

Geometric modeling of overlapped WAAM beads by Hermite curves

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Abstract

Purpose – In Wire Arc Additive Manufacturing, the size and shape of the deposited material are affected by heat input, material deposition rate, orientation with respect to gravity, substrate conformation and the sequence of beads. The paper aims to investigate the geometric behavior of adjacent overlapping beads and proposes a model to predict the shape assumed in case of multiple beads deposition according to the succession of accumulation.

Design/methodology/approach – Geometric curves such as parabolas, circular arcs, cosines and ellipses have been proposed in the literature to represent the resulting cross-section bead profiles, even though multiple beads deposition scenarios have been scarcely explored and, in most cases, restricted to simplified situations, such as planar substrates. Therefore, a generic geometric model for the bead profile based on a closed curve composed of two Hermite spans is proposed.

Findings – The algorithm reliability was tested on circular specimens realized with a Fronius TPS 320i welding generator and on a set of documented cases from the literature. The results demonstrate that the Hermite curve-based model and the proposed algorithm can accurately reproduce various bead profiles in multi-bead and multi-layer deposition sequences.

Originality/value – Thanks to the positioning of its control points, the Hermite curve exhibits the necessary variability in bead height, width, shape and contact angles with the substrate, offering greater flexibility in geometric approximation compared to existing models. As a further contribution, the devised algorithm enables the prediction of the shape of complex combinations of deposition according to process parameters.

Keywords Wire arc additive manufacturing, Bead geometric modeling, Weld deposition, Multi-layer deposition, Multi-bead deposition

Paper type Research paper

1. Introduction

Wire Arc Additive Manufacturing (WAAM) is a manufacturing option for the aerospace, mechanical and building industries, thanks to high deposition rates and the possibility of realizing or repairing large parts (Evans *et al.*, 2022; Jafari *et al.*, 2021). However, WAAM is a complex process that is difficult to model. The parts produced exhibit significant geometric irregularities, primarily due to two factors: the size and shape of the individual deposited beads, and, hence, the way the beads combine with each other to form the part being built.

On the one hand, the single bead is strongly related to the process parameters (Jorge *et al.*, 2020). Typically, a bead section is described as depicted in Figure 1, where the common measured parameters are illustrated. They include width (w), height (h), two contact angles (θ_1 and θ_2) and the area of the cross section (A). As illustrated, contact angles are measured at the base of the bead between the substrate profile and the

tangents to the edge of the bead cross-section at its points of contact with the substrate.

Various elements influence the shape and the sizes of the single weld bead. Its shape primarily depends on technological aspects such as material, chosen welding technology and shielding gas. The size of the bead can be modulated by controlling the amount of material being poured by the welder in relation to the speed of the robotic arm. The heat power provided and the dissipation rate due to the conformation of the underlying material have a strong influence. Additional aspects to be considered include gravitational effects, type and flow rate of shielding gas and direction of the welding torch (Mohd Mansor *et al.*, 2024).

The second relevant aspect in forecasting the shape of a WAAM manufactured part is related to the way successive

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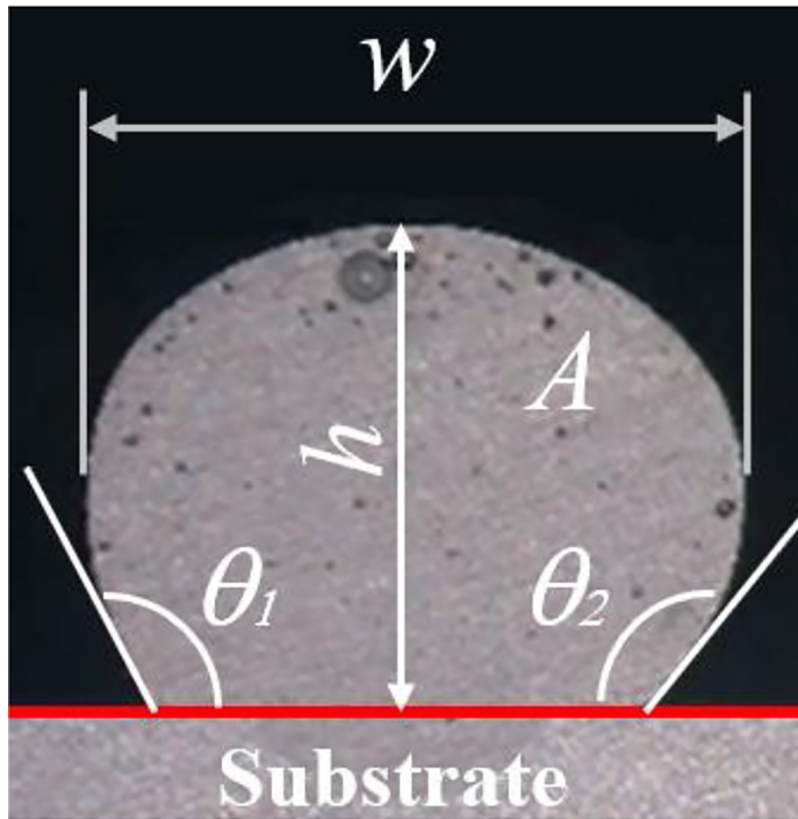
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Figure 1 Typical geometric parameters to describe a bead section

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deposited beads interact. In fact, the underlying beads partially melt when a new one is added and determine the shape of the deposited material. Furthermore, material accumulation effects arise from discontinuities in the deposition paths, such as in highly curved portions, and the overlap occurrences of adjacent or superimposed beads. While these effects depend on the process parameters, they are also significantly influenced by geometrical aspects of the deposition process. They can become particularly pronounced and challenging to be predicted when multiple adjacent beads (hereafter referred as *multi-bead* configurations) or successive overlapped layers (hereafter referred as *multi-layer* configurations) are deposited.

Mathematical models have been proposed in the literature to describe the profile of a bead cross-section and forecast the expected geometry of the deposited material. The main implemented functions are the parabola, the cosine, the circular arc and the elliptic curves (Ding *et al.*, 2015; Lambiasi *et al.*, 2022). The section area has been typically evaluated from the analytical formulation and compared with the deposited area, often measured from metallographic sections, representing an index of the model accuracy (Xiong *et al.*, 2013). Starting from these geometric models, attempts have been made to handle multi-bead and multi-layer depositions (Zhang *et al.*, 2024). The primary aim was the prediction of the geometry of adjacent or stacked beads, and to determine the optimal spacing between bead centers to allow homogeneous material distribution (Ding *et al.*, 2015). As an additional result, the expected waviness of the external surfaces of the model can be estimated.

However, the mathematical functions being used to represent bead cross-sections have some limitations. The parabola is a second order symmetric function, and it is not possible to adjust the two contact angles. Furthermore, the curve is open, which hinders proper stacking in multi-layer scenarios. The cosine function exhibits the same problems. In contrast, circular and elliptic curves are closed and define planar regions that can be composed through Boolean operations to geometrically represent the progress in bead deposition. Nevertheless, even with these curves, the contact angles cannot be varied. Furthermore, bead cross-section shapes actually vary from circular to parabolic forms, as observed by Zhang *et al.* (2022). Therefore, it is essential to develop a model capable of representing the various geometric shapes of bead cross-sections.

For these reasons, this paper presents a novel geometric representation for the cross-section of a bead using a closed curve composed of two cubic Hermite spans. This model is characterized by a set of length parameters and two contact angles, providing enhanced flexibility in its representation compared to the previous mathematical models. Furthermore, the closed curve configuration is beneficial to an algorithm which is here introduced to forecast the overlap of beads, managing multi-bead and multi-layer configurations that occur in manufacturing parts of a certain level of complexity. In contrast to the approaches developed in existing literature, where methods and algorithms are typically designed with the assumption of a planar substrate, the presented algorithm can

handle the deposition processes on nonplanar generic substrate shapes. In general, nonplanar depositions pose two challenges. The first is the complexity of paths found on curved surfaces. In addition, bead section shape and size should be continuously changed to adapt to varying layer thickness while following the 3D curved path and uneven substrate. Indeed, the Hermite model offers the required degree of adaptability to the numerous factors that influence the shape of the WAAM beads, such as temperature distribution. It can provide extensive support for validating the deposition strategies designed for the manufacturing process, generating realistic bead shapes from its application in discrete, possibly dense, points along the paths. The accuracy of the proposed model has been successfully tested by considering experimental trials and comparison with cases described in previous works.

The remainder of the paper is organized as follows: a literature review on bead modeling approaches is presented in Section 2 along with the causes that affect the shape. The proposed model is described in Section 3. The results of the algorithm application are analyzed in Section 4. Finally, conclusions and future works are outlined in Section 5.

2. Background on bead geometric modeling

Various mathematical functions have been proposed to describe the cross-sectional profile of weld beads in the WAAM process and to estimate the deposited material in terms of shape and dimensions (Lambiase *et al.*, 2022). The most used curves include the parabolic function, which describes a symmetrical concave profile; the elliptical function, representing an half-oval cross-section; the cosine function, adopted for smooth and symmetric transitions; and the circular arc curve, which has been used for sections resembling circular segments (Zhang *et al.*, 2024). Table 1 summarizes the features of the functions which have been leveraged in the proposed studies. The reported predicted area A_p represents the measure of the surface bounded by the mathematical function toward a planar substrate.

Table 2 summarizes the main studies on the representation of bead profiles in WAAM specimens, highlighting the types of mathematical models being used. In addition, the table reports whether the adopted model has been tested with multi-bead and multi-layer cases.

A significant variability of models results from Table 2. The main reason can be identified in the different shapes assumed by welding beads according to the experimental conditions, material, heat dissipation rate and technological parameters under investigation (Xiong *et al.*, 2013). Figure 2 shows how different models try to adapt to the shapes assumed by the beads.

From the analysis of existing models, the following main limitations emerge:

- existing models can only fit specific bead shapes that are obtained under restricted process conditions;
- the contact angles θ_1 and θ_2 show a strong variability, but the existing analytical formulations do not provide the necessary degree of freedom necessary to vary them;
- circle, parabola and ellipse are second degree curves that do not exhibit the required freedom of interpolation;
- cosine function introduces more computational expense and still lacks flexibility in setting tangents at the ends of the profile; and
- only closed curves can be used to stack sequences of successive bead profiles and compose them with Boolean operators.

From these considerations, the need for a more generalized model that reconciles previous efforts clearly emerges. It is necessary to have more flexibility in adjusting the curve representing the bead to the various process conditions and parameters.

In the literature, the single-bead models have been also extended in multi-bead overlapping scenarios. These studies have focused on the estimation of behavior when two or more adjacent beads are deposited (Figure 3), determining the optimal distance between beads to minimize excessive waviness of the top surface (Ding *et al.*, 2015).

However, multi-bead and multi-layer models have mostly been built on the parabolic function, as reviewed in Table 2. As shown in Figure 4, this formulation leads to an open curve, to a limitation in delimiting the profile and consistently represent the lateral surface of a stack of beads (Li *et al.*, 2018; Teixeira *et al.*, 2023).

As a result, a comprehensive model that accurately represents the profile of single bead, multi-bead and multi-layer configurations while accounting for all geometric variables is still missing.

In conclusion, bead modeling approaches in the literature often refer to standardized conditions: a single bead deposited on planar plates, or sequences of parallel equally spaced beads. Even if valuable, this does not represent the generic case. The realization of artifacts requires variable distances between beads, curved paths, different deposition patterns among successive layers, causing uneven support surfaces due to voids and material accumulation points. The Hermite model and the algorithm to compose beads presented in this work want to introduce a general approach to cope with any location of deposition on any substrate shape deriving from previously deposited layers of beads. Hermite curve provides flexibility in

Table 1 Details of the mathematical functions used in bead modeling. The height and width of the bead are reported in terms of the coefficients α and β appearing in the analytical expressions. The area A_p of the bead is formulated in terms of w and h

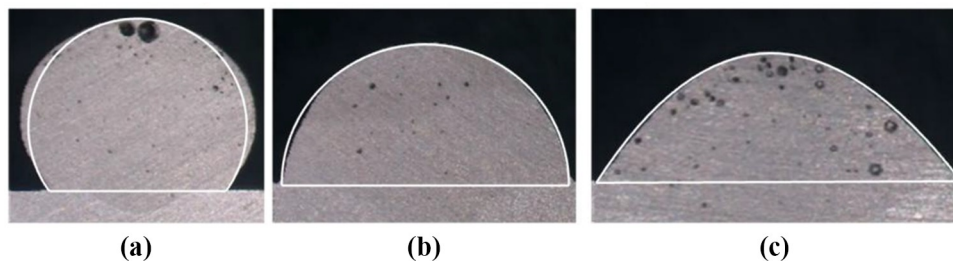
Model	Expression	Height (h)	Width (w)	Predicted area (A_p)
Parabola	$y = \alpha x^2 + \beta$	β	$2\sqrt{-\beta/\alpha}$	$(2/3) wh \cong 0.667 wh$
Cosine	$y = \alpha \cos(\beta x)$	α	π/β	$(2/\pi) wh \cong 0.637 wh$
Half ellipse	$y^2/\alpha^2 + x^2/\beta^2 = 1$	α	2β	$(\pi/4) wh \cong 0.785 wh$
Circular arc	$y = \sqrt{\alpha^2 - x^2} + \beta$	$\alpha - \beta$	$2\sqrt{\alpha^2 - \beta^2}$	$f wh : f \in [0.667, 0.785]$

Source(s): Table by authors

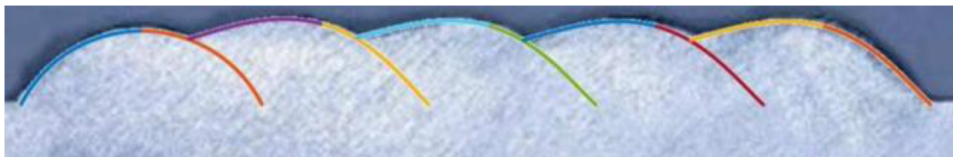
Table 2 Review of the main works proposing bead models applied to multiple beads

Work	Considered model	Application to multi-bead configurations	Application to multi-layer configurations
Banaee et al. (2023)	Parabola	✓	
Chen et al. (2023)	Parabola	✓	
Ding et al. (2015)	Parabola, cosine, arc	✓	
Hu et al. (2020)	Parabola, arc	✓	
Lambiase et al. (2022)	Parabola, arc, ellipse		
Li et al. (2018)	Parabola	✓	✓
Li et al. (2021)	Parabola	✓	✓
Mang et al. (2023)	Laplace equation		
Ocelik et al. (2014)	Parabola, arc, cosine, ellipse	✓	✓
Suryakumar et al. (2011)	Parabola	✓	
Wang et al. (2021)	Arc		
Xiong et al. (2013)	Parabola, arc, cosine	✓	
Zhang et al. (2024)	Parabola, arc, cosine, ellipse		✓

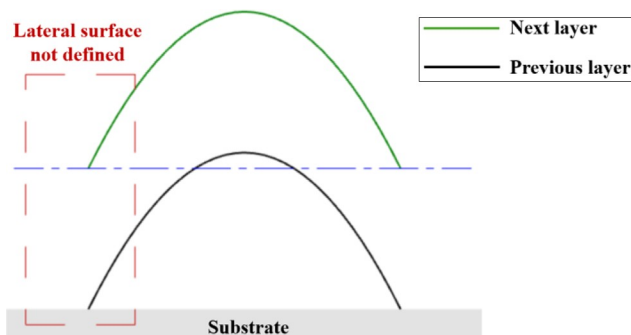
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Figure 2 Adoption of different bead representation curves: a) circular arc; b) elliptical arc; c) parabolic curve

Source: Figure adapted from [Zhang et al. \(2022\)](#)

Figure 3 Example of multi-bead algorithm: recursive axisymmetric drop shape deposition

Source: Figure adapted from [Chen et al. \(2023\)](#)

Figure 4 The parabola model cannot consistently represent the lateral surface of a WAAM wall

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the shape definition of the bead section according to experimental parameters and the algorithm gives the possibility to simulate how the bead interacts with the substrate.

2.1 Causes of the variability of the bead shape

To better contextualize the proposed model, this section provides a brief background on the various factors that influence the shape as well as the size of the weld bead. First, the Wire Feed Rate (*WFR*) determines the amount of filler material being deposited. An increase in *WFR* leads to larger bead sizes, as more material is supplied while energy input is raised accordingly. Thus, *WFR* is closely linked to both current (*I*) and voltage (*V*) provided by the heat source ([Zhang et al., 2022](#)). Besides, *WFR* should be considered against the Torch Travel Speed (*TS*). Increasing the *TS* causes the robot to move

faster, reducing the amount of material deposited in the unit length and diluting the energy provided in a certain amount of time over a longer length. This results in a cooler and narrower bead (Cao *et al.*, 2022). Furthermore, Xiong *et al.* (2013) have observed how the bead shape depends on the WFR/TS ratio. Specifically, higher values of this ratio correspond to a more elliptical or circular bead cross-section, while lower values produce a more parabolic or triangular shape.

The technological aspects of the welding process, i.e. material composition, wire diameter d_w , the chemical composition of the shielding gas and the type of welding technology influence the deposition (Kou, 2003; Veiga *et al.*, 2021). The type of material being processed (Zhang *et al.*, 2022) influences the morphology of the deposited bead as material properties such as thermal conductivity, viscosity and surface tension impact. Just the shielding gas flow rate, typically 13–15 l/min, has shown limited effects on the bead geometry (Dinovitser *et al.*, 2019).

In addition, the torch orientation and the gravitational force also affect the molten metal. The material tends to flow toward gravity, altering the bead shape (Mang *et al.*, 2023). If the substrate is not horizontal, the molten drops bend or even flow, influenced by gravity. The substrate temperature is equally important. Higher underlying temperatures generally cause the bead to spread, reducing its height. Such conditions vary during the deposition, as the part is built and the geometric conditions, temperature profiles and heat dissipation change. Consequently shape and size of the bead varies from bead to bead during the progress of the deposition, even if working parameters are left unchanged (Li *et al.*, 2018; Zhang *et al.*, 2024).

As regards the choice of technological process, Cold Metal Transfer (CMT) is frequently adopted, given the lower rate of heat being provided and the improved capability in controlling the deposition conditions. However, CMT introduces additional specific parameters to be considered. Arc length, defined as the distance between the electrode and substrate, affects the bead characteristics. Similarly, dynamic correction settings can influence heat input, as negative values increase the frequency of CMT cycles, potentially raising the amount of heat introduced into the weld. Polarity also impacts weld penetration. Keeping the wire as the positive electrode is preferable to reduce penetration, i.e. melting the material previously deposited.

In conclusion, these factors underline the need for defining an appropriate model to predict the final morphology of the beads accurately. The literature has shown that correlations can be established between process parameters and the obtained shape and feed mathematical models (Xiong *et al.*, 2013). Machine learning could also be useful to correlate process parameters to geometrical description to improve the possibilities given by regression models that often struggle to capture the complexity of the relations. Sensors can be employed to measure the deposited material, correct on the fly the working parameters with closed-loop approaches, stabilize the process, maintain target conditions and avoid material shortages or excesses (Kishor *et al.*, 2025).

Nevertheless, the scope of this paper is not related to how certain shape and dimensions are obtained. The goal of the work moves from the geometrical description of the bead as it comes from the chosen setup and selected parameters. The

focus is set on an algorithm to combine experimental data and forecast the geometry of several overlapped beads. The proposed simulation can anticipate defective situations in the process planning phase to limit the need for corrections during the actual deposition and provide a synergistic effect.

3. Bead section modeling approach

Cubic Hermite curves are parametric curves used in computer graphics and geometric modeling. They are defined by cubic polynomials specifying end points of the curve along with the tangents at these points (Yong and Cheng, 2004), that can be conveniently used to control the shape of the curve, as shown in Figure 5.

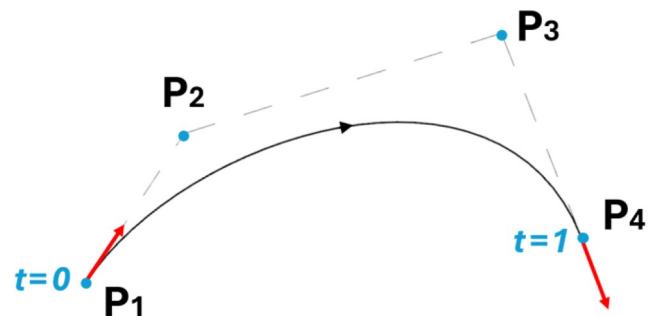
The proposed bead model is based on a closed curve composed of an upper Hermite span (continuous portion in Figure 6) and a twinned lower Hermite curve (dashed portion in Figure 6).

A cubic Hermite curve is equivalent to a third degree Bézier curve defined by four control points. In the proposed model, the endpoints of the two Hermite portions coincide, and the tangents of the two spans are aligned. Therefore, the position of the control points P_1 to P_6 is determined by the independent parameters $\Gamma = [t_1, t_2, t_3, t_4, b, \theta_1, \theta_2]$ as defined and illustrated in Figure 6. These parameters being defined, the enclosed area between the Hermite curve and the substrate A_h represents the nominal deposited area.

The dashed curve portion should not be intended as bead penetration (Dinovitser *et al.*, 2019), neither the bounds of the so-called Heat Affected Zone (HAZ) that appears in metallographic sections. The dashed curve is designed to create a closed loop, which defines a planar domain representing a bead section to be handled by Boolean operations. The shape of the inferior dashed curve comes to a role when the union of the described shape with a particular substrate leaves some portion of it exposed. Thus, the lower curve is just a profile completing the upper shape of the bead and guaranteeing a closed profile to be obtained.

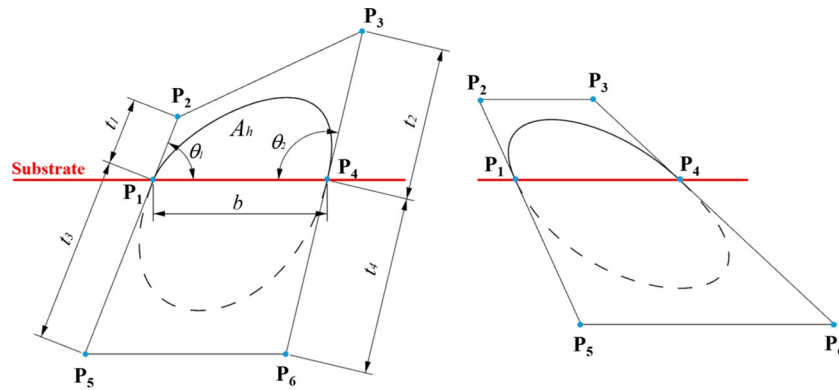
The proposed profile has a high level of flexibility, mostly given by the possibility of adjusting the values of the two angles θ_1 and θ_2 , and modulating the distances t_1 and t_2 . This flexibility allows for the representation of various bead shapes depending on the physical and technological aspects of the deposition

Figure 5 Third degree Hermite curve and the control points of its Bézier form. The curve parameter t varies from 0 at the start point P_1 to 1 at the end point P_4



Source: Authors' own work

Figure 6 Bead model based on a closed curve composed of two Hermite spans. On the right side the special case when P_3 - P_2 and P_6 - P_5 are parallel to P_4 - P_1



Source: Authors' own work

process, as described in the previous section. Conversely, the role of t_3 and t_4 is connected to the wanted shape of the lower portion of the sides of the beads and can be conveniently assumed to be proportional to the respective counterparts of the upper profile.

As illustrated in Figure 7, different configurations of the weld bead are depicted, demonstrating the versatility of the model in capturing the dynamic nature of weld formation compared to previous models in the literature.

In the case where P_2 and P_3 are aligned with the direction of the segment P_1 - P_4 , the shape of the beads appears as in Figure 6 on the right. As shown in the following such configuration exhibits some useful properties to facilitate computational aspects.

3.1 Proposed model properties

The proposed modeling approach relies on a framework composed of three major steps, as depicted in Figure 8.

Initially, the set of parameters $\Omega = [h, w, A, \theta_1, \theta_2]$ is derived from tests according to deposition and process conditions (WFR, TS, voltage). These correlations are based on empirical evidence and experimental data. Then, correspondences are established between the parameters Ω and the parameters Γ defining the Hermite bead profile. Explicit analytical expressions are desirable to accelerate the application of the proposed representation in modeling overlapping beads for composing the shape of the desired part. Such relations allow for easier inversion of the equations, enabling more effective use in the modeling algorithm presented in the subsequent

sections. To this end, certain approximations are introduced to enhance algorithm efficiency while having minimal impact on the overall accuracy of the results. Finally, the third step leverages the parametric model for composing beads according to the slicing and deposition paths and, hence, for simulating the geometry of the part under realization.

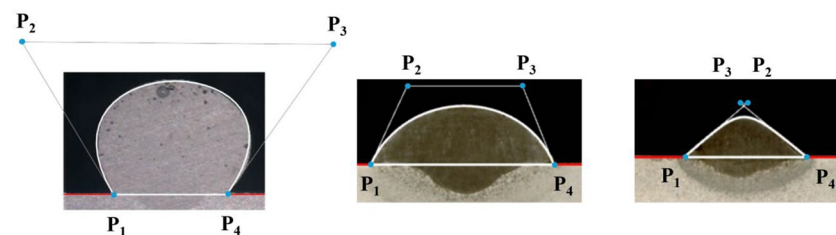
Figure 9 represents a Hermite curve with the control points P_1 to P_4 foreseen by the equivalent Spline form and some geometrical constructions derived from the application of the De Casteljau's algorithm, which are useful to show a few meaningful properties (Kreyszig, 2015). Point C is defined as the midpoint of the segment connecting points P_1 and P_4 , and M is the midpoint of the segment connecting P_2 and P_3 . Point H^* represents the intersection of the Hermite curve with the line segment joining C and M , while H is defined as the point of maximum height of the Hermite curve relative to the segment P_1 - P_4 . The quantities h and h^* represent respectively the distance of H and H^* from the line P_1 - P_4 .

The following equation (1) allows the calculation of the height h of the bead based on the parameters of the Hermite curve representation under an acceptable approximation:

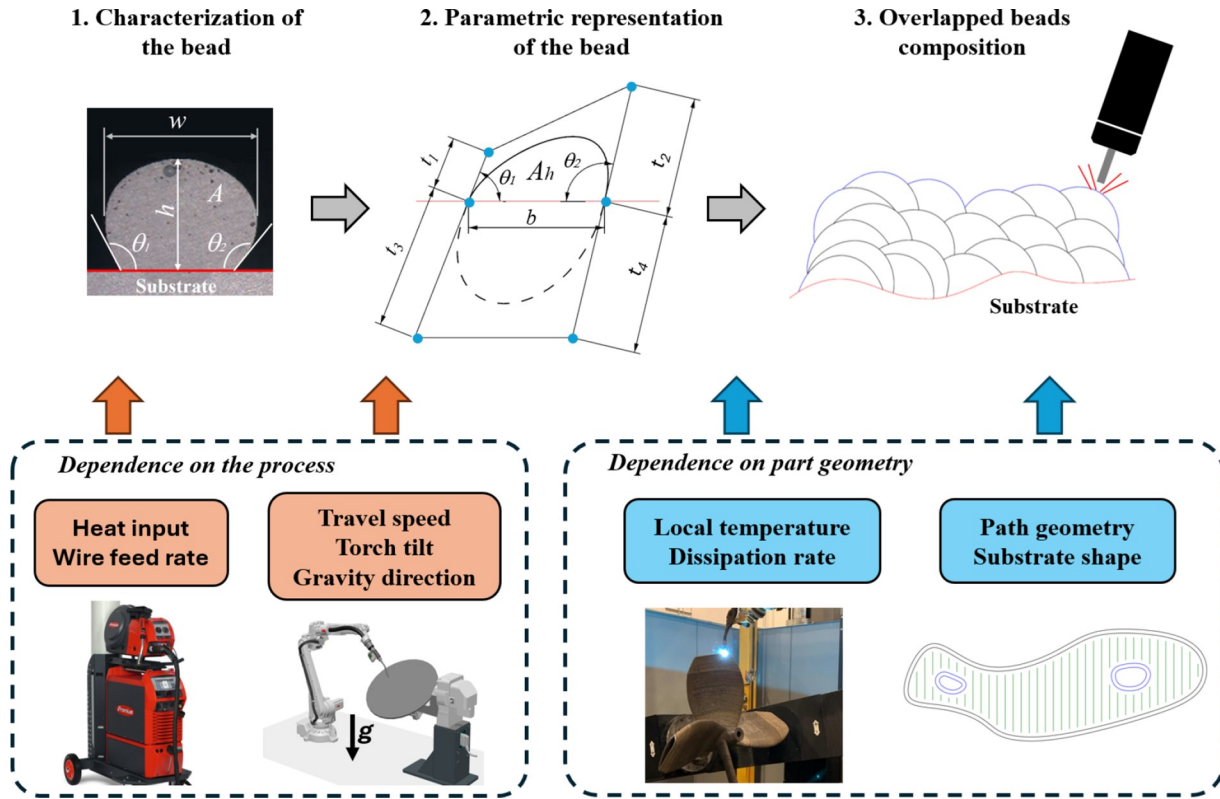
$$h \cong h^* = \frac{3(h_1 + h_2)}{4} = \frac{3}{8}(t_1 \sin \theta_1 + t_2 \sin \theta_2) \quad (1)$$

Notably, when points P_2 and P_3 are aligned with the direction of the segment P_1 - P_4 , the height h is equal to h^* , zeroing the error in the calculation of the bead height. In this case, h computation reduces to the following expression [equation (2)]:

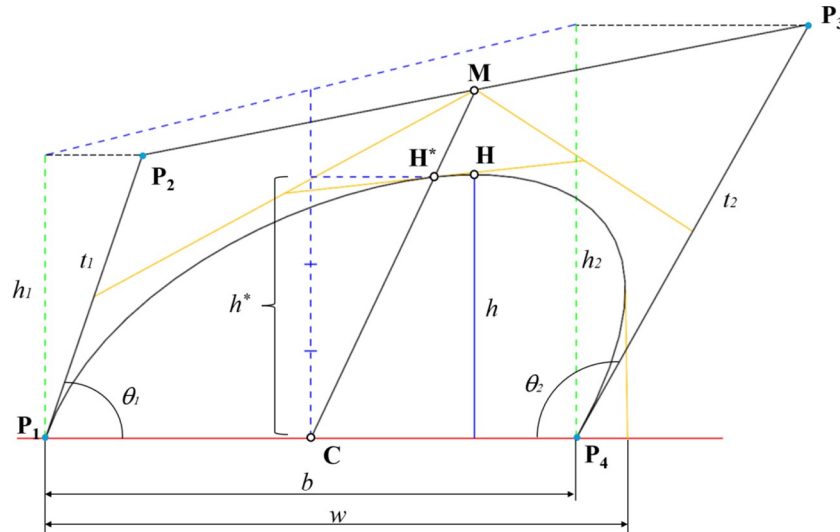
Figure 7 Representation of diverse bead shapes and the position of the relative control points



Source: Authors' own work

Figure 8 Overall framework for bead composition modeling

Source: Authors' own work

Figure 9 Definitions and properties of a bead section represented by a Hermite curve

Source: Authors' own work

$$h = \frac{3}{4}h_1 = \frac{3}{4}h_2 = \frac{3}{4}t_1\sin\theta_1 = \frac{3}{4}t_2\sin\theta_2 \quad (2)$$

In the following, to correlate experimental parameters Ω to the representation ones Γ , the Hermite angles θ_1 and θ_2 in the model

are assumed to be equal to those experimentally measured. This ensures consistency between the experimental data and the parameters used in the Hermite curve representation. The width w is equivalent to b when both angles θ_1 and θ_2 are less than or equal to $\pi/2$. In the other cases, w is still approximated with b , though it could be obtained from experimental data.

Finally, it is useful to estimate the area A_h of the bead represented by a Hermite curve as a function of the curve parameters. Even though the area can be computed by numerical integration, the following equation (3) provides an easy explicit formulation as a function of the bead parameters listed in Ω :

$$A_h = 0.8 \nu h b \quad (3)$$

where ν depends on θ_1 , θ_2 , and h/b . Given a certain ratio h/b , ν it is equal to 1 if $\theta_1 = \theta_2 = \pi / 2$. In the other cases, it varies according to θ_1 and θ_2 as in Figure 10. The depicted analysis focuses on the contribution of a singular angle when P_2 and P_3 are aligned with the direction of segment P_1 - P_4 . Additionally, it is possible to aggregate the individual contributions to evaluate the cumulative effect of both angles.

As can be seen, the trend of the curve is mostly linear, and the slope coefficient m of the interpolating line has been evaluated for the contact angles included in the interval $[45^\circ, 135^\circ]$. The graph in Figure 10 refers to a bead section where the height-to-base ratio h/b is 0.5. Different ratios lead to different slope coefficients which have been plotted in Figure 11. Once again, the behavior is linear with a slope coefficient k of 0.00672, while the intercept is zero.

In conclusion, assuming $h_1 = h_2$ the area can be estimated from the h/b (or h/w ratio under the conditions discussed above) by the following equation (4):

$$\nu = k \frac{h}{b} (\theta_1 + \theta_2) + 1 \quad (4)$$

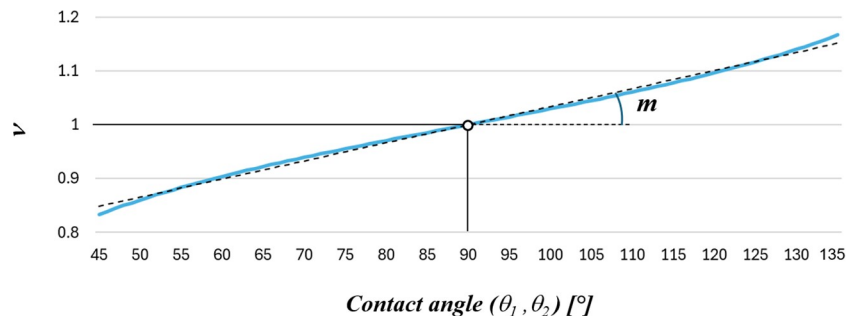
Once w , h and the deposited area A_h are known, one possibility is to determine $(\theta_1 + \theta_2)$ so that the area of the bead exactly matches the desired value.

3.2 Algorithm to compose beads

The proposed geometric representation forms the basis of an algorithm that allows for the geometrical simulation of overlapping beads, effectively managing scenarios involving multi-bead and multi-layer configurations. The algorithm has been developed in a planar bidimensional domain, paving the way for further work to extend it to the three-dimensional space. Figure 12 outlines the workflow of the algorithm.

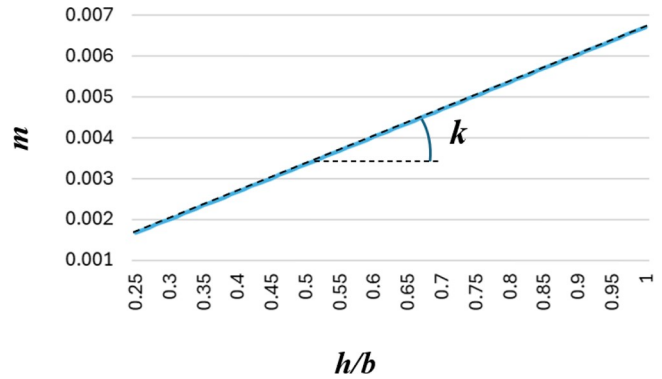
The inputs of the algorithms are the initial substrate profile S , the directions of the deposition d_i , the list of points T_i representing the welding torch positions, and the set of parameters Γ determined by the manufacturing process choices.

Figure 10 Factor ν by varying the contact angle. The ratio $h/b = 0.5$ and m turns to be 0.00336



Source: Authors' own work

Figure 11 Plot of the slope coefficient of the interpolated line vs h/b ratio



Source: Authors' own work

The outputs are the shape and orientations of the sections of the deposited beads, as well as the resulting external profile, which represents the new substrate for subsequent depositions.

3.2.1 Steps of the algorithm

Starting from the first bead, point Q is determined as the intersection between the initial substrate S and the line l oriented along direction d_i , passing through point T_i , which defines the location of the torch. A circle with center at Q and a diameter b is then constructed. The substrate segment enclosed within this circle is discretized into a series of points spaced at regular distance δ . A least-squares fit line m is subsequently computed through these points, and the closest point C from Q with respect to m is identified (Figure 13).

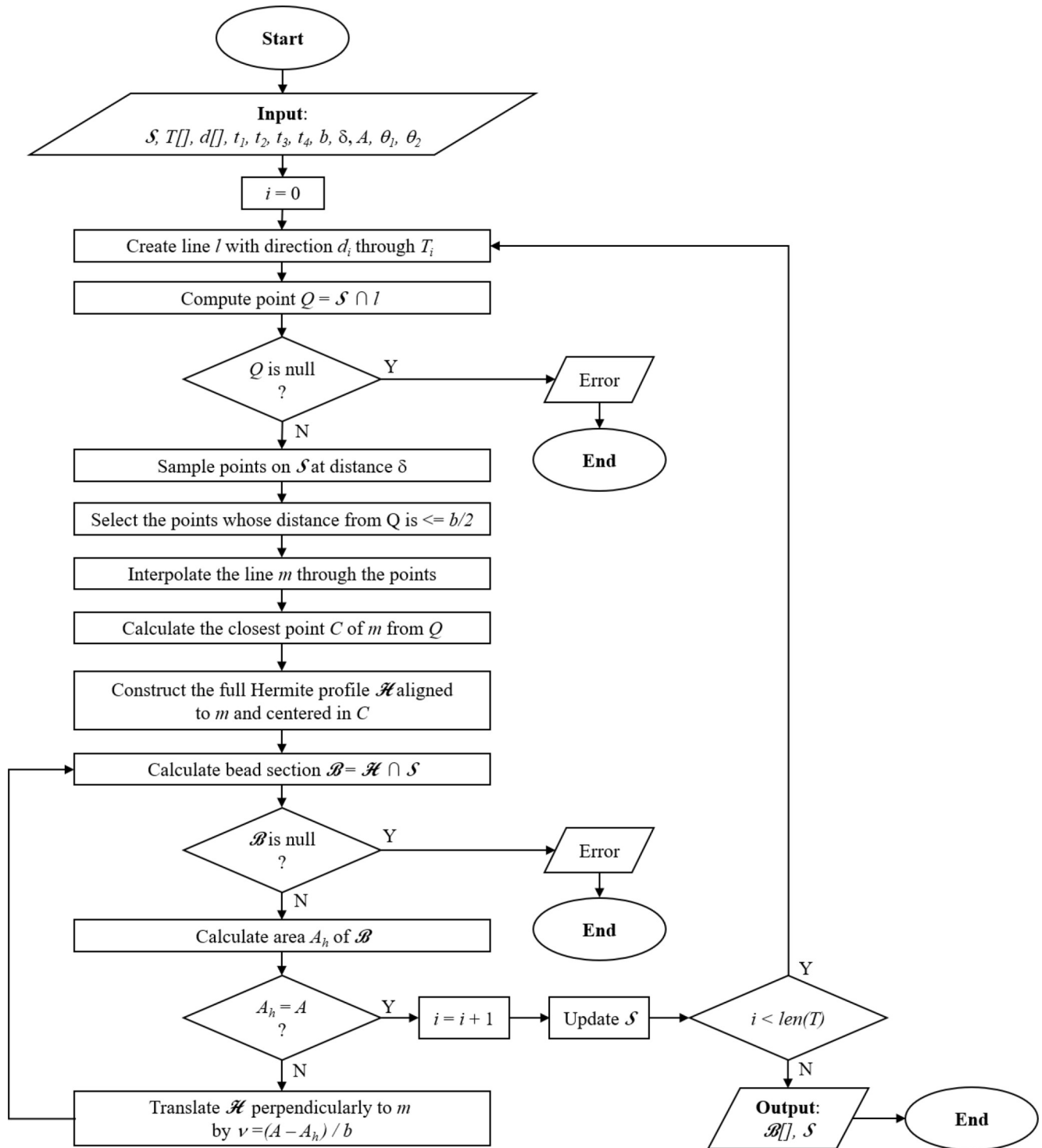
Then, the closed curve H , composed of two Hermite portions, is generated from the fit line m and point C according to the geometric parameters Ω (Figure 14, left). The curve is intersected with the substrate S , creating a region B of area A_h representing the geometry of the deposited bead (Figure 14, right).

Next, the area of the deposited bead A_h is compared with the target A . If discrepancies are found, the curve H is translated by a distance v , determined by equation (5):

$$v = \frac{A - A_h}{b} \quad (5)$$

This process is iterative until the areas coincide within a defined tolerance (Figure 15). Once the desired match is achieved, the substrate curve S is updated, and the algorithm proceeds to the next deposition point T_{i+1} (Figure 16).

Figure 12 Flowchart of the algorithm

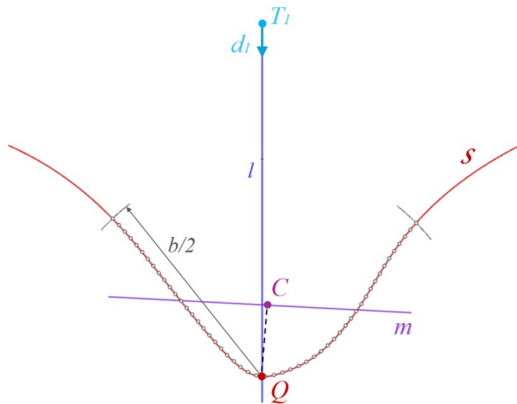


Source: Authors' own work

As can be observed from Figure 17, the iteration of the procedure allows the overlap of adjacent weld beads to be simulated. The approach also applies to multi-layer scenarios, enabling geometric modeling of bead interactions for generic sequences of bead depositions.

4. Validation of the proposed algorithm

The proposed bead section model and composition algorithm were tested on single-pass, multi-bead and multi-layer cases. The test cases aimed to evaluate the performance of the model

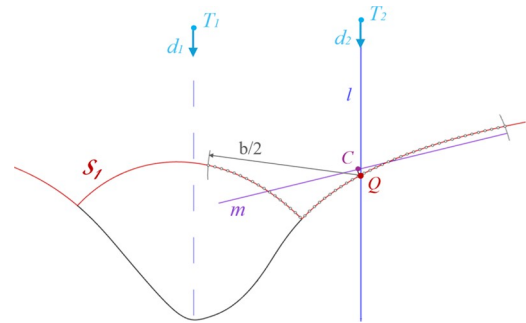
Figure 13 Substrate segmentation and construction of the fit line m 

Source: Authors' own work

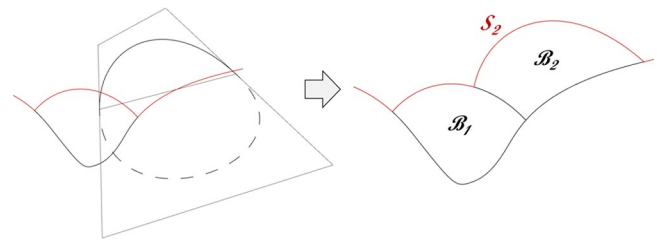
under different deposition conditions, ensuring its effectiveness across varying levels of complexity.

Five different test cases were analyzed. The first one focused on single-bead deposition (Kindermann *et al.*, 2020), the second included both single-bead and multi-bead scenarios (Xiong *et al.*, 2013), while the third test case (Lettori *et al.*, 2025) concentrated on multi-layer deposition. The fourth one combined single-bead, multi-bead and multi-layer conditions (Ding *et al.*, 2015). Finally, the fifth case study examined multi-bead and multi-layer depositions (Li *et al.*, 2018).

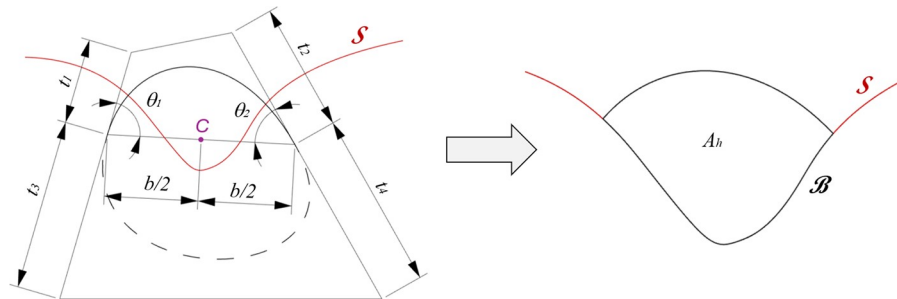
The validation has focused on the following aspects:

Figure 16 Interpolation of the new line m for the subsequent torch point T_2 

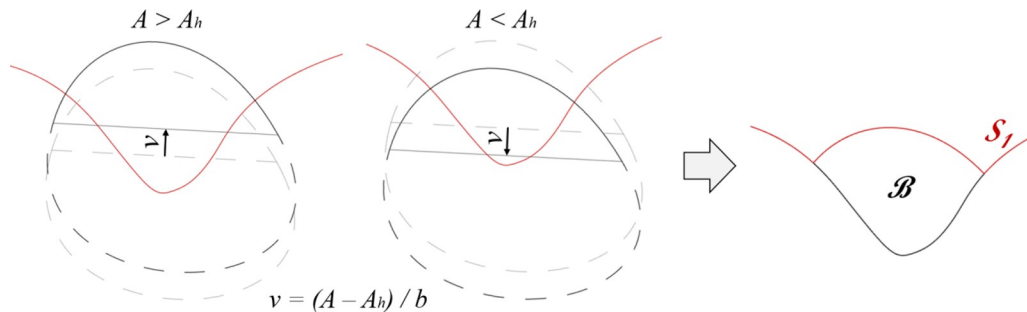
Source: Authors' own work

Figure 17 Construction of the second Hermite profile and result of the new Boolean intersection

Source: Authors' own work

Figure 14 Construction of the closed curve H (left) and Boolean intersection defining the deposited bead B of area A_h (right)

Source: Authors' own work

Figure 15 Left: positive translation of the bead to increase area A_h . Middle: negative translation of the bead to decrease area A_h . Right: final configuration of the bead and S curve redefinition

Source: Authors' own work

- evaluation of the ability to reproduce the profile of a single bead section;
- evaluation of the modeling and overlapping approach in reproducing the quantity of the deposited material through an assessment of the errors in the obtained section area compared to other modeling approaches and the nominal amount of material deposited by the torch; and
- assessment of the overall profile of the section in multi-bead and multi-layer scenarios with focus on the ability to reproduce observed shapes depending on the sequence of deposition.

An analysis of the welding process, combined with a volume balance of the material deposited per unit of time, yields the relationship defined in [equation \(6\)](#), which allows the cross-sectional area A_d of the bead to be expressed as a function of the deposition parameters ([Ding et al., 2015](#)):

$$A_d = \frac{\pi}{4} \frac{WFR}{TS} d_w^2 \quad (6)$$

The relative error Ed . [[equation \(7\)](#)] assesses the deviation between A_d and the area A_p of the section represented by mathematical models from the literature, i.e. parabola, cosine and ellipse:

$$e_d = \frac{A_p - A_d}{A_d} \quad (7)$$

It is observed that A_d depends on process parameters, which are known but can be modulated by the welding source according to mechanisms predefined by the manufacturers. Specifically, it includes heat input by adjusting voltage and/or current, but also the WFR within predefined ranges according to arc formation conditions. Establishing a clear relationship between welding parameters, metal transfer, and heat input becomes challenging ([Selvi et al., 2018](#)). Moreover, the motion parameters, i.e. TS , are influenced by the intrinsic constraints and capabilities of the anthropomorphic robotic system ([Zhang et al., 2021](#)).

Since A_d is subjected to such uncertainties, the measured area A_m is also introduced, which is obtained by measuring the

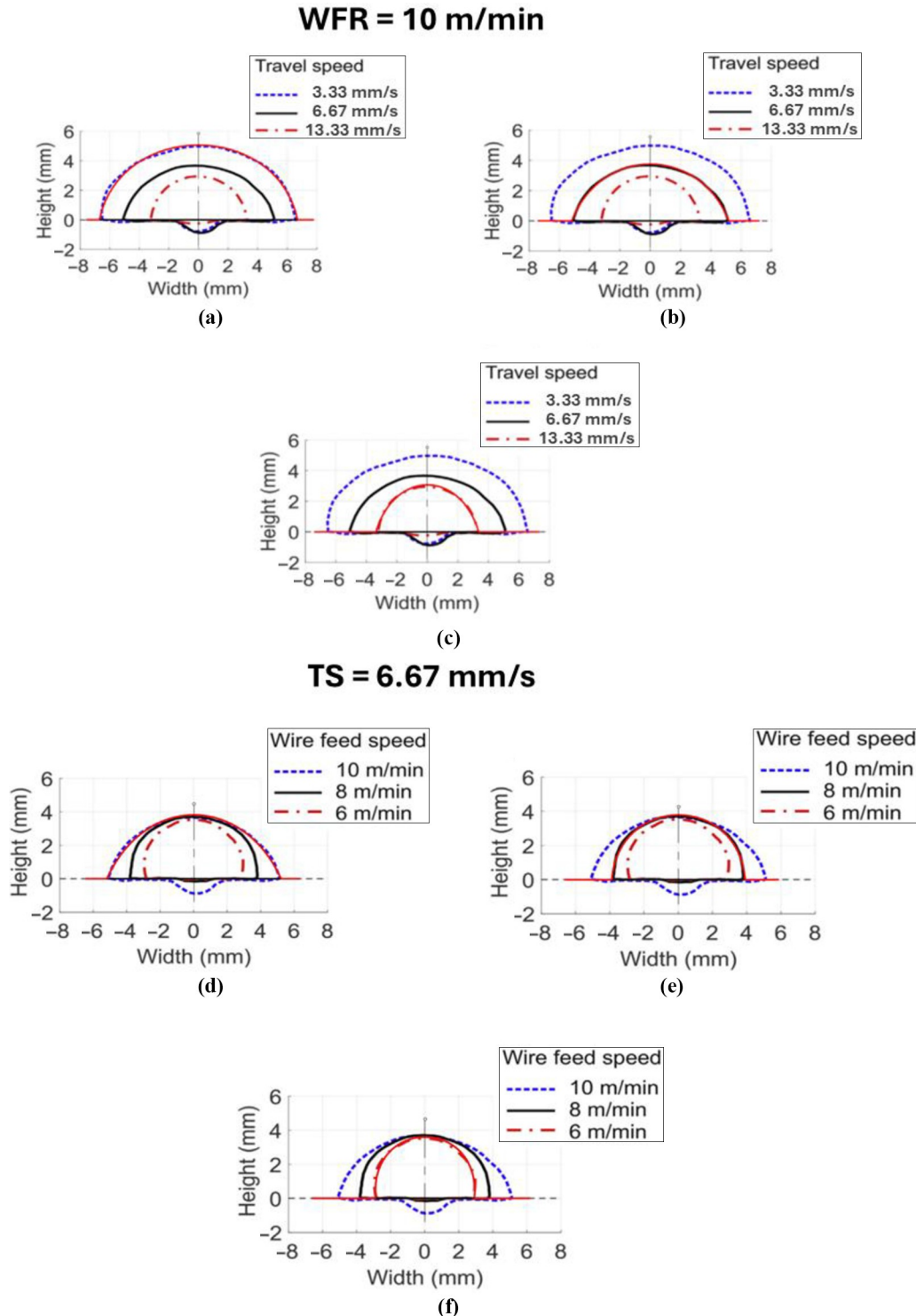
Table 3 Summary of the application of the Hermite representation to the experimental data reported by [Kindermann et al. \(2020\)](#)

Experimental setup	Generator	Fronius CMT 4000					
	Gas type	Ar 100%					
	Gas flow rate [l/min]	15					
	Wire diameter (d_w) [mm]	1.2					
	Wire material	Low carbon steel					
Means of measure	MeltView MIRA2						
Cases reported in Figure 18		a	b	c	d	e	f
Deposition parameters	Multi-bead	–	–	–	–	–	–
	Multi-layer	–	–	–	–	–	–
	Power [W]	4700	4717	4642	2700	3557	4717
	Wire feed rate (WFR) [m/min]	10	10	10	6	8	10
	Travel speed (TS) [mm/s]	3.33	6.67	13.33	6.67	6.67	6.67
	WFR/TS	50	25	12.5	15	20	25
Obtained bead sizes	Width (w) [mm]	13.39	10.35	6.7	5.8	7.79	10.35
	Height (h) [mm]	5.05	3.74	3.02	3.61	3.78	3.74
	h/w	0.38	0.36	0.45	0.62	0.49	0.36
	Deposited area (A_d) [mm ²]	56.55	28.26	14.14	16.96	22.61	28.26
	Measured area (A_m) [mm ²]	51.23	28.13	15	17.29	22.95	28.23
	$A_m - A_d$ [mm ²]	−5.32	−0.13	0.86	0.33	0.34	−0.03
	$(A_m - a_d)/a_d$	−9.4%	−0.5%	6.1%	2%	1.5%	−0.1%
Parabola model	A_p [mm ²]	45.08	25.81	13.49	13.96	19.63	25.81
	e_d	−20.3%	−8.7%	−4.6%	−17.7%	−13.2%	−8.7%
	e_m	−12%	−8.3%	−10.1%	−19.3%	−14.5%	−8.6%
Cosine model	A_p [mm ²]	43.05	24.64	12.88	13.33	18.75	24.64
	e_d	−23.9%	−12.8%	−8.9%	−21.4%	−17.1%	−12.8%
	e_m	−16%	−12.4%	−14.1%	−22.9%	−18.3%	−12.7%
Ellipse model	A_p [mm ²]	53.08	30.39	15.88	16.44	23.12	30.39
	e_d	−6.1%	7.5%	12.3%	−3.1%	2.2%	7.5%
	e_m	3.6%	8%	5.9%	−4.9%	0.7%	7.6%
Proposed Hermite model	A_h [mm ²]	51.27	28.13	14.98	17.32	22.95	28.13
	Contact angle θ_1 [°]	78	69	76	95	85	69
	Contact angle θ_2 [°]	78	69	76	95	85	69
	e_d	−9.3%	−0.5%	5.9%	2.2%	1.5%	−0.5%
	e_m	0.1%	0%	−0.1%	0.2%	0%	−0.4%
Source(s): Table by authors							

scanned profile from metallographic sections or scans, thus decoupling the geometric assessment from the experimental unknowns. Images from the analyzed papers have been processed in the Rhinoceros® 3D CAD system (McNeel Inc.) using appropriate scaling factors drawn from known lengths

reported by the authors of the works. Then, the reconstructed profiles and torch positions are based on available data and could be subject to minor biases and errors due to the image resolution, scale reconstruction procedures, inaccuracies in the interpolation of curves on the images, or small inconsistencies

Figure 18 Overlay of the Hermite-based bead section curves (continuous red lines) to the images from [Kindermann et al. \(2020\)](#)



Source: Figure adapted from [Kindermann et al. \(2020\)](#)

found in the numbers. This leads to the definition of the relative error e_m , which is reported in [equation \(8\)](#) as an additional measure of the accuracy of the representation model:

$$e_m = \frac{A_p - A_m}{A_m} \quad (8)$$

Based on the properties of the Hermite curves reported in Section 3.1, control points P_2 and P_3 have been assumed to be

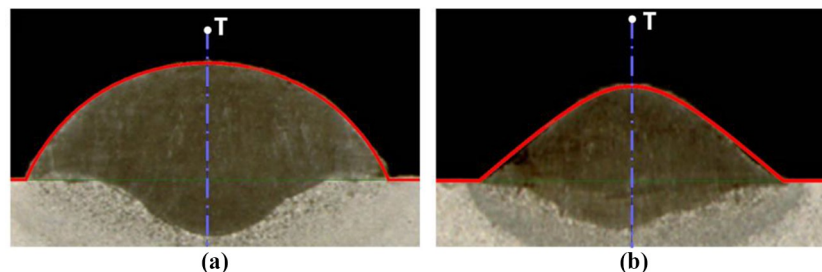
aligned along the P_1 - P_4 direction. For individual beads, h and w are known from experimental data. Angles θ_1 and θ_2 are assumed to be equal and selected to match the experimental data, while minimizing the difference between A_h and the measured area A_m . In multi-bead and multi-layer cases, h will be appropriately estimated as reported in the following, starting from the measured A_m . Since the considered cases primarily involve θ_1 and θ_2 being less than or equal to $\pi/2$, b is taken

Table 4 Summary of the elaboration of data reported by [Xiong et al. \(2013\)](#)

Experimental setup	Gas type Gas flow rate [l/min] Wire diameter (d_w) [mm] Material Means of measure	Ar 95%, CO2 5% 18 1.2 Copper coated steel Laser vision sensor					
		a	b	a	b	c	d
Cases reported in Figure 19							
Cases reported in Figure 20							
Deposition parameters	Multi-bead	–	–	✓	✓	✓	✓
	Multi-layer	–	–	–	–	–	–
	Wire feed rate (WFR) [m/min]	4.4	4.4	4	4	4	4
	Travel speed (TS) [mm/s]	2.5	7.5	3	3	6.5	6.5
	WFR/TS	29.3	9.8	22.2	22.2	10.3	10.3
Obtained bead sizes	Width (w) [mm]	11.76	7.54	9.20	9.20	7.19	7.19
	Height (h) [mm]	3.80	2.34	3.71	3.60	2.53	2.53
	h/w	0.32	0.31	0.40	0.39	0.35	0.35
	Deposited area (A_d) [mm ²]	33.18	11.06	25.13	25.13	11.60	11.60
	Measured area (A_m) [mm ²]	32.14	10.51	26.40	28.47	12.10	10.53
	$A_m - A_d$ [mm ²]	–1.04	–0.55	1.27	3.34	0.50	–1.07
	$(A_m - a_d)/a_d$	–3.1%	–5%	5.1%	13.3%	4.3%	–9.2%
Parabola model	A_p [mm ²]	29.76	11.77	22.75	22.08	12.15	12.15
	e_d	–10.3%	6.4%	–9.5%	–12.2%	4.7%	4.7%
	e_m	–7.4%	12%	–13.8%	–22.4%	0.4%	15.4%
Cosine model	A_p [mm ²]	28.42	11.24	21.73	21.08	11.60	11.60
	e_d	–14.3%	1.6%	–13.5%	–16.1%	0%	0%
	e_m	–11.6%	6.9%	–17.7%	–26%	–4.1%	10.2%
Ellipse model	A_p [mm ²]	35.04	13.85	26.79	26.00	14.30	14.30
	e_d	5.6%	25.3%	6.6%	3.5%	23.3%	23.3%
	e_m	9%	31.8%	1.5%	–8.7%	18.2%	35.8%
Proposed Hermite model	A_h [mm ²]	32.14	10.50	26.41	28.46	11.87	10.54
	Contact angle θ_1 [°]	67	40	74	74	46	46
	Contact angle θ_2 [°]	67	40	74	74	46	46
	e_d	–3.1%	–5.1%	5.1%	13.2%	2.3%	–9.1%
	e_m	0%	–0.1%	0%	0%	–1.9%	0.1%

Source(s): Table by authors

Figure 19 Approximation by Hermite curves of the metallographic sections reported by [Xiong et al. \(2013\)](#)

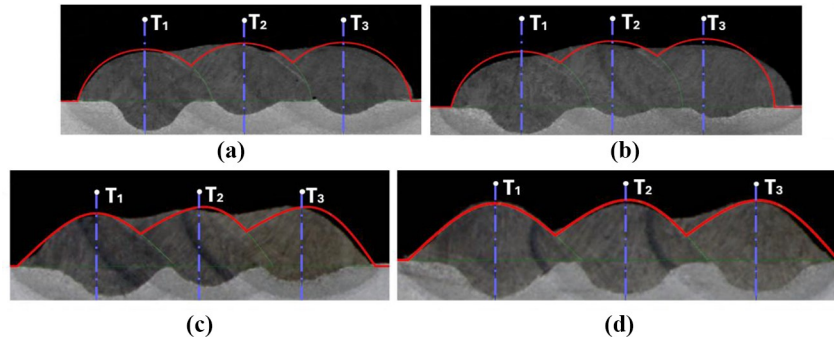


Source: Figure adapted from [Xiong et al. \(2013\)](#)

equivalent to w . Finally, concerning the bottom part of the bead profile, the direction defined by the points P_5 and P_6 aligns with the P_1 – P_4 direction, t_3 is set as double of t_1 , while t_4 is derived accordingly (Figure 6, right).

The generation of the Hermite profiles and the bead composition algorithm have been implemented as a Rhinoceros® Plug-in. The RhinoCommon SDK was used to translate the geometric procedures described above using the

Figure 20 Application of the algorithm to adjacent beads sections reported in Xiong *et al.* (2013)



Source: Figure adapted from Xiong *et al.* (2013)

Table 5 Summary of the elaboration of data from performed multi-layer deposition experiments

Experimental setup	Generator	Fronius TPS 320i			
	Gas type	Ar 85%, CO2 15%			
	Gas flow rate [l/min]	15			
	Wire diameter (d_w) [mm]	1.2			
	Material	Carbon steel			
	Means of measure	3D scanner			
Cases reported in Figure 20		a	b	c	d
Deposition parameters	Multi-bead	–	–	–	–
	Multi-layer	✓	✓	✓	✓
	Wire feed rate (WFR) [m/min]	2	1.5	1.5	2
	Travel speed (TS) [mm/s]	5	4	5	5
	WFR/TS	6.7	6.3	5	6.7
Obtained bead sizes	Width (w_{av}) [mm]	5.40	4.35	3.57	4.68
	Height (h_{av}) [mm]	1.48	1.54	1.46	1.82
	h/w	0.27	0.35	0.41	0.39
	Deposited area (A_d) [mm ²]	7.54	7.07	5.65	7.54
	Measured area (A_m) [mm ²]	7.49	6.34	5.02	8.20
	$A_m - A_d$ [mm ²]	–0.05	–0.73	–0.63	0.66
	$(A_m - A_d)/A_d$	–0.7%	–10.3%	–11.2%	8.8%
Parabola model	A_p [mm ²]	5.33	4.47	3.47	5.68
	e_d	–29.3%	–36.8%	–38.6%	–24.7%
	e_m	–28.9%	–29.6%	–30.8%	–30.8%
Cosine model	A_p [mm ²]	5.09	4.26	3.32	5.42
	e_d	–32.5%	–39.7%	–41.3%	–28.1%
	e_m	–32.1%	–32.7%	–33.9%	–33.9%
Ellipse model	A_p [mm ²]	6.27	5.26	4.09	6.69
	e_d	–16.8%	–25.6%	–27.7%	–11.3%
	e_m	–16.2%	–17.1%	–18.5%	–18.5%
Proposed Hermite model	A_h [mm ²]	7.49	6.34	5.02	8.2
	Contact angle θ_1 [°]	90	90	90	90
	Contact angle θ_2 [°]	90	90	90	90
	e_d	–0.7%	–10.3%	–11.2%	8.8%
	e_m	0%	0%	0%	–0.1%

Source(s): Table by authors

C# language. As a result, in the Rhinoceros environment, the user has commands available to enter the parameters and positions of the beads, draw the Hermite profiles and update the substrate curve. The profiles obtained through the algorithm are represented in the figures by a continuous red curve, while the single deposited beads are depicted with a continuous green curve. A dash-dot blue curve highlights the position and direction of the torch wire.

In the application of the approach to multi-layer test cases, constant parameters have been used to determine the shape of the Hermite bead section, even though the approach is conceived to vary such parameters during the simulation of the deposition. The choice was made for the sake of simplicity and for the unavailability of detailed data to correlate shape parameters to the different process conditions.

4.1 Application to the single-bead case

Table 3 summarizes the results of the application of the proposed representation to the experiments reported in Kindermann *et al.* (2020). First, it is noted how A_d differs from

A_m , confirming the experimental uncertainties in the deposition parameters.

The error e_m turns to be close to zero demonstrating the accuracy of the proposed model in approximating the experimental bead sections under different conditions. The resulting curves are depicted in Figure 18. The upper part reports experiments with WFR equal to 10 m/min and TS respectively equal to 3.33, 6.67 and 13.33 mm/s. In the bottom part of the figure, TS has been kept constant, while WFR varied as reported.

4.2 Application to single-bead and multi-bead cases

Table 4 summarizes the key characteristics of the experiment proposed by Xiong *et al.* (2013) on single and multi-bead depositions with different WFR to TS ratio, thus leading to significant changes in the section shape. As before, A_m has been taken as a target for the Hermite model, so that the relative e_m turns to be close to zero.

Figure 19 confirms how the proposed model adapts well to various bead geometries due to its inherent flexibility.

Figure 21 Elaboration of specimen section obtained from GOM ATOS 200 3D blue-light scans

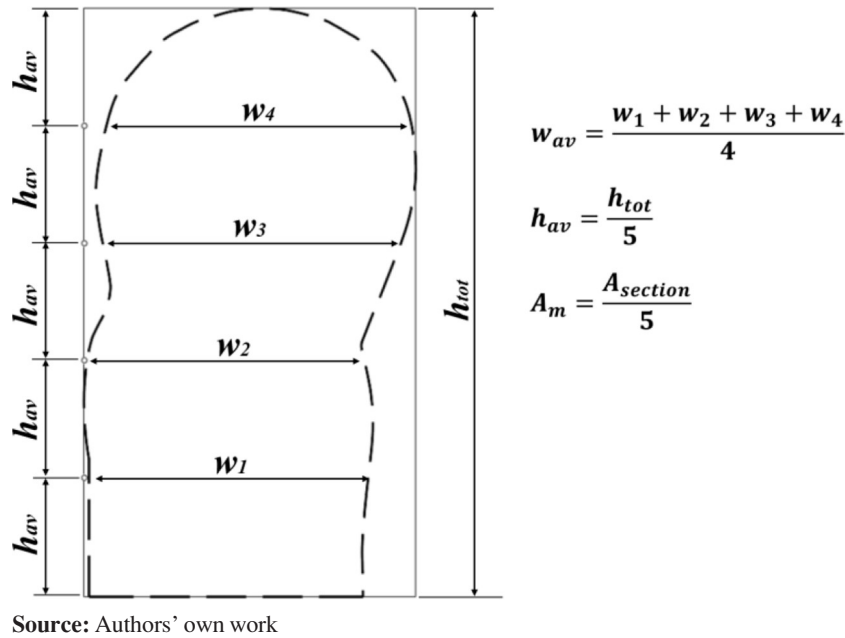
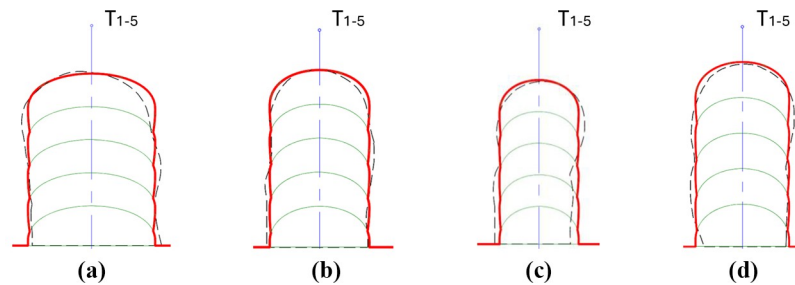


Figure 22 Results of the geometric approximation given by the proposed algorithm on a stack of five beads with varied deposition parameters. The dashed line is the section of the specimen



Moreover, Figure 20 illustrates the results of the algorithm for overlapping adjacent beads. In these cases, an average value for the area A_m has been drawn from the profile of the three beads. The distances between the deposition locations T_1 , T_2 and T_3 were available in the analyzed paper.

The results produced by the overlapping algorithm are promising. Although some differences in the sections can be observed, they can be further minimized by adjusting the values of the contact angles.

4.3 Application to multi-layer

The algorithm was applied to a multi-layer case based on an experiments developed in Lettori *et al.* (2024) and Lettori *et al.* (2025). Circular samples of 5 layers were developed by varying WFR and TS as reported in Table 5. Average height h_{av} , average width w_{av} , and average area A_m of the beads were estimated from sections of meshes obtained from 3D optical laser scans, as illustrated in Figure 21.

The proposed model leverages the closed profile of the bead and the iterative process to adjust the overall area A_m (Figure 15). These elements ensure a reliable outcome, showcasing the capability of the approach in reducing the

inherent discrepancies between the real bead profile and the generated one.

Figure 22 illustrates the four sections analyzed. As can be seen, the proposed model provides an accurate approximation of the actual profile, particularly in terms of the specimen height. The results can be further improved by adjusting the bead section parameters Γ from layer to layer in consideration of the varying heat dissipation conditions, which are more pronounced in the first layers in contact with the supporting plate.

4.4 Application to single-bead, multi-bead and multi-layer

The algorithm was applied to the case study developed by Ding *et al.* (2015). First, Table 6 summarizes the results concerning the deposition of a single bead.

Also in this case, the adoption of appropriate contact angles allows a good representation of the bead shape, as visible in Figure 23.

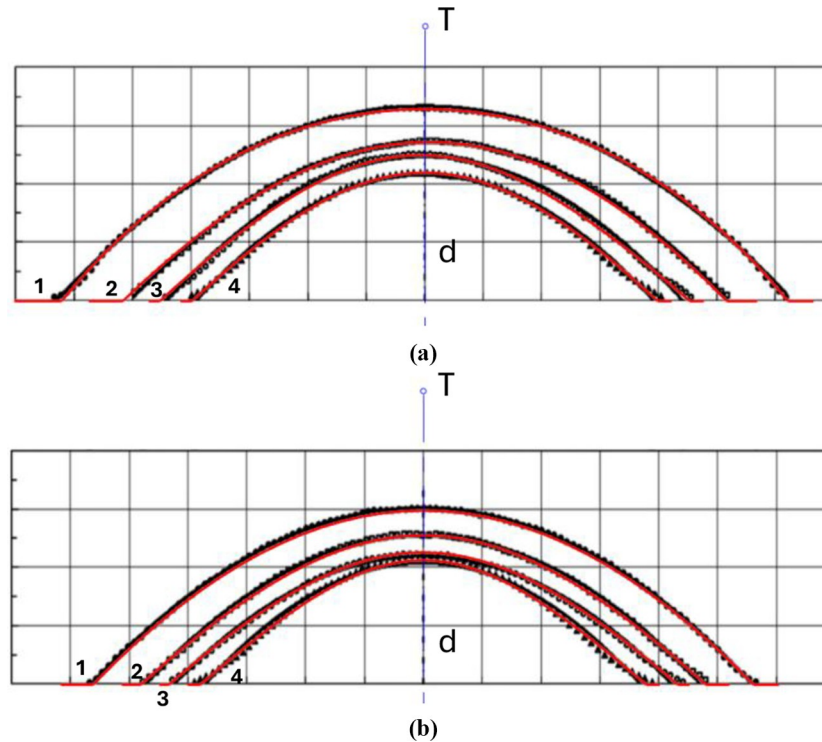
Subsequently, the algorithm was tested on multi-bead and multi-layer cases as in Table 7.

Figure 24 presents the results for two adjacent beads deposited at varying distances. The proposed model does not

Table 6 Summary of the experiments from Ding *et al.* (2015): single bead cases

Experimental setup	Gas type	Ar 82%, CO2 18%							
	Gas flow rate [l/min]	22							
	Wire diameter (d_w) [mm]	1.2							
	Material	Copper coated steel							
Means of measure	3D laser profiler (0.002 resolution)								
Cases reported in Figure 23		a1	b1	a2	b2	a3	b3	a4	b4
Deposition parameters	Multi-bead	—	—	—	—	—	—	—	—
	Multi-layer	—	—	—	—	—	—	—	—
	Wire feed rate (WFR) [m/min]	5	5	5	5	5	5	5	5
	Travel speed (TS) [mm/s]	3.33	4.17	5.00	5.83	6.67	7.50	8.33	9.17
Obtained bead sizes	WFR/TS	25	20	16.7	14.3	12.5	11.1	10	9.1
	Width (w_{av}) [mm]	12.45	11.33	10.36	9.67	9.06	8.73	8.06	7.67
	Height (h_{av}) [mm]	3.28	3.00	2.74	2.56	2.50	2.21	2.17	2.09
	h/w	0.26	0.26	0.26	0.26	0.28	0.25	0.27	0.27
	Deposited area (A_d) [mm ²]	28.27	22.62	18.89	16.17	14.14	12.55	11.31	10.29
	Measured area (A_m) [mm ²]	27.49	22.14	18.25	15.75	14.17	12.12	10.76	9.81
	$A_m - A_d$ [mm ²]	−0.78	−0.48	5.01	5.00	5.00	5.00	5.00	5.01
	$(A_m - a_d)/a_d$	−2.8%	−2.1%	26.5%	30.9%	35.4%	39.8%	44.2%	48.7%
Parabola model	A_p [mm ²]	27.22	22.66	18.92	16.50	15.10	12.86	11.66	10.69
	e_d	−3.7%	0.2%	0.2%	2%	6.8%	2.5%	3.1%	3.8%
	e_m	−1%	2.4%	3.7%	4.8%	6.6%	6.1%	8.4%	8.9%
Cosine model	A_p [mm ²]	26.00	21.64	18.07	15.76	14.42	12.28	11.13	10.21
	e_d	−8.1%	−4.3%	−4.3%	−2.6%	2%	−2.2%	−1.6%	−0.8%
	e_m	−5.4%	−2.3%	−1%	0.1%	1.8%	1.3%	3.5%	4%
Ellipse model	A_p [mm ²]	32.06	26.68	22.28	19.43	17.78	15.15	13.73	12.58
	e_d	13.4%	18%	18%	20.2%	25.8%	20.6%	21.4%	22.3%
	e_m	16.6%	20.5%	22.1%	23.4%	25.5%	25%	27.7%	28.3%
Proposed	A_h [mm ²]	27.50	22.13	18.25	15.75	14.16	12.12	10.77	9.81
Hermite model	Contact angle θ_1 [°]	51	46	44	42	41	40	42	40
	Contact angle θ_2 [°]	51	46	44	42	41	40	42	40
	e_d	−2.7%	−2.2%	−3.4%	−2.6%	0.2%	−3.5%	−4.8%	−4.7%
	e_m	0%	−0.1%	0%	0%	−0.1%	0%	0.1%	0%

Source(s): Table by authors

Figure 23 Approximation of the experimental sections of single beads reported by Ding *et al.* (2015)

Source: Figure adapted from Ding *et al.* (2015)

achieve the same level of accuracy as the approximation proposed by the original authors, as it was specifically designed to handle multiple adjacent bead passes. Discrepancies are primarily due to the surface tension effects of the molten drops, which are not adequately captured. This suggests that the algorithm needs to be improved in this direction, although it still shows a good level of approximation given its generality and applicability to a wider variety of cases.

Finally, Figure 25 shows the result of the algorithm applied to the multi-bead and multi-layer case study. As mentioned, the parameters Γ were kept constant for each bead. The algorithm can handle properly these complex configurations, and the results are highly encouraging. The ability to capture asymmetric heights of the samples depending on the deposition sequence is remarkable.

4.5 Application to multi-bead and multi-layer

The final case study reported in Table 8 and Figure 26 involves multi-bead and multi-layer specimens. The experimental profiles were obtained from the work of Li *et al.* (2018).

Figure 26 illustrates the results of the algorithm for two strategies based on different sequences of the deposition points T_b , i.e. alternating the direction at each layer (Figure 26) or maintaining the same direction (Figure 26). The algorithm demonstrates a high level of accuracy in estimating the growing trend of the layers profile in both cases. In the simulation, the parameters Γ were kept constant, and the distances between the points T were taken from the reference work. This consistency highlights the robustness of the algorithm in foreseeing the

effects of the deposition sequence, which appears as irregularities and asymmetries in the final shape.

4.6 Final remarks

The variability of the proposed cases is quite broad. Cases 1 and 3 were made using a Fronius CMT welding process, a technology widely adopted in WAAM. Unfortunately, all the experiments refer to 1.2 mm carbon steel, which proves to be a common choice regarding the material used. However, in the reported experiments *WFR* overall varies between 1.5 and 10 m/min, while *TS* varies between 2.5 and 13.33 mm/s.

It is observed that the proposed bead composition algorithm works on a simplified geometric basis. The bead shape deposition follows a principle based on Boolean shape operations that ensures the preservation of the area of the bead. For example, the complex melting effects due to temperature, which cause some of the observed discrepancies, are not fully captured. Minor discrepancies can be accepted provided with the higher level of genericity of the proposed modeling approach that can cope with much wider cases. Furthermore, the model allows for bead-to-bead section parameters to be adjusted, and this allows for more sophisticated thermophysical effects to be included, if adequate simulations are available.

Finally, the Hermite model requires more computational cost compared to other formulations in the literature, as some calculations cannot be performed analytically but require iterative evaluation. Properties and simplifications are introduced to easy calculations while still allowing for

Table 7 Summary of the experiments from Ding et al. (2015): multi-bead and multi-layer cases

Experimental setup	Gas type	Ar 82%, CO2 18%				
	Gas flow rate [l/min]	22				
	Wire diameter (d _w) [mm]	1.2				
	Material	Copper coated steel				
Means of measure	3D laser profiler (0.002 resolution)					
Cases reported in Figure 24		a	b	c		
Cases reported in Figure 25					a	b
Deposition parameters	Multi-bead	✓	✓	✓	✓	✓
	Multi-layer	–	–	–	✓	✓
	Wire feed rate (WFR) [m/min]	5	5	5	5	5
	Travel speed (TS) [mm/s]	6.67	6.67	6.67	8.33	8.33
	WFR/TS	12.5	12.5	12.5	10	10
Obtained bead sizes	Width (w) [mm]	9.55	8.68	9.44	7.62	8.00
	Height (h) [mm]	2.46	2.48	2.37	2.21	2.36
	h/w	0.26	0.29	0.25	0.29	0.30
	Deposited area (A _d) [mm ²]	14.13	14.13	14.13	11.31	11.31
	Measured area (A _m) [mm ²]	14.50	14.59	14.71	11.38	11.06
Parabola model	A _m – A _d [mm ²]	2.62	3.25	4.10	0.58	–2.25
	(A _m – a _d)/a _d	18.5%	23%	29%	5.1%	–19.9%
	A _p [mm ²]	15.66	14.35	14.92	11.23	12.59
	e _d	10.8%	1.6%	5.6%	–0.8%	11.3%
	e _m	8%	–1.6%	1.4%	–1.4%	13.8%
Cosine model	A _p [mm ²]	14.96	13.70	14.24	10.72	12.02
	e _d	5.9%	–3%	0.8%	–5.3%	6.2%
	e _m	3.2%	–6.1%	–3.2%	–5.8%	8.7%
Ellipse model	A _p [mm ²]	18.44	16.90	17.56	13.22	14.82
	e _d	30.5%	19.6%	24.3%	16.8%	31%
	e _m	27.2%	15.8%	19.4%	16.2%	34%
Proposed Hermite model	A _h [mm ²]	14.51	14.59	14.71	11.37	11.06
	Contact angle θ ₁ [°]	39	50	40	40	40
	Contact angle θ ₂ [°]	39	50	40	40	40
	e _d	2.7%	3.3%	4.1%	0.5%	–2.3%
	e _m	0.1%	0%	0%	–0.1%	0%

Source(s): Table by authors

considerable flexibility. The bead composition algorithm also introduces an iterative procedure for convergence. However, a few iterations are sufficient, and tests demonstrate that the process is rapid. Although in practical parts the required calculations must be repeated in many sections, the computational cost is comparable to standard geometric procedures in CAD systems and remains much lower than finite element approaches. Therefore, the approach is considered to represent a good compromise between performance and accuracy.

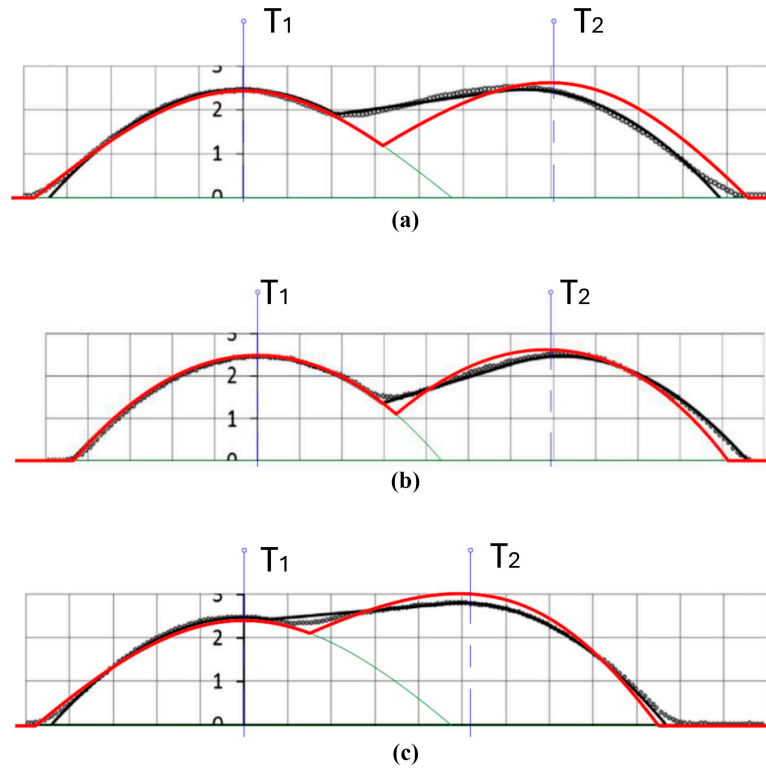
5. Conclusions and future directions

The paper proposes a novel geometric model for the 2D representation of weld bead profiles based on a closed curve formed by two Hermite curves. Unlike existing models in the literature, such as parabola, cosine or arc, the Hermite curve offers greater flexibility, representing various shapes that a weld bead may assume. The proposed model has also been implemented in an algorithm to estimate the interaction between adjacent beads and to handle multi-layer case studies. The algorithm was validated on significant case studies from

literature, including one study developed by the authors. The results show good geometric approximation, given the flexibility provided by the model. Furthermore, the outcomes of the algorithm handling the overlapping of multiple beads are encouraging, as multi-bead and multi-layer scenarios are successfully foreseen, also including effects due to the sequence of deposition.

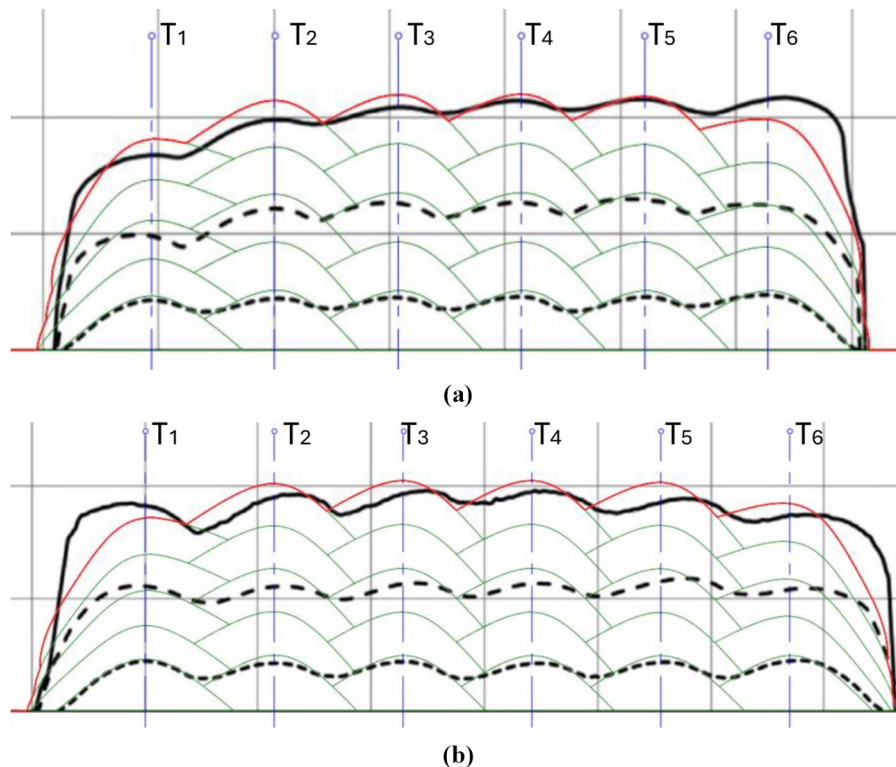
Future work includes the creation of correlations to identify the Hermite curve shape based on process parameters, such as *WFR*, *TS*, input power and substrate temperature resulting from heat dissipation. In practical applications, such knowledge would improve the ability of the approach to vary the bead profile parameters during deposition from layer to layer and, also, from one point to another. The proposed model and algorithm can also be extended to 3D cases to estimate the behavior of weld bead cross-sectional profiles and the interaction between the deposited material volumes. These contributions would enable a more comprehensive design of the deposition process and provide an accurate prediction of the resulting shapes, thus enabling optimized material deposition for near-net-shape while minimizing material waste.

Figure 24 Results on the experiments performed by [Ding et al. \(2015\)](#). The black curve is the result of the algorithm proposed in the original paper, the red curve the proposed algorithm in this paper, while the dots represent the measured profile of the specimen



Source: Figure adapted from [Ding et al. \(2015\)](#)

Figure 25 Results of the geometric approximation of the proposed algorithm considering the experiment (multi-layer) in [Ding et al. \(2015\)](#). The black dashed and continuous bold curves are the actual profile of the specimen

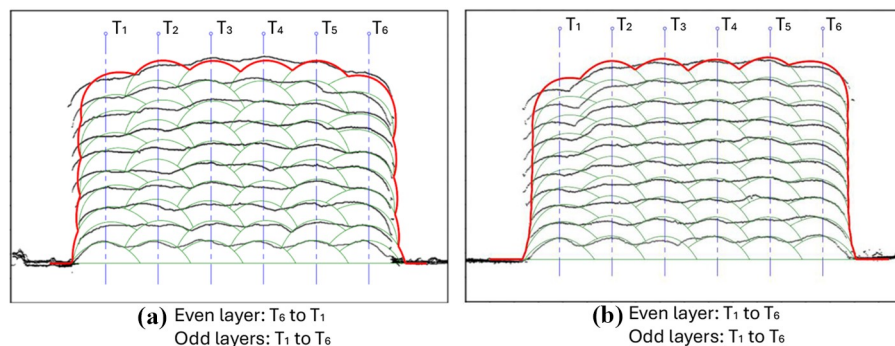


Source: Figure adapted from [Ding et al. \(2015\)](#)

Table 8 Summary of the experiment taken from Li *et al.* (2018)

Experimental setup	Generator	Panasonic YD-500FR	
	Gas type	Ar 95%, CO2 5%	
	Gas flow rate [l/min]	18	
	Wire diameter (d_w) [mm]	1.2	
	Material	Copper coated steel	
	Means of measure	Structured light sensor	
	Voltage [V]	22	
	Current [A]	150	
	Power [W]	3300	
Cases reported in Figure 26		a	b
Deposition parameters	Multi-bead	✓	✓
	Multi-layer	✓	✓
	Wire feed rate (WFR) [m/min]	3.73	3.73
	Travel speed (TS) [mm/s]	6	6
	WFR/TS	10.36	10.36
Obtained bead sizes	Width (w) [mm]	7.327	7.327
	Height (h) [mm]	2.304	2.304
	h/w	0.314	0.314
	Deposited area (A_d) [mm ²]	11.72	11.72
	Measured area (A_m) [mm ²]	10.82	10.7
Parabola model	$A_m - A_d$ [mm ²]	−0.90	−1.02
	$(A_m - A_d)/A_d$	−7.7%	−8.7%
	A_p [mm ²]	11.25	11.25
	e_d	−4%	−4%
	e_m	4%	5.2%
Cosine model	A_p [mm ²]	10.75	10.75
	e_d	−8.3%	−8.3%
	e_m	−0.7%	0.4%
Ellipse model	A_p [mm ²]	13.25	13.25
	e_d	13.1%	13.1%
	e_m	22.5%	23.9%
Proposed Hermite model	A_h [mm ²]	10.82	10.7
	Contact angle θ_1 [°]	57	57
	Contact angle θ_2 [°]	57	57
	e_d	−7.7%	−8.7%
	e_m	0%	0%
Source(s): Table by authors			

Figure 26 Results of the application to the experiment reported in Li *et al.* (2018) reproducing the original alternating deposition sequence. The black curves are the measured growing profiles of the specimen layer by layer



Source: Figure adapted from Li *et al.* (2018)

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