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Mathematical discussion in problem solving activities supported by technology: An achievable goal in primary school

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The use of technologies to foster the mathematical discussion has been largely investigated in the last decade. In particular, Padlet - a digital space where users can collaborate by uploading, organizing, and sharing content - demonstrated its suitability in this context. In this paper, we exploit Padlet in a primary school, where students working in small groups approach the same mathematical problem. Our results highlight that Padlet is an effective means even for younger students and allows for supporting the mathematical discussion and for promoting the comparison of different strategies in problem solving activities, encouraging the participation of students with special needs.

Keywords: Primary school mathematics, mathematical discussion, problem solving, technology.

Introduction

Nowadays, especially after the pandemic, students are immersed in different learning environments that often involve the use of technology. In such a context, teachers should set up classroom environments characterized by collaborative learning models in which students access information from a variety of media, producing and sharing their work (Collins & Halverson, 2018). To make the most of the opportunities that technologies offer, it is good to bring them into teaching practices as a means of promoting and encouraging mathematical discussion among those involved in the teaching-learning process: “if we do not manage to incorporate digital technology in students’ mathematical practices in a way that they experience as meaningful, it is a useless enterprise” (Drijvers, 2019, p.24). Recent studies in mathematics education, focused on middle school students, have shown that problem-solving activities can be supported using online platforms, such as Padlet¹. Padlet is a cloud-based software that allows users to collaborate by uploading, organizing, and sharing content using virtual boards called “padlets”; indeed, in the context of mathematics education, Padlet represents a digital space in which students can post their hypotheses and solutions, share and comment on their ideas with peers, and discuss (Giberti, Arzarello, Bolondi & Demo, 2022; Giberti, Arzarello & Bolondi, 2022). In particular, it emerged that the use of Padlet changes the dynamics of the mathematical discussion in the classroom during problem solving activities: the discussion is based on different levels of interaction constantly intertwining and the posts in the padlets are used by the teacher to foster interaction between students. In that paper, the authors highlighted the potentialities of sharing students’ solutions with Padlet, asking them to comment on others’ ideas, to provide each student with the time they need to reflect and then overcome specific difficulties emerging in this kind of mathematical discussion. Finally, the use of Padlet fostered the inclusion and participation of all students in the problem-solving activity and in the related mathematical discussion, the analysis of divergent hypotheses, and the promotion of a positive culture of errors.

¹ <https://padlet.com/>

Several studies have indicated that the utilization of interactive digital technologies can aid and improve the understanding of mathematics in the earlier grades (Larsen et al., 2018), thus we aim to investigate if and how a similar activity could be proposed also to younger students.

Theoretical framework

Problem solving is a fundamental issue in mathematics education, several studies suggest how to promote mathematical problem solving and Kilpatrick (1985) identified five promotional approaches to problem solving activities: *Osmosis, Memorization, Imitation, Cooperation and Reflection*. Within our research, we consider particularly important the last two: *Cooperation* which suggests promoting problem solving activities in small groups of students, and *Reflection*, following which problem-solving abilities develop thanks to action-oriented activities and through reflection of different approaches (Liljedahl et al., 2016), thus communication and discussion between students and between teachers and students are fundamental. Mathematical discussion can be defined as a “purposeful talk on a mathematical subject in which there are genuine pupil contributions and interactions” (Pirie & Schwarzenberger, 1988, p.461). Following Bartolini Bussi and colleagues it can also be defined as a “polyphony of voices articulated on a mathematical object, which constitutes a reason of the teaching-learning activity” (1995, p.7), and it is essential at every school level (Cockcroft, 1982). In a mathematical discussion, there are some fundamental issues (Bartolini et al., 1995): (i) the definition of a specific focus; (ii) the interaction and engagement of the students considering their different points of view; (iii) the role of the teacher to involve all students in the discussion through questions that arouse interaction, without influencing them. Starting from a classroom problem-solving activity, a ‘comparative discussion’ consists in comparing multiple solutions/strategies proposed by the students when facing the problem. The discussion led to constructing a representation of the problem, which can converge in the formalization of new mathematical concepts. Richland and colleagues (2017) highlighted how the mathematical discussion based on the comparison of several solutions to the same mathematical problem is an incisive pedagogical practice. Indeed, comparing different representations and finding relationships between them obviously requires a mental exercise that develops relational thinking in mathematics (Skemp, 1976) and then allows students to reach a deeper mathematical understanding of the concept. Gadgil and colleagues (2012) have also analyzed how ‘conceptual change’ is facilitated and deepened by holistic comparison and contrast through a system of relationships and differences and not just on an accumulation of beliefs and false myths self-explained by students. Moreover, Richland and colleagues (2017) focused on two important concepts: the Working Memory (WM), which is “the cognitive resource that enables humans to hold information in mind and to manipulate that information without losing it” (p. 43), and the Executive Functions (EF), which control the information that must be stored in the WM. Teachers must consider the differences between the EF students and the different needs of learners in terms of discussion time and support. Moreover, the great deal of effort required by EF and WM might lead students into distraction errors, as they struggle to relate different representations and also to identify fewer relationships between compared solutions (Richland et al., 2017). The support of a digital platform, such as Padlet, allows a thorough investigation of problem-solving activities by (i) collecting student’s answers before the mathematical discussion, (ii) sharing these answers with the other students and asking them to read and comment on them, (iii) displaying all students’ answers in the whiteboard during the discussion. Considering in particular the last point, Padlet is indeed a useful

tool to implement the strategies suggested by Richland and colleagues to overcome the main obstacles emerging in this kind of mathematical discussion (Giberti, Arzarello & Bolondi, 2022).

Research Questions

In this research we investigate the interactions emerging in a group work activity and the intertwining in the following mathematical discussion in a grade 5 class with the support that educational technologies can provide, and in particular the use of Padlet, for comparing different ideas and solutions to the same problem (Richland et al., 2017). The integration of digital tools in mathematics education is a “nontrivial issue” because it affects all aspects of education (Drijvers et al., 2013) and, as Jacinto and Carreira state “the role and impact of digital tools in mathematical problem-solving processes remains an under-explored topic” (Jacinto & Carreira, 2022, p.2560). It is even less so if we consider the use of technological tools within the primary school setting. Then our research questions are: RQ1) How does the Padlet tool foster student interaction and participation of primary school students during the problem-solving activity and in the related discussion? RQ2) How does this instrument promote inclusion in primary school?

Data and methods

The experiment involved 20 grade 5 students of the same class in Italy. The class is composed of 11 males and 9 females, including 4 students with special needs. In particular, 1 student has relevant school difficulties, 1 foreign student, recently arrived in Italy, has strong language-related difficulties, and 2 students are certified for Specific Learning Disorders (specifically, one is certified for dyslexia, dysorthography, and dysgraphia, and the other one for dyscalculia).

The problem proposed belongs to an international problem-solving competition between classes called “Rally Matematico Transalpino” and, in this context, was already administered to 1653 classes. In this research, we consider the results of the international competition as a benchmark, the strategies that emerged from the data collected by the group work were identified and ordered on the basis of the classification proposed in the international report, also with regard to how technology influenced the strategies used². The problem³ requires the comparison of the area inside and outside a polygon drawn on a 6 x 6 grid square. The resolution process could be based on two different approaches: (i) *Triangles approach*: students might divide the polygon and the outside area in triangles and make a comparison between the triangles’ areas. In this case, the students can calculate/count the area of the triangles or base their solution on the direct comparison of equivalent triangles (e.g., using arrows to link equivalent triangles, cutting and overlapping); (ii) *Grid square count approach*: students might compare the inside and outside area counting the number of squares; in this case, they have to also consider the squares’ fractions.

² http://www.projet-ermitage.org/ARMT/navi_fic2.php?code=gp81-it&flag=1&langue=it&w=

³ “Veronica made a drawing of a heart shape in her notebook. She colored the heart in red and the remaining part of the square in blue. Which is the larger part? The red or the blue one? Explain how you came up with your answer by adding one or more posts on Padlet and try to convince the other groups that your solution is the best one. To add a post in Padlet, click on the “+” button in the column of your group. You can add to the post text, pictures, audio, videos, and any other material that can help you explain to others your way of thinking.” (Translated from Italian by the authors).

In our experimental plan, we proposed this problem as a post in Padlet (first column in Figure 1). Students were divided into 4 heterogeneous groups in terms of mathematical skills and the composition of the groups also considered the special needs of students and students' personality so as to promote the participation of the shy. The teacher assigned a name to each group but the names were shared with the other groups only at the end of the experiment consisting of 3 phases:

- Phase 1 (1 hour): The teacher reads the problem and the assignment; the students work in groups to solve the problem and post their hypotheses in Padlet. Each group accesses the Padlet through a tablet; the Padlet configuration is organized by columns, the first column reports the problem, and the other columns are named after the groups and will collect the group's posts. The teacher also gives each group 3 printed copies of the problem and a box with many materials (colors, scissors, glue, tissue paper, etc.). Padlet is set to hide other groups' posts: each group only sees its posts. At the end of this phase, the teacher makes all posts visible also for the other groups.
- Phase 2 (1 hour): Each group reads and comments on the other groups' solutions.
- Phase 3 (1 hour): The teacher shows all padlets on the multimedia whiteboard and discusses with the students the different solutions proposed and the comments collected.

The experiment was conducted by two researchers (authors of this paper) and the teacher, who also orchestrated the final mathematical discussion. Data collected consisted of Padlet posts and comments, using a Padlet backpack for schools account; Audio-recording and video recording of the groups' work (transcribed); Video recording of the final discussion (transcribed).

Results and discussion

In Phase 1, each group posted at least one solution and explained how they worked to tackle the problem. A variety of solutions, strategies, and arguments were collected by the teacher from the Padlet (Figure 1).

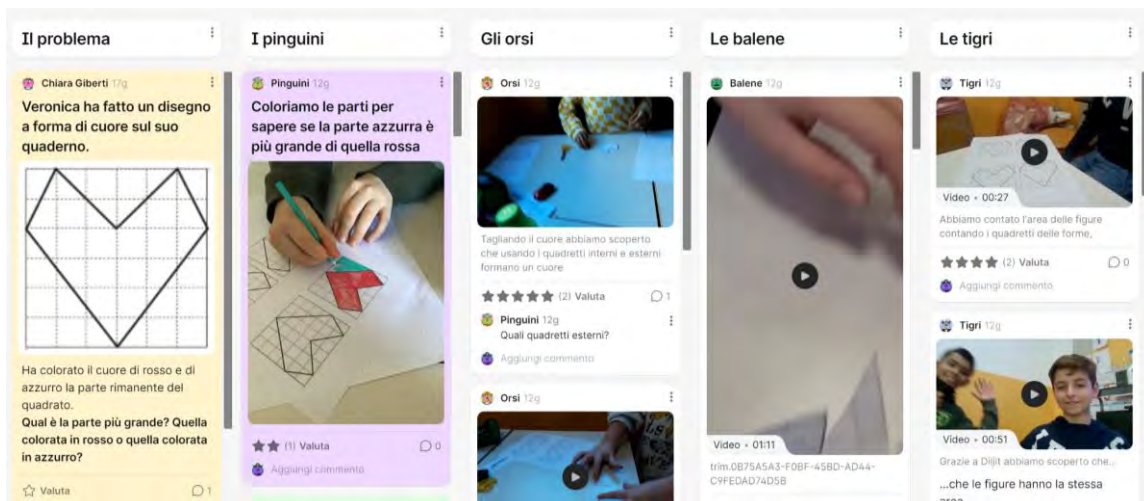


Figure 1: Completed Padlet. In the first column, the teacher posted the text of the problem together with the assignment; the other four columns collected all posts from each group

Almost all groups posted their solutions using videos and/or photos, often coupled with textual descriptions aimed at illustrating what they were doing; for instance, in the second column in Figure

1, the students added the following caption to their video: “coloring the inner and outer parts of the heart to understand which one is bigger”. All students involved in the activity participated personally and gave their contributions to the group work. Even if Padlet was new for all students, being a usable digital tool, it allowed each group to immediately understand its functioning without the teacher’s intervention; indeed, they independently and easily uploaded to Padlet all files including the identified solutions together with all necessary explanations and arguments.

Then the teacher made all posts visible and asked all groups to observe and comment on all other’s solutions (Phase 2). Even if all group members examined the other solutions, some difficulties emerged in commenting on them. Most of the groups misunderstood the teacher’s assignment and evaluated others’ posts rather than reviewing them; in most of the cases, the students only voted the other solutions with the “reaction modality” or stated if they considered the proposed strategy right or wrong. Furthermore, due to the use of videos, the anonymity of the groups had fallen at the end of Phase 1; therefore, the comments could have been influenced by the knowledge of the groups’ members. Padlet also fostered the interaction of students with special needs as they participated proactively in the group work, explaining their opinions in their respective groups. For instance, in one of the groups (Orsi), the student with relevant school difficulties took the responsibility upon himself of recording the videos, taking photos, and uploading them to the platform; this led him to actively participate also in the mathematical discussion within the group. Similar experiences were documented in the other groups that included students with special needs: one of them had the role of writing the comments of the group, and another explained one of the group’s solutions in a video.

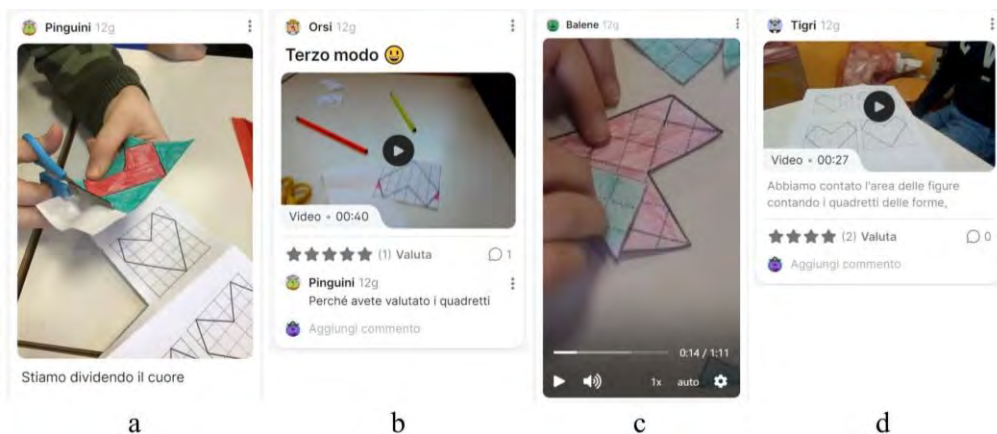


Figure 2: Strategies posted on Padlet explained by means of photos and videos

Going deeper into the padlets analysis and referring to the strategies classification proposed in the previous section, the first group (Pinguini) used the triangles approach (Figure 2a), cutting and overlapping the red and blue parts; they also observed that the heart has a symmetry axis but they did not explicitly refer to the concept of symmetry when they explained their reasoning. The second group (Orsi) used the same strategy and, in one of their videos they use the cut parts to recompose the heart and explain “So, we cut out the outer part of the heart that, in fact, was inside the square and, by assembling it, we realized that it formed another heart identical to the original”. Furthermore, the same group (Orsi) also proposed a grid square count approach in a video (Figure 2b), but they also stated that they had a problem with this method because some parts of the drawing did not include

entire grid squares. It is interesting to notice that this group posted their reasoning even if they did not overcome the limitation encountered and that the first group (Pinguini) commented on their post trying to help them in refining their strategy by specifying that the problem is the evaluation of the (not-entire) squares. The third group (Balene) adopted a two-step strategy: the triangle approach, realized by cutting and overlapping the blue parts on the heart, was realized after dividing the heart into different parts by drawing darker lines on the grid (Figure 2c). This group posted more than one video of the same strategy, each one filming a different member of the group explaining their reasoning; this helped them in enhancing their argumentation. The last group (Tigri) integrated the two approaches (Figure 2d): they adopted the grid square count approach considering that not all the squares included in the heart are entire, stating that the red area is 18 squares; secondly, they observed through the triangles approach that the two areas are equal and then they concluded that also the blue area must be 18 squares. A summary of all strategies and comments posted by the four groups is reported in Figure 3. Despite the already explained difficulties encountered in commenting on other strategies, we observed that the mathematical discussion in terms of comparing different solutions started during Phase 2: some groups explicitly evaluated the others' strategies by posting comments like, for instance, "Nice idea, we didn't have it!!" or "The reasoning is great, it's almost like ours, but yours is colored".

























Pinguini 	Orsi 	Balene 	Tigri 
Coloring parts	Cutting parts commented by  	Cutting parts commented by  	Counting squares commented by  
Cutting parts commented by   	Counting squares commented by 	Coloring parts commented by  	Cutting parts commented by 
Evaluating symmetry commented by  	Coloring parts commented by 	Overlapping parts commented by  	
Overlapping parts commented by  			

Figure 3: Summary of the strategies proposed by each group. The icons denote the comments posted by the groups on others' strategies

The classroom discussion (Phase 3) lasted approximately one hour and was orchestrated by the mathematics teacher. The Padlet, in which all the strategies and comments were collected, was displayed on the whiteboard; the teacher and the students decided to consider the posts of each group following the chronological order because students explained that this ordering corresponded to the evolution of their reasoning. Then, the teacher started considering the cutting and overlapping strategy proposed by the first group (Pinguini) and, almost immediately, encouraged students to reflect on similar strategies also proposed by other groups "Are there other groups that have used a similar strategy?". The Padlet helped the teacher and the students in comparing groups strategies as highlighted also in previous research (Giberti, Arzarello & Bolondi, 2022); other two groups then participated in the discussion stating:

- Balene: We colored and then cut the blue part into pieces, which is the outer part, and recreated the heart. We then compared it with the red part and they had exactly the same size.
- Tigri: That's how we did it, too! We cut the outer part into pieces equal to the heart. Then to verify, we paired the same blue and red cut parts.

Then the first group intervened again to further specify their approach:

Pinguini: We did something somehow similar. First, we sketched but then cut out the pieces and laid the blue pieces on top of the red ones to see whether they really were congruent. We cut out the pieces because it is more practical.

The process of comparing and contrasting different solutions could require a huge effort in terms of WM and EF (Richland et al., 2017) but Padlet supported this process during the whole mathematical discussion. Indeed, the teacher and the students often refer to specific posts to identify analogies and compare different strategies and the fact that the students already had analyzed others' ideas helped in following and participating in the discussion, especially for the students with special needs. Furthermore, a challenging but important goal of the mathematical discussion is also to highlight the relationships between more and less efficient strategies (Carpenter et al. 2014). This goal is particularly challenging for young students but in a few extracts of the discussion, this kind of competency emerges:

Orsi: We evaluated the squares that were not complete, the pink ones were not complete. Using colors we highlighted the little pieces that completed each other and made an entire square, so we counted them.

Pinguini: We didn't do it that way; we were counting the squares of both the heart and the other part to see if one was bigger than the other. However, it wasn't the right solution because there were squares that weren't all entire.

Conclusions and further issues

With our experiment we verified that for Primary school children, the use of a tool such as Padlet allowed making proposed solutions visible in different registers of representation and easily accessible and recoverable throughout the discussion. As stated by Gadgil and colleagues (2012), this fact actually promoted pupils' reasoning and participation during the different phases of the classroom activity, and in particular during the discussion. Padlet allowed sharing of not only verbal statements but also videos, images, sketches, and documentation of the processes of the groups during the problem solving activity. Gadgil also argues for the importance of constant and solid work to develop teaching interventions that can lead to solid conceptual change. In this research, this aspect represents a limitation due precisely to the lack of continuity in proposing problem-solving through co-constructed mathematical discussion within the daily teaching practice of the class under consideration. Students with special needs participated in the group's work proactively, managing technological tools and discussing their ideas with their peers. As a result, they easily followed the final mathematical discussion and they intervened to explain their ideas. As a further issue, we plan to investigate more articulated situations (as it has been done with older students), where the interaction between groups may be fostered by using Padlet also during the group problem solving phase. Moreover, it will be interesting to consider the different ways a teacher can intervene in the Padlet discussion, searching for specific modalities of orchestration.

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