students to refer to the posts and comments collected, basing their discussion on the previously collected information. The anonymity continued for the first part of the discussion and then nicknames were spontaneously revealed; students' voice recordings and writings are reported in the results section, using their nicknames.	Students participate in the classroom discussion orchestrated by the teacher. They can refer to the posts and comments in the Padlet shared on the interactive whiteboard and have the possibility to use their personal devices to access the Padlet.	Video recording of the classroom discussion and its transcript. The final discussion was videotaped by one of the researchers who did not intervene at all in the discussion.

Table 1 (continued)

Steps	Lesson	Description	Teacher activity	Students' activity	Data collected
Step 9	Lesson 2	Survey and interview	At the end of the experiment, the teacher proposed to all students an online anonymous survey (Appendix A, Table 11) designed by the researchers, with the aim of collecting their opinions on how the use of the Padlet improved/hindered their participation in the mathematical discussion. The teacher's point of view on the development of the discussion as compared with previous discussions in the same class was collected through a semi-structured interview. The teacher was not aware of the questions included by the researchers in the survey until the interview.	Students answer the final survey and describe how the use of Padlet improved/hindered their participation in the mathematical discussion.	Students' written responses to the final survey. Video recording of the final interview with the teacher and its transcript.

Table 2 Representation of each type of comment in the diagram

Symbol	Type of comment
Yellow rectangle	Rejection of the post, supported by constructive feedback
Green rectangle	Endorsement of the post, supported by constructive feedback
Orange rectangle	Constructive comments which are neither explicitly in favor nor contrary to the post
Red rectangle	Rejection of the post, not supported by constructive feedback
Light blue rectangle	Endorsement of the post, not supported by constructive feedback
Gray rectangle	Comment not connected to the mathematical problem
Black dot	Explicit reference to the reasoning of his/her own personal thought or answer
Black triangle	Explicit comparison with other answers (even if not specified in some cases, e.g., "among all the answers, this is the only one…")
Black star	Indirect reference to previous reasoning (not explicit)

with previous mathematical discussion. We analyzed the information gathered by the students in the fourth column of the Padlet (Step 7—Final comments) and the survey, which included 9 open-ended questions on metacognitive aspects related to how Padlet fostered/hindered their reasoning and participation during the problem-solving activity (Appendix A, Table 11).

5 Results

5.1 Padlet to promote students' participation

In our experiment, the mathematical discussion begins in the Padlet and then develops in the classroom. Thus, as observed in previous research (Giberti et al., 2022b), we have to consider that participation of students could emerge in different ways. First, we consider the way students participate in the Padlet, posting their hypotheses and commenting on other students' posts (Table 1, steps 1 to 7). The graph in Fig. 3 represents students' interactions within the Padlet, considering the three different plots proposed. As described in the previous section and in particular in Table 2, for each plot, we report all students' posts and classmates' comments. The nickname reported in each rectangle is the one of the author of the post/comment.

All students posted their solution and commented on other posts for each plot: all the nicknames are present in the three columns of the Padlet (Fig. 3) and included both in a post (dark blue rectangle) and in a comment (other rectangles). Thus, everyone commented by following and/or reacting to other posts, and we can consider this as evidence of the first two levels of participation proposed by Cohen and Lotan (2014). Furthermore, in this peer-discussion with no intervention by the teacher, we observe an evolution in the way students argue their reasoning and comments. If we observe the Padlet after step 2 (Fig. 3) of our experiment (thus in relation to Plot A), the comments are mainly concentrated in some of the posts: 4 out of 22 posts received more than 50% of the comments and 4 posts have no comments. On the contrary, in the following steps (especially considering Plot C), almost all posts received comments, and thus, almost all students received feedbacks from their classmates. Only one post related to Plot C did not receive any comment (ThebIEM's post), and 11 posts out of 21 received 4 comments or more. Moreover, from Plot A to Plot C, the total number of comments increased, and the number of comments which rejected

Fig. 3 Schematic representation of posts and comments in the Padlet, undersigned with the students' nicknames

the post but proposed an argument which could help the classmate to improve his/her reasoning increased (Table 3 and yellow rectangles in Fig. 3). Thus, an informal process of mentorship between peers begins and evolves in the Padlet and emerges spontaneously; the teacher just observed the students' work and managed the time between the different steps through the Padlet.

This evolution in the way that students comment is accompanied by a change also in the way they post their ideas, as highlighted by the students themselves. For instance, in the last column of the Padlet (students' final comment, Step 7), Vespasiano wrote:

> I think that, thanks to the activity, there was a positive improvement in answers. Answers improved from the argumentation point of view but also from an aesthetic point of view. Moreover, plot by plot, the inclusion of data and mathematics calculations were more frequent and analysed more in depth.

Table 3 Number of each type of comment in relation to each plot/column

Type of comment	Plot A	Plot B	Plot C
Rejection, supported by constructive feedback (yellow)	9	13	20
Endorsement, supported by constructive feedback (green)	18	20	17
Constructive comments which do not explicitly reject or endorse what is written in the post (orange)	15	10	12
Rejection, not supported by constructive feedback (red)	1	4	3
Endorsement, not supported by constructive feedback (light blue)	12	9	15
Comments non-connected to the mathematical problem (gray)	0	0	4
Total number of comments	**55**	**56**	**71**

Similar considerations were made also by Anastasia:

> I noticed that my answers (like those of many other students) ‘evolved’, in the sense that while in the first plot I had not paid attention to the concrete data of the plot but had just expressed a sort of hypothesis and an idea of how I would perform the calculations, by the last one my answers were including data, calculations and results.

Furthermore, the use of the Padlet also enabled wide participation of students during the classroom discussion: almost all the students actively participated following others’ interactions or reacting to others’ solicitations (first two levels of participation proposed by Cohen & Lotan, 2014). Only four students did not speak (Cappuccetto, Kebab, Temperatura, and Maria, who were not “previously fragile” students), but, as in previous research (Giberti, 2022b), we observed that the use of the Padlet also allows indirect participation of students in the classroom mathematical discussion. This can be considered as a first affordance of the instrument emerging from the students’ activity: the reasonings collected in the Padlet were discussed also through classmates’ and the teacher’s voices (in the sense of Bartolini Bussi and colleagues, 1995) even if the person who proposed a particular case of reasoning did not actively speak up. Other new ways of participation also emerged, some of which possibly are due to students’ habits in social interactions in the web: for instance, one girl (Kebab) actively participated in the Padlet discussion, but during the final discussion, she was using her smartphone to search on the Padlet for specific posts related to the current classroom discussion and did not therefore speak up. Kebab then used the Padlet as a personal support during the classroom discussion. A second affordance which emerged was related to the possibility for students to look at the Padlet on their personal devices: this helped them to check for specific posts discussed in the classroom or other posts that they remembered from the previous lesson and then easily understand the connections between different solutions overcoming obstacles due to an excessive cognitive load, in terms of WM and EF (Richland et al., 2017); we will discuss this more in-depth in the next paragraph. Other evidence of this form of participation, even if only as “onlookers” in the final discussion, emerged in students’ answers to the final survey. For instance, Maria, who did not take active part in the final discussion, stated with reference to Padlet: “It was useful for me; in fact, in this way it was possible to talk about the subject of the conversation in a clearer way.” Maria’s statement highlights the fact that even if some students did not take the floor during the mathematical discussion, this does not mean that they were not following and learning from it; this is explicitly confirmed in the final survey by Temperatura, a girl who actively participated in the Padlet discussion but did not speak during the classroom discussion: “Having already read the classmates’ solutions helped me, in that I already had a general idea of the different points of view, and so could follow the class discussion more easily”.

Moreover, we also observed that there were students more inclined to participate during classroom discussion and reaching the highest levels of participation proposed by Cohen and Lotan (2014). For instance, Guardia was the main protagonist of the classroom discussion, he led most of the discussion and put forward new issues, but he limited his participation in the Padlet to a few mostly non-constructive comments that were superficial and judgmental (see, for instance, the response to Mongolfiera’s post in the second situation “I consider your method very rough”). In this case, part of the teacher’s role in the classroom discussion is usually to manage students like Guardia to ensure that they do not dominate, but we will observe that, in this case, other students then directly reacted to Guardia’s voice.

Higher levels of participation were reached also by other students who took the floor to explain their reasoning, sometimes merely through verbal explanation and referring to what they wrote in the Padlet and sometimes directly using the Padlet to explain. In the following extract (Table 4), Carota, one of the "previously fragile" students, leads part of the discussion, explaining her hypothesis which put forward the idea of using straight lines to compare the plots (highest levels of participation proposed by Cohen & Lotan, 2014), and she is encouraged by the teacher to use Padlet as a support for her pitch (Fig. 4).

In this episode, the role of the teacher is crucial in orchestrating the different voices (in the sense of Bartolini Bussi and colleagues, 1995): the idea of comparing the areas under the curves develops in the discussion and the teacher also allows other students to participate and reflect on it. Furthermore, in this episode, we observed in the didactical performance a third affordance of the instrument which arises from the teacher's intuition and was not considered in the pedagogical design structure: the teacher proposed to Carota the possibility of drawing on the screen as if it is a picture to add new elements supporting the argumentation process.

Later during the discussion, the idea proposed by Carota becomes the focus of the discussion between two other students (Table 5). During the interview, the teacher described

Table 4 Extract of the mathematical discussion in which Carota explains her idea verbally and then with the support of the Padlet

57	CAROTA: Yes, but let's say that I saw it in a different way, because I said to take a line that represented the averages so that the area below the curve would be the same… as… that is, the areas above and under the line that are included in the plot would be the same…
58	TEACHER: ok. So when you say "I imagine drawing a line that passes like this [pretends to trace the plot on the interactive board with a pen] in such a way that the area…?
59	CAROTA: …that it be comprised between the line and the graph, above and below, be equal
60	TEACHER: Ok. And why do we imagine this line like this?
61	CAROTA: To express the idea of the average value
62	TEACHER: To express the average value…
63	GUARDIA: If I have understood, also due to the graph, we can see (I mean, it's easy to see) a line because I can replace the part above with that below, so I can estimate a value…
64	CECILIA [addressing GUARDIA]: But, like, representing an average? I don't get it…
65	TEACHER: Oh, I don't know, Carota, try and explain it…
66	CAROTA: Yes, the line represents the average value of the graph
67	TEACHER: So, in your opinion, the line can represent the average value of the graph, talking about the average value of what?
68	CAROTA: The temperature
69	TEACHER: …of the temperature, so, in your opinion, we can put a line that represents the average temperature, which would be here. If I've understood you, we have to put it so that…
70	CAROTA: That the areas above and below are equal!
71	TEACHER: That the areas below and above are equal… What do you think? [Speaking to Anastasia]
72	ANASTASIA: I can't fully understand well what she means when she says the area below must be the same as the area above compared to…
73	TEACHER: Do you want to step up and show us? [turning to CAROTA]
74	CAROTA: I'll try…
	[the teacher sets the interactive board to drawing mode on the Padlet screen, allowing details to be added to the plot—Fig. 5]
75	CAROTA: So, I imagined drawing a line. Taking, for example, the red function, tracing a horizontal line like this, so that the area between the line and this part of the plot here and the area here below, be instead equal, to display the average…

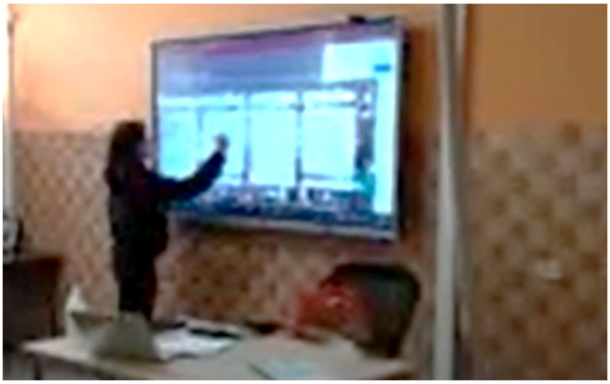

Fig. 4 Carota explaining her solution, with the support of the Padlet

these two students as follows: Guardia is one of the best students in the class, and he is confident about his mathematical skills and always takes a lead role during discussions; Stampella is one of the "previously fragile" students identified by the teacher and, during 5 years of school, has almost never participated actively in classroom mathematical discussion. In the following episode, Stampella supports Carota's strategy and she successfully defends the fact that the line could be horizontal, arguing against Guardia.

The dialog continues and Stampella prevails over Guardia; this episode is a paradigmatic example of how the use of the Padlet enhances the participation of all students in the mathematical discussion: in a previous similar discussion, Stampella never participated but, in this case, she leads part of the discussion and actively defends Carota's hypothesis against one of the best students of the class. This is evidence of how the possibility to take time to reflect and read other solutions and comments in the Padlet is a huge support for some students, such as Stampella, who in this case, was more self-confident in speaking during the discussion and in questioning the ideas of one of the best students in the class.

Table 5 Extract of the dialogue between Guardia and Stampella to discuss Carota's idea

116	GUARDIA: but, if I took even one line […] anyway, I want to give a good estimate, to fill the part below with the part above, so I need to take a line that is oblique and so I would have two oblique lines in opposite directions so they cross like this. In any case, there would also be another one then to estimate. So, if I take the line, then it makes sense to calculate the area below and not just look at the line…
117	STAMPELLA: Why oblique?
	[…]
120	GUARDIA: in city X you must take it a little like this… so as to estimate well
121	STAMPELLA: No, not necessarily, if you consider a horizontal line, take the area below to the left and right to find a balance
122	GUARDIA: yeah, but if I want to fill the area on the left with that of the right, if I want to do it well, I have to still make it oblique. If I want…
123	STAMPELLA: No, not really! [laughs] I mean, I don't think so, anyway

The wider participation of this kind of discussion is explicit in many students' answers to the final survey; in particular, they stated that they felt more comfortable thanks to the anonymity of the posts and comments:

> It may be that in class a student does not feel comfortable or thinks that his/her response is banal and so doesn't voice it to classmates; in this way (also thanks to use of nicknames), the student can express his/her ideas without any problems.

They stated also that the discussion supported by the Padlet was more engaged and freer:

> [The work was] much more engaging and interesting, it stimulated discussion. Usually, in class we wait for the cleverest student in that subject to answer and then we follow. Instead, I think in this case it was more active with lively discussion… more constructive!

5.2 Padlet as a support to foster relational thinking of all students

Furthermore, it should be noted that the comparison between different strategies, a key element to improving relational thinking (Richland et al., 2017), emerges spontaneously in the Padlet. We report here (Fig. 5) a paradigmatic example in which the strategy proposed by Stampella regarding Plot B is compared to other strategies in both Anastasia and Maria's comments; other comments focusing on the comparison of strategies are also present in relation to Plot A and Plot C columns.

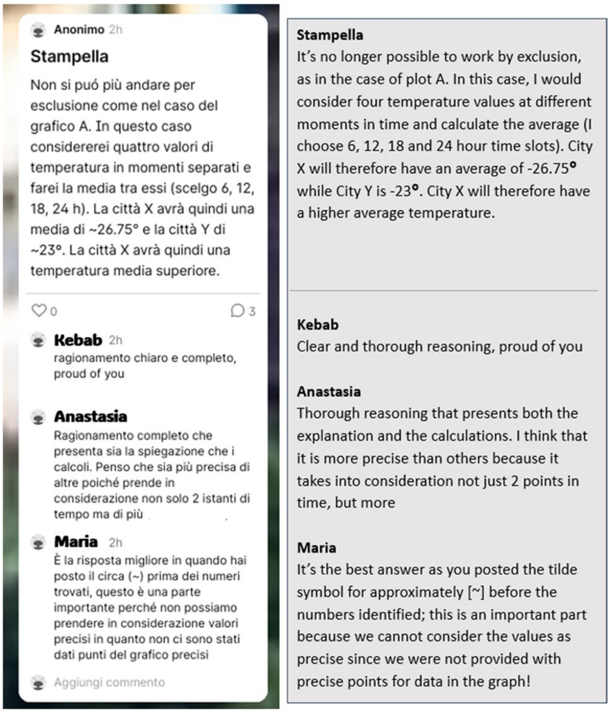

Fig. 5 Example of a post with comments reporting the comparison of multiple strategies (Steps 3 and 4)

Indeed, taking into account Padlet data, the evolution in the way students make inputs (such as written posts and comments) is accompanied by an evolution of written references between posts and comments (Fig. 3). In particular, if we consider the number of references between posts and comments in the Padlet:

- In the first situation tackled (Plot A), only two references are made by students when commenting on their peers' posts: one student refers explicitly to her own personal thoughts (black dot) and one student explicitly compares her classmate's post to all the others
- In the second one (Plot B), six references are identified, almost all reporting an explicit comparison with other answers (one black dot and four black triangles)
- In the third situation (Plot C), the number of references increases to 11, and almost all the references are not explicit (1 black dot, 1 black triangle, and 9 black stars)

This increased number of references from the first to the last situation faced by the students suggests a greater use of relational thinking, and the transition from explicit to non-explicit referencing highlights a first act of generalization regarding what they read in their classmates' posts and comments. This process occurred spontaneously, without teacher intervention.

The explicit request to compare the multiple solutions observed in the Padlet is made by the teacher at the beginning of the classroom discussion, as decided in the pedagogical design structure: the solutions are different not only in terms of different approaches proposed by the students but also concerning the three different situations proposed. As highlighted by Richland and colleagues (2017), this request could require an important effort in terms of WM and EF, especially if the solutions are explained by other students verbally, thereby requiring them to pay particular attention to grasp the emerging information, think it over, and retrieve it for future consideration (thus through the *updating* EF).

In our experiment, the earlier collection of students' strategies and the reflection already made by students reading and commenting on the Padlet helped them in managing this process, and this is enhanced by the frequent references they make to the Padlet from the beginning of the discussion. This is the case of Guardia (Table 6), who, at the beginning of the classroom discussion, compares the strategies in the Padlet and, of his own volition, does not refer directly to the people who wrote these strategies.

This episode could be interpreted in terms of the "distancing effect" (Bartolini Bussi et al., 1995) and highlights a fourth affordance of the Padlet: this process becomes spontaneous thanks to the Padlet, while in a traditional mathematical discussion, the intervention of the teacher to promote such effect is often needed. We observed that a fifth affordance specific of the use of Padlet is related to time management: as Bartolini Bussi and colleagues (1995) highlighted, in a balance discussion, the time given by the teacher between the lesson in which students face the problem and the classroom discussion allows the teacher to collect and read the solutions proposed to plan the discussion and allows the students to distance themselves from their own product. This happened also in our study, but we observed that this time allowed students to take the point of view of other students. The reference to

Table 6 Guardia's comparison of different strategies at the beginning of the classroom discussion

11	GUARDIA: […] as we saw on the Padlet, maybe many people who considered the values of a plot but did not perfectly describe the process… for example, in the third plot, it was very variable… for the first and second, it could be ok to consider the values but for the third…[…] in the first and second case we saw that it can also work but in the third, which is already more complicated, you need to proceed more precisely

Table 7 Extract of the classroom discussion as an example of implicit reference to solutions explained verbally and in the Padlet

172	MONGOLFIERA: I agree with what those two said: about the fact that all the methods are 'approximate', in a manner of speaking. Like, in the third graph, I remember that the answers gave different results when using different methods; some said that X had a higher temperature, while others said Y did, and others said they were the same, so it all depends on the values taken from the estimation

the Padlet could be explicit, as in the previous extract, or implicit, as in the following one (Table 7) in which the student, later in the discussion, compares different hypotheses and comments on the correctness of the results, referring both to the previous intervention of classmates in the discussion and to the posts and comments uploaded in the Padlet.

The comparison process (thus the *shifting* between solutions) is supported by the possibility of revisiting the Padlet during the classroom discussion, the frequent reference to what is written in the Padlet is suggested by the teacher in several moments of the classroom discussion, but some students consult spontaneously the Padlet on their personal devices (see for instance Kebab's episode described in the previous section). Referring to the posts help students in this process, supporting their WM and EF, as this student stated in the final survey: "It was useful to see again the comments if I couldn't remember them."

At the end of the discussion, the comparison of solutions focuses on the issue of whether it is correct to provide an approximate result for the proposed problem; this issue has already emerged in the comments to Plots B and C in the Padlet and is explored further in the discussion, thanks to both teacher and students' interventions such as this contribution from Tramonto (Table 8).

The support given by the use of Padlet in terms of WM and EF is not just related to the possibility to easily compare different solutions, which requires the activation of *shifting* and *updating* processes; a higher effort in terms of EF (in particular during the *inhibition* process) is required also when, during the discussion, a student explains verbally his/her own solution and corrects the given solution while making the pitch (Richland et al., 2017), as in the following episode (Table 9).

In this episode, the requirement in terms of WM and EF for the students who are listening to Girasole is lower than if the same solution and relative correction were discussed only in classroom discussion. The first part of the discussion made in the Padlet makes this correction not "on-air." Thus, the EF overload of other students is mediated by the fact that

Table 8 Extract of the classroom discussion: Tramonto and the teacher compare different solutions, considering the way they approximate the result

162	TRAMONTO: In my opinion, there is not one best method, just easier ones and less easy ones. Because, with not having precise values, in the end, for every method, we have to estimate some values and even just because of the fact that the squares are big, we can't do it… that is, we are forced to estimate a value. So, whether we take the subtended area or whether we select obviously quite a few points to come up with an average, in any case, it is always just an estimate. So, we can find it just as easy to calculate the area as to calculate the points and find the straight line
163	TEACHER: Ok, So, in your opinion, we don't have… there's not a better or worse method, we are always talking in terms of estimation and, in any case, the result we get…
164	TRAMONTO: …it's always something approximate, so in the first two, any method seems the same, while the problems emerge with the third where the curves are similar because they overlap more than in the other plots and so, in any case, by estimating we are lining up a mistake which will come out in the result, which appears very similar, and we can't say with certainty what the greatest average temperature is

Table 9 Extract of the classroom discussion: Girasole corrects her solution while making the pitch

25	GIRASOLE: Maybe taking the values of our minimum numbers in general, beyond the fact, that is, even if you could read the plot precisely, maybe it is not enough because… to give a practical example, let's say that one measures the trend, that is in five hours of temperature, if the temperature is around 20 degrees for four hours but then in the fifth hour it goes down to 15 degrees, the average is not 17.5 because the average will be higher, I mean, the average takes into account all the values, not just the maximum and minimum values, so maybe the area, calculating the area, is more logical even if it's not very practical
26	TEACHER: ok, So, in your opinion, just the maximum and minimum values don't work because…
27	GIRASOLE: There are too few values. That is, even taking the beginning, the end, let's say… even 4 or 5 values is still few
28	TEACHER: and here [indicating the Padlet] someone wrote something like that, or not?
29	GIRASOLE: I wrote it in a comment but then I saw the others'… […] maximums and minimums… [laughs]

Table 10 Extract of the classroom discussion: Carota recalls her idea in the end of the discussion

213	CAROTA: But why do we say that, given that the problem asks you to compare two trends, the line was a simple way to see… that is, to imagine, which line was above and which was below

they have already read Girasole's solution and have the possibility to retrieve it on their own devices while she is speaking, and the teacher supports this process explicitly in referring to the Padlet (thereby supporting the *shifting* process).

Finally, it emerges that the mediator role of the Padlet enables a more dynamic discussion. Traditionally, during a classroom discussion, the introduction of a new issue often hides the issue discussed previously, while the support of the Padlet helps the teacher and students in managing a discussion with more interludes, and the same issue can be faced and explored several times during the discussion. This is the case of the episode in the previous paragraph when Carota proposes the use of a line to approximate the plots, which is also echoed later in the debate (Table 10).

The necessity for the students to recall what Carota proposed could require a strong effort in terms of WM and EF, but the Padlet works as a support where all the ideas discussed are collected and can be easily located.

6 Conclusions and further perspectives

In this paper, we described an experiment based on the use of a digital platform (Padlet) for supporting mathematical discussion in the classroom. We observed that the use of Padlet enables the teacher in promoting a discussion which is not "linear," as a usual verbal discussion in the classroom necessary is, while it allows the interweaving of "simultaneous" voices; this has brought to examples of participatory culture emerging within the classroom, in the sense of Jenkins (2009), in particular in the creation and sharing of problem-solving strategies, and in the emergence of informal mentorship among students in an inclusive perspective.

Indeed, the pedagogical structure that we designed, including Padlet to collect students' ideas and comments, allowed the development of a preliminary peer-discussion with no intervention by the teacher (steps 1 to 6). The role of the teacher is to introduce the task to the students and then manage the time between the steps of the pedagogical design structure. Time management was an important issue also in the balance discussion proposed by Bartolini Bussi and colleagues (1995), but, in that case, the teacher had only the possibility to decide the time between the lesson in which students face the problem and the discussion in the classroom. In our study, the teacher also has the possibility to manage the time, while students are solving the problem observing the development of the Padlet, in order to ensure that all students have the necessary time to reflect and work on each step (fifth affordance). As described above in Table 1, students tackled only one situation (plot/column) at a time. This implies the possibility of developing the first part of the mathematical discussion in Padlet in a precise way: contrary to a "linear" verbal orchestrated discussion in the classroom, each student can intervene in his/her own time, retrieving sentences, comments, and stimuli posted in previous steps of the discussion (Giberti et al., 2022a, b).

The interactions between the students (their initial posts and subsequent related comments) show how convergence and contrasts between the proposed solutions evolve spontaneously: the number of comments increased, comments were spread among almost all the posts so that almost all students had feedback from their peers, and comments rejecting a post became more frequently constructive, engaging students in a peer-mentorship process. Moreover, the permanence on the Padlet of the posts allowed good control of previous posted answers in the different stages of the task, making the weaving between the different stages more effective and supporting students the *shifting* and *updating* EF. Indeed, references to other comments or posts increased, and thus we observed that the comparison of different strategies and the process of generalization began spontaneously in the Padlet.

The role of the teacher then became crucial during the classroom mathematical discussion (Step 8). The materials collected in Padlet became a tool in the teacher's hands for stimulating and orchestrating a physical discussion in the classroom. Indeed, the use of Padlet allows the teacher to structure the discussion in a more complex and effective way as compared to the balanced discussion described by Bartolini Bussi and colleagues (1995) in which the instrument used by the teacher was the whiteboard. In our study, the process of instrumental genesis is managed by the teacher orchestrating the use of the Padlet in order to manage the mathematical discussion in the classroom through this tool: for this, she encourages explicit reference to what is written in the Padlet and enhances the affordances of the tool during the discussion. Moreover, as in previous balance discussions, the teacher influences "the development of the discussion with his/her own interventions" (Bartolini Bussi et al., 1995, p.11), but, in this case, the teacher's intervention activity is not exclusively oral but also related to the use of technology. A paradigmatic example is the episode involving Carota: the teacher proposed a new way of instrumentalization (thus enhances a-third-new affordance of Padlet) to support Carota in explaining her idea.

The use of Padlet promoted a balance discussion which included several perspectives and students' voices and fostered students' participation and the inclusion of more "previously fragile" students. This participation was both in the work on Padlet (steps 1 to 6) where all students actively participated by following or reacting to other posts and comments and in the classroom discussion phase where all students, except four, took the floor. The four students who did not take the floor during the discussion gave us the possibility to observe new ways of participating which emerged as affordances of Padlet, identified by the teacher or the students themselves (first and second affordances): for instance, some students followed the discussion while looking at their smartphones and monitoring the Padlet, possibly according to their habits while using online social media and mobile technology, while in other cases, a student was indirectly involved by the teacher through the reading of his/her post.

The wider participation also of more "previously fragile" students both in the Padlet activity and in the classroom discussion emerged thanks to the fact that the Padlet supported the students' WM and supported the use of principal EF, thus reducing the cognitive load required in several steps of the discussion. This was explicitly referenced firstly by the teacher but also by many of the students themselves, who refer to the Padlet as a support to recall others' comments and to shift between different solutions helping them during the discussion (thus supporting the *shifting* EF). Padlet was also observed as a support for WM and EF in the analysis of the final discussion, as in the specific episode where Girasole corrects her own strategy while speaking; indeed, in this episode, the EF *inhibition* and *updating* are necessary to decide what is important to retain in the WM and what can be forgotten. The support given by the Padlet in terms of WM and EF is demonstrated also by the fact that it was frequently mentioned and used as a support for students' reasoning during the classroom discussion, promoting the comparison between different solutions and thus relational thinking (*shifting* EF). Reference to the Padlet could be both explicit and implicit, and a spontaneous "distancing effect" emerges in the students' contributions (fourth affordance).

While all the students reached the first two levels of participation as described by Cohen and Lotan (2014) following and reacting to others' solicitations in the Padlet activity and/or in the classroom discussion, the highest levels of participation were reached by students who were not necessarily the best performing or the most self-confident in mathematics. Two episodes of the classroom discussion involving two "previously fragile" students illustrate this: the first is the one in which Carota leads the discussion and proposes the visualization of the average as a straight line in the plot and the latter is when Stampella supports this idea against Guardia.

In this research, we observed that Padlet supports teachers' and students' instrumentalization towards participation and inclusion in the mathematics classroom, making the discussion more inclusive than traditional balance discussions due to (i) time management, (ii) possibility to participate in the discussion at different levels and through new ways, (iii) ease for students and teacher to promptly retrieve and compare students' productions, (iv) spontaneous activation of relational thinking, and (v) higher engagement of students. Thus, the results of our previous research are confirmed and explored further in this different environment and considering a problem which requires more structured argumentation and the need to refer to explicit mathematical elements of the problem.

It should be noted that Padlet is a general-purpose technology which is not directly designed for mathematics education purposes and, because of this, a double instrumental genesis framework was considered: the pedagogical design structure of Table 1 is the result. One limitation of this study is that it was the first time that the tool was used by both the teacher and the students (Ball & Barzel, 2018). Hence, there might have been a positive impact due to newness of the activity on students' engagement and interest, as well as a negative impact due to the inexperience of the users. Starting from students' answers to the final survey and teacher consideration in the interview, we consider that the inexperience did not influence the experiment indeed, the teacher did not report any specific issue in using this tool and it was the same also for students. Moreover, students appreciate the way to communicate used in Padlet because it was recognized as close to social media they use. Padlet may also offer researchers and teachers a rich and ready-to-analyze corpus of contributions by students, where individual and collective actions interlace. In a further perspective, we think that it would be very interesting to perform a detailed analysis of how students' language changes and evolves during such digitally-mediated discussion, from an informal register to a more mathematically-structured one (Barwell, 2016).

In summary, we think that this research is a possible answer to some of the questions posed by Engelbrecht and colleagues (2020) about the new social interactions, the design of new teaching settings, and the new ways of thinking in the use of digital tools: in particular, we have shown an example of "how mathematics educators can develop research-based principles of design regarding new teaching contexts that digital tools provide" (Engelbrecht et al., 2020 p. 838) and of "how technologies in online contexts support interaction among participants as a medium to support mathematical knowledge construction and teaching competences" (Engelbrecht et al., 2020 p. 827).

Appendix

Table 11 Online survey proposed to students at the end of the experiment

Italian version of the survey	English translation made by the authors
Che cosa ne pensi del fatto di inserire nel Padlet le tue ipotesi e idee relativamente a un problema di matematica? Come ti sei sentito/a nel postare la tua idea?	How did you feel about posting your hypotheses and ideas regarding a math problem in the Padlet? How did you feel about posting your idea?
La possibilità di commentare le idee degli altri e di leggere i commenti degli altri alla tua idea ti ha aiutato/a oppure ti ha ostacolato/a nel riflettere sui grafici successivi? In che modo?	Has the ability to comment on others' ideas and read others' comments on your idea helped or hindered you in thinking about subsequent plots? In what ways?
Nella discussione in classe del problema, ti è stato utile oppure non ha avuto rilevanza avere già letto/visto le soluzioni dei tuoi compagni e delle tue compagne? Perchè?	During the class discussion of the problem, was it helpful or irrelevant for you to have already read/seen the solutions of your classmates and peers? Why?
Nella discussione in classe del problema, è stato utile/inutile/diintralcio avere il Padlet a disposizione per visualizzare le idee discusse?	During the class discussion of the problem, was it helpful/unhelpful/disadvantageous to have the Padlet available to display the ideas discussed?
Immagina di aver affrontato lo stesso problema in classe individualmente sul quaderno: dopo che l'hai risolto, l'insegnante chiede a ciascuno di spiegare le proprie ipotesi e le discutete insieme. Quali differenze/somiglianze pensi ci sarebbero tra la discussione così condotta e quella che si è avuta in questa esperienza? Quali sono le possibili cause delle differenze secondo te?	Imagine that you have tackled the same problem in class individually in your notebook: after you have solved it, the teacher asks each person to explain their hypotheses and you discuss them together. What differences/similarities do you think there would be between the discussion thus conducted and the one that took place in this experience? What do you think are the possible causes of the differences?
Come ti sei sentito/sentita durante la discussione in classe? Ci sono state differenze rispetto ad altre discussioni di matematica avvenute in passato?	How did you feel during the class discussion? Were there any differences from other math discussions that have occurred in the past?
C'è un episodio, un momento della discussione o un pensiero che ti è rimasto particolarmente impresso e che vorresti condividere con noi?	Is there an episode, a moment in the discussion or a thought that particularly stuck with you that you would like to share with us?
Indica tre aspetti di questa esperienza che ti sono piaciuti di più. Se lo ritieni, motiva brevemente la tua risposta	Name three aspects of this experience that you liked best. If you think so, briefly justify your answer
Indica tre aspetti di questa esperienza che ti sono piaciuti di meno. Se lo ritieni, motiva brevemente la tua risposta	Name three aspects of this experience that you liked least. If you think so, briefly justify your answer

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Data availability The data that support the findings of this study are available from the University of Bergamo, but restrictions apply to the availability of these data, which were used under license for the current study and so are not publicly available. The data are, however, available from the authors upon reasonable request and with the permission of the University of Bergamo.

Declarations

Conflict of interest The authors declare no competing interests.

Ethics approval Approval was obtained from the local ethics committee of the University of Bergamo (Italy).

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