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Pickup and delivery with lockers

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ABSTRACT

We define a pickup and delivery routing problem with time windows that arises in last-mile delivery. A customer can be served either directly at home, by one of the available capacitated trucks, or via lockers, that allow a self-service option. On the same route, the couriers must deliver the parcels and collect the packages that the customers intend to return. The returned parcels can be picked up directly at the customers' homes or at a locker. Customers can select home service, self-service at one of the nearby lockers with a discount, or let the logistics company decide. All services must be performed within a given time window.

We propose three formulations, two branch-and-cut algorithms, and some valid inequalities. We also investigate the case with a single vehicle, with different types of time windows, including no time windows. Moreover, we show how to accommodate simultaneous pickup and delivery and multiple requests from a customer.

1. Introduction

Retail e-commerce sales worldwide have grown steadily from 2014 to 2021, with forecasts predicting that sales values will more than double in 2022 with respect to 2017 and will be more than three times as high in 2025 (see Statista (2022b) and Risberg (2022)). The main challenges introduced by the e-commerce boom that retailers and logistics companies must overcome are the need to quickly deliver a large number of parcels and the need to quickly collect returns at a minimum cost. To respond to these challenges, the aforementioned companies are implementing alternative delivery methods; one of these is the use of *lockers* (see Mitrea et al. (2020)) located in places that are convenient to the customers and in which couriers deliver the parcels that will be collected by the customers at a later time, or, in reverse, in which customers can drop their returns that couriers will collect at a later time. The introduction of lockers for last-mile deliveries and collections offers two main advantages, with respect to home service: the reduction of traveling costs and external costs (such as pollution, congestion, noise, etc.), due to the fact that more parcels are delivered and/or collected at the same place in batches, and thus the couriers can travel shorter routes. Moreover, the lack of a need for synchronization between the couriers and the customers allows more flexibility in both delivery and collection times, which hence favors a decrease in the number of failed deliveries and pickups. For these reasons, the introduction of lockers is gaining popularity among logistics and e-commerce companies (see, e.g., Lin et al. (2022)) and among customers: it has been shown that their introduction improves the online shopping experience (see, e.g., Vakulenko et al. (2018)). Indeed, according to the research on parcel lockers and last-mile delivery by JLL (2022) and to the study on logistics customer satisfaction by Logmore (2022), lockers are getting more attention thanks to the fact that "consumers have more control over when they pick up their shopping, rather

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than having to wait for deliveries or risk parcels being left in the wrong place'', lockers ''give customers greater flexibility over their shopping, especially as lockers located in transport hubs often offer public access'', and ''retailers want to enable a more seamless, easy delivery and parcel lockers are one part of the solution''. According to Ghaderi et al. (2022), lockers for pickup and drop-off are a ''flexible solution for different business modes and actors'', ''they can be used for parcels that cannot fit conventional mailboxes'', and they are ''easy to access by both the couriers and the customers''. The same studies report that distribution companies, online retail companies, but also grocery companies, among others, are eager to use parcel lockers, either in a collaborative way with other companies or by investing in their own locker networks (see e.g. Ghaderi et al. (2022)). Some examples are the following: in 2017, Deutsche Post DHL had more than 3400 lockers across Germany, meaning there were 340,000 compartments available to customers in 1600 cities and municipalities (see, e.g., Rabe et al. (2020), Morganti et al. (2014), and Symonds (2022)); in 2019, Decathlon started installing lockers in each of its 1500 stores worldwide (see, e.g., Bug et al. (2018) and Kerr and Różycki (2022)); Amazon currently has several Amazon Hub Lockers in over 900 cities in the US (see, e.g., Amazon (2022) and Kim and Wang (2022)).

If the parcels to be delivered account for the majority of the logistics operations due to online shopping, logistics and e-commerce companies have to deal with the problem of collecting returned parcels as well. According to the recent Statista's Global Consumer Survey (Statista, 2022a), between 40% and 50% of respondents in Western countries declared having returned at least one item purchased online in the 12 months prior to the survey that was conducted in 2021/2022, while this practice is even more common for Indian and Chinese users, with a 73% and 66% positive response to the same question, respectively. Moreover, the ReBound report on returns (rebound, 2022) shows that 25% of UK customers return between 5% and 15% of the items they buy online. Returns are very important and costly: in 2020, the amount of merchandise returned by consumers to retailers in the US was $428 billion (i.e., 10.6% of total sales) (see, e.g., Jena and Meena (2022)). For this reason, logistics and e-commerce companies also need to optimize returns and sometimes offer the possibility to buy online and return to stores (see, e.g., Yang and Ji (2022)) or to lockers (see, e.g., Vakulenko et al. (2018)).

In this paper, we tackle a problem that arises with the introduction of lockers for both deliveries and returns in the routes of the couriers as an alternative to the home service: the *pickup and delivery vehicle routing problem with lockers and time windows*, a last-mile problem in which identical capacitated vehicles perform both pickup and delivery services, either directly to the customers or to nearby capacitated lockers. When purchasing a service, the customers can select either home service, self-service at one of the nearby lockers, earning a discount, or let the courier company, and thus our model, decide between home service and self-service at one of the selected lockers. The problem aims to minimize the traveled distance and the cost incurred when applying a discount to the customer served via locker while respecting the given constraints. We study the case with two types of time windows: for home service only or for both home and self-service. We propose three formulations and valid inequalities. Two of the proposed formulations need to be solved via a branch-and-cut algorithm, described in the following. Furthermore, we simplify these models for the single vehicle case and the case with no time windows. We also show how to adapt our methods to solve simultaneous pickup and delivery problems with lockers and multiple requests from the same customer. In this paper, we propose, for the first time, compact models and methods to solve problems with lockers, including pickup and delivery, a homogeneous capacitated fleet, time-windows, and a single visit to all nodes.

In the remainder of the paper, we first propose a detailed literature review in Section 2. Section 3 provides an explanation of the problems considered. Several formulations are proposed to model these problems in Section 4. The implementation of the branch-and-cut algorithm is defined in Section 5, while Section 6 presents computational results comparing the proposed formulations and provides a comment on the impact of the introduction of lockers in the last-mile delivery process. A comparison with similar methods from the literature is shown in Section 7. Section 8 concludes the paper.

2. Literature review

From an academic point of view, despite the existence of papers that have studied the decoupling of couriers and customers with systems that are lockers alike (see, e.g., the modular bentobox proposed in Dell'Amico et al. (2011)) and some others that have suggested analyzing the use of lockers coupled with low environmental impact vehicles for urban freight transportation (see, e.g., Perboli and Rosano (2019)), most of the studies that consider the use of lockers have been published in the last five years. For a wide classification of recent studies, we refer the reader to the survey by Rohmer and Gendron (2020).

In the following, we report the main contributions of the literature that considers the use of lockers by first presenting papers that address problems that deal only with delivery services. We first analyze the cases where deliveries are performed with one single vehicle and then those that use multiple vehicles. After that, we consider papers that give importance to the service preference of the customers (service via lockers or other types of service). Eventually, we describe those papers that consider both pickups and deliveries, which is also the focus of this paper. For each paper, we describe the main characteristics of the studied problems, the methods used to solve them, and the largest instance size. To summarize this and to highlight the differences between our methods and those from the literature, in Table 1 we show a summary of the main characteristics of the relevant works. Column *#truck* shows if the problem solved makes use of a single vehicle (1) or multiple vehicles (+). For the following characteristics, we use (Y) if included and (N) if not considered. Column *heter.* shows if a heterogeneous fleet is considered, 2 *sets* indicates that two different sets of vehicles are used to visit lockers and customer homes. Columns *cap. truck* and *cap. lock.* show if trucks and lockers are capacitated, respectively. Under column *radius*, we report if the correspondent work considers a problem in which only customers within a certain radius can be served by a locker (*close* means that the customers that allow shipping at lockers can only use the closest one). Column *tw* indicates if time windows for pickup and/or delivery services are included. We also show if time windows are soft time windows or if they are imposed only for a subset of customers. Column *pref.* indicates whether customers can specify

Table 1
Summary of the main characteristics of the routing problems including *lockers* studied in the literature divided by paper.

Paper	#truck	heter.	cap. truck	cap. lock.	radius	tw	pref.	max time	P&D	Of	Form.	Method	Inst.
Jiang et al. (2019)	1		N	Y	Y	Y(soft)	N	Y/N	N	D + L	MILP	MILP, VND	E(60/6), H(100/10)
Oliveira and dos Santos (2020)	+	2 sets	Y	Y	Y	N	N	N	N	D	MILP	MILP, VND	E(75/10), H(150/10)
Orenstein et al. (2019)	+	N	Y	Y	N	N	Y(0–1) (lock.)	Y	N	D + T + F	MILP	MILP, C&W + TS, Petal + TS	E(200/20), H(1500/50)
Wen and Li (2016)	+	N	Y	N	N	N	N	N	N	D + L + T	ILP	C&W	H(12)
Dumez et al. (2021)	+	N	Y	Y	N	Y	Y	Y	N	D + T	MINLP	LNS	H(200)
Tilk et al. (2021)	+	N	Y	Y	N	Y	Y	Y	N	D + T	SP	B&P&C	E(60)
Mancini and Gansterer (2021)	+	N	N	Y(∞)	Y	Y	Y(0–1)	Y	N	D + L + T	MILP	MILP, LNS, ILS	E(50), H(75)
Grabenschweiger et al. (2021)	+	N	N	Y	Y(close)	Y	Y(0–1)	Y	N	D + L	MILP	MILP, ALNS	E(25), H(75)
Sitek and Wikarek (2019)	+	Y	Y	Y	N	N	Y(0–1)	Y	Y	D + L	ILP	ILP + CLP, NN + CLP	E(200), H(200)
Sitek et al. (2021)	+	Y	Y	Y	N	Y(some)	Y(0–1)	Y	Y	D + L	ILP	ILP + CLP, GA + CLP	E(200), H(200)
Yu et al. (2022)	+	N	Y	Y(∞)	N	Y	Y(0–1)	Y	Y	D	MILP	MILP, SA	E(25), H(100)
Buzzega and Novellani (2022)	1/+	N	Y/N	Y	Y	Y/N	N	Y/N	N	D + L	MILP	MILP, B&C	E(100)
This paper	+	N	Y	Y	Y/N	Y	Y(0–1)	Y/N	Y	D + L	MILP	MILP, B&C	E(100)

a preference for alternative delivery locations: (0–1) shows that the preference is defined as a yes or no answer for each delivery option, $lock.$ means that lockers are the only delivery option. Column $max\ time$ shows if a maximum time or length is imposed on the routes. Column Of shows the objective function cost components, being D the distance or transportation costs, L the locker service cost (or other type of location service cost) or the penalty for serving with alternative methods, T the truck cost or the labor cost, and F the cost of failed deliveries. Afterwards, we display the type of formulation presented (if any) and the method(s) used to solve the problem. Let us define the acronyms used in the table and also in the rest of the paper. ILP stands for integer linear programming, while MILP stands for mixed ILP, and MINLP stands for mixed integer non-linear programming. CLP stands for constraint logic programming. Among the constructive heuristics, C&W represents the Clarke and Wright Savings heuristic, NN the Nearest Neighbor one, and Petal the Petal heuristic. TS stands for Tabu Search, VND for Variable Neighborhood Descent, LNS for Large Neighborhood Search, ALNS for adaptive LNS, ILS for Iterated Local Search, and GA for Genetic Algorithm. B&C stands for Branch-and-Cut and B&P&C stands for Branch-and-Price-and-Cut. Eventually, we show the size of the instances solved by the exact (E) and heuristic (H) algorithms, being the first the number of customer nodes and the second the number of lockers. In the case where only one number is reported, it represents the most characteristic one.

2.1. Delivery problems with lockers and a single vehicle

The first works that consider the use of lockers for delivering parcels only (no pickup of returned parcels) make use of a single vehicle. Jiang et al. (2019) study the *traveling salesman problem with time windows for the last-mile delivery in online shopping*, a generalization of the *traveling salesman problem* (TSP) *with time windows* in which customers can either receive their parcels at home or at nearby lockers. Lockers have a maximum radius in which customers can be served and a maximum capacity. No preference between the two types of service is considered. However, the objective function aims to minimize the truck traveling costs, the traveling cost of the customer to reach the locker, and the cost of opening a locker. The authors propose a MILP formulation and a VND. The MILP formulation models hard time windows, however, in order to improve efficiency, the authors solve the case with soft time windows. The largest instance solved to optimality counts 60 customers and six lockers. Buzzega and Novellani (2022) propose a set of MILP formulations and B&Cs for a similar problem that they call the *vehicle routing problem with lockers*, which may or may not consider (hard) time windows and/or multiple vehicles. Lockers have a maximum radius and a maximum capacity. No preference is considered, but the cost of servicing a customer via locker is included in the objective function together with the traveling costs. The authors improve with respect to Jiang et al. (2019) by solving instances that include 100 customers and 10 lockers.

2.2. Delivery problems with lockers and multiple vehicles

The majority of the other works only considers multiple vehicles. Oliveira and dos Santos (2020) present a multiple vehicle problem in which one set of trucks is devoted to home delivery only and another set to servicing lockers. The problem aims to minimize routing costs, with the routing costs of the trucks servicing the lockers being cheaper than the others. The authors propose a MILP formulation and solve only three instances with 75 customers and 10 lockers exactly; however, they also introduce a VND metaheuristic based on the well-known C&W algorithm (see, e.g., Clarke and Wright (1964)). They solve instances with up to 150 customers and 10 lockers heuristically. Orenstein et al. (2019) study a vehicle routing problem (VRP) called the *flexible parcel delivery problem* for delivering parcels from the depot to the lockers. This work includes several differences with respect to the others. No customer can be served directly, but only via lockers. In fact, each parcel is associated with a set of possible locker destinations. Moreover, failed deliveries are allowed. Parcels have different sizes, and both lockers and trucks are divided into cells of different sizes. No more than one parcel can be accommodated in one cell, and a cell must be sufficiently large to contain a parcel. The objective is to minimize the traveling costs, the cost of using each truck, and the penalties incurred in the event of failed deliveries. The authors propose a MILP formulation and two metaheuristics based on the C&W and the Petal heuristics (see, e.g., Foster and Ryan (1976)), whose solutions are improved by a TS algorithm. They consider instances with up to 50 lockers and 1500 parcels, but they solve to optimality only three instances with 20 lockers and 200 parcels. Wen and Li (2016) compare two multiple vehicle delivery modes. The first case is traditional delivery, which is carried out by a group of tricycles, each of which delivers one parcel at a time. Soft time windows are considered in this case. The objective function includes the transportation cost, the emission cost, and the penalty for exceeding the time windows. The second case considers that each customer has a delivery locker in front of their home. The problem is defining the routes of multiple capacitated vans to deliver the parcels to the customers. No time window is considered in this case because parcels are delivered to lockers. The objective function minimizes the transportation and emissions costs of the vans, the cost of leasing the lockers, the labor cost, and the van purchase cost. The authors model the problem with MILP formulations and present a C&W-based heuristic to solve a real-world instance with 12 customers.

2.3. Delivery problems with service preference using lockers and multiple vehicles

The works that follow allow customers to express their service preferences to some extent, such as allowing delivery only to a specific set of lockers or choosing between home delivery and locker delivery. All these works consider multiple vehicles and limit the truck routes by a maximum duration. Dumez et al. (2021) present the *VRP with delivery options* (VRPDO), a VRP with time windows in which several delivery options such as home, workplace, locker, or car trunk are considered. Customers define their delivery preferences among the options. Both vehicles and lockers are capacitated, and lockers can be visited by more than one truck. The objective is to define the truck routes that minimize the traveling costs while guaranteeing a minimum level of satisfaction for the customers. The authors propose a MINLP formulation, but they solve the problem with a dedicated LNS, in which a set-partitioning is used to reassemble routes, that solved instances with up to 200 customers. Tilk et al. (2021) also solve the VRPDO. A homogeneous fleet of trucks can be used and a fixed cost is considered for each truck used. Furthermore, the traveling cost must be minimized and a certain level of service, depending on the preference of the customers, achieved. The authors propose a (B&C&P) algorithm and solve instances with up to 60 customers. Mancini and Gansterer (2021) propose the *VRP with private and shared delivery locations*, where customers can either be served at home within a given time window or at a locker, in both cases with an unlimited homogeneous fleet of uncapacitated trucks. Customers can select one of the two types of delivery services or let the company decide. The service via locker needs compensation to customers and thus it is a cost for the company. They define a maximum capacity for the lockers, but the used instances have lockers with a capacity larger than the number of customers. The authors propose a MILP model, a LNS-based matheuristic, and an ILS algorithm. They study different time windows preferences, different radiuses of service, and different compensation schemes. They could solve exactly most of the 50 customer instances and heuristic solutions are provided for the 75 customer instances. Grabenschweiger et al. (2021) propose the *VRP with heterogeneous locker boxes*, where the customers have multiple requests that can be delivered either at home (respecting a time window) or at a locker by an unlimited homogeneous set of uncapacitated trucks. Customers can be served by the closest locker (if they allow so), for which they receive compensation, which is a cost for the company. If the delivery happens at a locker, then all the parcels for the same customer are stored at the same locker. The authors consider the case with lockers having boxes of a single dimension or having different dimensions. They propose two MILP models, each of which considers one of the two last features and an ALNS-based metaheuristic. They could solve exactly most of the 25 customer instances, and heuristic solutions are provided for the 75 customer instances, improving some large solutions with respect to Mancini and Gansterer (2021).

2.4. Pickup and delivery problems with lockers

Despite the fact that returning parcels has become a more and more common habit among customers purchasing online, only a few papers consider using lockers also for picking up the returned packages. Sitek and Wikarek (2019) consider both pickup and delivery services, proposing a *capacitated VRP* (CVRP) called *CVRP with pick-up and alternative delivery*, in which heterogeneous vehicles are used to pick up items from or deliver items to customers at home, at a post office, or at a capacitated locker. Multiple items can be delivered to or picked up from the same customer. Vehicles must respect maximum capacity and maximum shift duration. The objective is to minimize the distance traveled by the trucks and the penalties linked to customer preferences. The preference is defined as a binary constant for each item and alternative delivery point. All types of node can be visited multiple times and by different vehicles. The authors propose an ILP and a NN-based heuristic algorithm, both enhanced by a CLP preprocessing. Sitek et al. (2021) extend the previously presented work by imposing time windows on a subset of delivery nodes. They extend the ILP model and propose a genetic algorithm, both improved by a CLP prepossessing. In both cases, they could solve instances with up to 20 vehicles, 200 delivery or pickup points, and 2000 items. Yu et al. (2022) solve the *VRP with simultaneous pickup and delivery and parcel lockers*, where a homogeneous, capacitated, and limited set of vehicles is used to service the customers either at home or at a locker by minimizing the travel cost. All customers have both a pickup and a delivery request and must be visited only once. A time window is imposed for those customers served at home. Customers can select home service, self-service at at most one of the lockers, or let the company decide. The authors propose a MILP model and a SA algorithm. The exact method could solve instances with up to 25 customers.

In this paper, we consider both pickups and deliveries performed by a homogeneous fleet of capacitated trucks, allowing the customers to indicate a preference between one or more lockers for self-service or home delivery, and considering different types of hard time windows for each service, as indicated by the customers. We propose three MILP formulations, two of which are solved via B&C algorithms. We propose several valid inequalities and procedures to separate dynamically those that are non-polynomial in number. A set of variations is considered, such as the single vehicle problem, the case without time windows, or the adaptation to the simultaneous pickup and delivery problem. Previous papers proposing exact methods for the single vehicle case or homogeneous multiple vehicle cases did not consider pickups. In those papers that considered pickups, heterogeneous vehicles are used, and nodes could be visited multiple times to respect the vehicles' capacity and the time windows, while we provide more compact models and algorithms for the problem with a homogeneous set of vehicles, where all nodes can be visited at most once. Moreover, we consider several types of time windows for the service of customers. We empirically demonstrate that our methods improve with respect to results from the literature.

3. Problems definition

In this section, we formally describe the problems that we will tackle in the remainder of the paper.

3.1. The pickup and delivery vehicle routing problem with lockers and time windows

The *pickup and delivery vehicle routing problem with lockers and time windows* (PDVRPLTW) is modeled on a digraph $G = (V, A)$. The set of vertices V is partitioned as $V = \{0\} \cup V_C \cup V_L$, where 0 represents the depot, $V_C = \{1, 2, \dots, n\}$ is the set of the n customers, and $V_L = \{n+1, n+2, \dots, n+m\}$ is the set of the m available lockers. The set of customers is also partitioned into the set of delivery and pickup customers, namely, $V_D = \{1, 2, \dots, n_d\}$ and $V_P = \{n_d + 1, n_d + 2, \dots, n\}$, such that $V_C = V_D \cup V_P$ and $V_D \cap V_P = \emptyset$. $A = \{(i, j) : i, j \in V\}$ is the given set of arcs, each of which is associated with a distance d_{ij} and a truck traveling time τ_{ij}.

The problem is to find the routes of an unlimited fleet of identical vehicles with a capacity Q to serve customers at a minimum cost. Each customer has a demand $q_i, i \in V_C$, $q_i > 0, i \in V_P$ for pickup customers and $q_i < 0, i \in V_D$ for delivery customers. We consider pickup and delivery customers as two separate sets, and one customer either requires a pickup or a delivery service. In the following, we show, however, how to accommodate simultaneous pickup and delivery customers or multiple requests from the same customer. The fleet size is not binding, although a maximum number of trucks can be imposed by just adding a limit on the arcs leaving the depot in the models we present in the following. The trucks used must leave from and return to the depot, and they can serve the customers either directly at home (home service) or via a set of lockers selected by the customers (self-service). A self-service (pickup or delivery) produces a fixed discount k for the customer and thus a cost of the same value that is included in the objective function. Each customer must be served, but both customers and lockers can be visited at most once by the trucks. Note that restricting the number of visits to the lockers to one is a typical feature of the relevant literature. Each locker $\ell \in V_L$ has a maximum capacity defined in number of cells c_ℓ. This means that a locker $\ell \in V_L$ can serve a maximum of c_ℓ delivery customers and c_ℓ pickup customers. The number of delivery and pickup customers are considered separately because the courier that visits the locker first empties the cells by picking up the pickup parcels and then can reuse the same cells to leave the parcels to be delivered. $V_L(i)$ is the set of lockers chosen by customer $i \in V_C$ as a possible delivery/pickup location, while V_H is the set of customers that can be served at home. Note that some customers may select more than one locker, but some may select none, and thus no self-service is possible for them. Some customers may allow both home and self-service, and some self-service only.

Each customer $i \in V_C$ must be served within a $[e_i, l_i]$ time window. A truck can arrive at a customer i before its earliest time window e_i, but it cannot serve it until e_i. The depots' earliest visit time e_0 is set to equal the start of the time horizon 0, so the truck cannot leave the depot before 0, while l_0 defines a maximum time that each route cannot exceed. Lockers have time windows $[e_\ell, l_\ell], \ell \in V_L$. Note that restrictive time windows for lockers can be used to model counters, i.e., man-operated lockers, typically within shops. In the following, we consider customer time windows in two ways: for home service only and for both home service and self-service.

The objective function to be minimized is made up of two components: the truck transportation costs and the cost (for the company) due to the discount guaranteed to the customers that collect or leave their parcels in a locker. Each one of these components is multiplied by a user-defined coefficient, which are α_1 and α_2, respectively.

3.1.1. Time windows for home service only

In the first version of the problem that we consider, the time windows must be respected (for both pickup and delivery services) only when the customer is served directly at home, but they do not need to be fulfilled when the customer is served via a locker. As shown in the objective function, in the latter case, the customers incur a discount, but they are required to collect the parcel at the locker after the truck visits it, or to leave the return parcel before the truck visits the locker. That information is shared with customers in advance.

3.1.2. Time windows for all type of services: home and self-service

In the second version, we replicate and extend the approach used in Jiang et al. (2019). In such a case, if a delivery customer is served at a locker, then the corresponding parcel must be delivered early enough to allow the customer to travel and collect it within her/his time window; for example, if customer $i \in V_D$ is assigned to locker ℓ, ℓ must be visited within $l_i - \tilde{\tau}_{i\ell}$, being $\tilde{\tau}_{i\ell}$ the time needed by the customer i to reach locker ℓ (provided by the customer or estimated by the company). Furthermore, if a pickup customer is served through a locker, then the corresponding parcel must be collected late enough to allow the customer to travel and leave it within his/her time window; for example, if customer $i \in V_P$ is assigned to locker ℓ, ℓ must be visited no earlier than $e_i + \tilde{\tau}_{i\ell}$.

We can thus define the PDVRPLTW *all services*, in which the time windows are considered when the customers are served both at home or via a locker, and the PDVRPLTW *home service*, in which the time windows are considered only when customers are served at home.

A feasible solution for the PDVRPLTWs is depicted in Fig. 1, in which the square is the depot, the circles are the delivery customers, the diamonds are the pickup customers, and the triangles are the lockers, which are represented together with the set of customers that can be served from them. The arrows represent the truck route, and the dashed arrows indicate customers collecting their parcels or leaving their returns at the lockers.

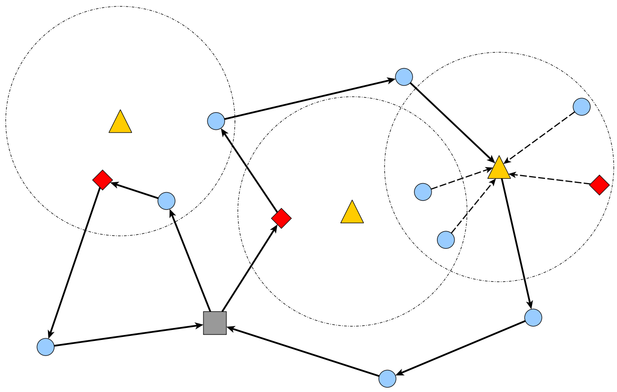


Fig. 1. A feasible solution for the PDVRPLTWs, considering that all customers are visited within their time window.

3.2. The pickup and delivery vehicle routing problem with lockers

We define the *pickup and delivery vehicle routing problem with lockers* (PDVRPL) as the version of the PDVRPTW in which customers must not be served within a given time window.

3.3. The single vehicle cases: the PDTSPL and the PDTSPLTWs

The single vehicle counterparts of the PDVRPL and of the PDVRPLTW are the *pickup and delivery traveling salesman problem with lockers* (PDTSPL) and the *pickup and delivery traveling salesman problem with lockers and time windows* (PDTSPLTW), respectively. In both cases, only one uncapacitated vehicle is used to perform pickups and deliveries. The truck is uncapacitated, and thus we do not need to consider customer requests. On the other hand, customers still need to be served either at home or via a locker. For the PDSTSPLTW, we can have two versions: the one in which time windows are considered only for the home service (PDSTPLTW *home service*), and the one in which time windows are considered also when the customers are served at the lockers (PDSTPLTW *all services*).

In Table 2, one can find a summary of all the symbols used.

4. Formulations

In this section, we first propose a set of different MILP formulations for the PDVRPLTWs and then we explain their adaptation to the PDVRPL and to the single vehicle cases.

4.1. Formulations for the PDVRPLTWs

In the following, we propose three MILP formulations for the PDVRPL.

4.1.1. The vehicle flow formulation

The first MILP formulation for the PDVRPLTWs that we present is called the *vehicle flow formulation* (VFF), being based on the well-known vehicle flow formulation for the VRP (see, e.g., Toth and Vigo (2014)). The VFF employs two sets of binary variables: variables x_{ij}, which equal 1 if the truck travels arc $(i,j) \in A$, 0 otherwise; and variables $y_{i\ell}$, which equal 1 if the customer $i \in V_C$ travels to locker $\ell \in V_L$ to collect her/his parcel or to leave her/his return, 0 otherwise. Note that $x_{ii} = 0$, variables x_{ij} and x_{ji} are set to 0 for all $j \in V$ if $i \notin V_H$, and variables $y_{i\ell}$ are set to 0 if $\ell \notin V_L(i)$. These settings are valid for this formulation and for those defined in the following. Moreover, a non-negative continuous variable t_i is used to represent the time at which the truck can leave the node $i \in V$, which can be either a customer or a locker. t_0 represents the return at the depot, because the starting time at the depot is fixed to $e_0 = 0$. We emphasize that if $e_0 > 0$, we can shift all time windows by e_0 and thus set $e_0 = 0$ without losing generality. Note that t is not the arrival time at a certain node because the truck is allowed to arrive and wait at a customer node to match its time window.

$$min\ z = \alpha_1 \sum_{i\in V}\sum_{j\in V} d_{ij}x_{ij} + \alpha_2 k \sum_{i\in V_C}\sum_{\ell\in V_L} y_{i\ell} \tag{1}$$

$$\sum_{j\in V} x_{ji} = \sum_{j\in V} x_{ij} \quad i \in V \tag{2}$$

$$\sum_{i\in V} x_{ij} + \sum_{\ell\in V_L} y_{j\ell} = 1 \quad j \in V_C \tag{3}$$

Table 2
Symbols used to describe and formulate the problems.

Sets	
V	Set of vertices, $V = \{0\} \cup V_C \cup V_L$
V_C	Set of customers, $V_C = \{1, 2, \dots, n\}$, $V_C = V_D \cup V_P$
V_D	Set of delivery customers, $V_D = \{1, 2, \dots, n_d\}$
V_P	Set of pickup customers, $V_P = \{n_d + 1, n_d + 2, \dots, n\}$
V_L	Set of lockers, $V_L = \{n + 1, n + 2, \dots, n + m\}$
$V_L(i)$	Set of lockers by which customer $i \in V_C$ agreed to be served
V_H	Set of customers that agreed to be served at home
A	Set of arcs, $A = \{(i, j) : i, j \in V\}$
S	A subset of customers and/or lockers
P	An elementary truck path $\{P(0), \dots, P(\|P\| - 1)\}$ that originates at the depot
$B_{P(h)}$	The set of customers served by a locker $P(h) \in V_L$ along a path P
P^-	A partial solution made of truck path P and customers sets $B_{P(h)}$ served by lockers $P(h) \in P \cap V_L$
$\mathcal{P}$	The set of all infeasible partial solutions P^-
Parameters	
n	Number of customers
n_d	Number of delivery customers
m	Number of available lockers
d_{ij}	Distance associated with each arc $(i, j) \in A$
τ_{ij}	Time needed by a truck to travel an arc $(i, j) \in A$
$\tilde{\tau}_{i\ell}$	Time needed by customer $i \in V_C$ to travel to locker $\ell \in V_L$
c_ℓ	Capacity of locker $\ell \in V_L$
k	Cost for performing a service to a locker
Q	Vehicles capacity
q_i	Request of customer $i \in V_C$, $q_i < 0, i \in V_D$ and $q_i > 0, i \in V_P$
$[e_i, l_i]$	Time window for vertex $i \in V$
α_1	Weight associated with the truck transportation costs
α_2	Weight associated with self-service costs
Variables	
z	Objective function value
x_{ij}	Binary variable that equals 1 if the truck travels arc $(i, j) \in A$, 0 otherwise
$y_{i\ell}$	Binary variable that equals 1 if the customer $i \in V_C$ travels to locker $\ell \in V_L$ to collect its parcel or to leave its return, 0 otherwise
f_{ij}	Non-negative continuous variable that represents the load of parcels of pickup customers on the truck on arc $(i, j) \in A$ in the TCF and the flow (both pickup and delivery) carried by the truck on the arc $(i, j) \in A$ in the OCF
g_{ij}	Non-negative continuous variable that represents the load of parcels of delivery customers on the truck on $(i, j) \in A$
t_i	Non-negative continuous variable that represents the time at which the truck can leave the node $i \in V_C \cup V_L$, which can be either a customer or a locker. t_0 represents the return at the depot

$$\sum_{j \in V} x_{j\ell} \leq 1 \quad \ell \in V_L \tag{4}$$

$$\sum_{j \in V} x_{j\ell} \leq \sum_{i \in V_C} y_{i\ell} \quad \ell \in V_L \tag{5}$$

$$\sum_{j \in V} x_{j\ell} \geq y_{i\ell} \quad i \in V_C, \ell \in V_L \tag{6}$$

$$\sum_{i \in V_P} y_{i\ell} \leq c_\ell \quad \ell \in V_L \tag{7}$$

$$\sum_{i \in V_D} y_{i\ell} \leq c_\ell \quad \ell \in V_L \tag{8}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - \sum_{i \in S \cap V_C} \sum_{\ell \in S \cap V_L} y_{i\ell} - \max \left\{ 1, \left\lceil \frac{|\sum_{i \in S \cap V_C} q_i|}{Q} \right\rceil \right\} \quad S \subseteq V_C \cup V_L \tag{9}$$

$$x_{P(0)P(1)} + \sum_{i=1}^{|P|-2} \sum_{j=i+1}^{|P|-1} x_{P(i)P(j)} + \sum_{P(h) \in P \cap V_L} \sum_{i \in B_{P(h)}} y_{i,P(h)} \leq \sum_{P(h) \in P \cap V_L} |B_{P(h)}| + |P| - 2 \quad P^- \in \mathcal{P} \tag{10}$$

$$x_{ij} \in \{0, 1\} \quad (i, j) \in A \tag{11}$$

$$y_{i\ell} \in \{0, 1\} \quad i \in V_C, \ell \in V_L \tag{12}$$

The objective function (1) minimizes two components, each one multiplied by its coefficient: the distance traveled by the trucks and the fixed cost of serving customers via locker. Constraints (2) ensure flow conservation at every vertex, while constraints (3) require that every customer is served either at home by a truck or at a locker. Constraints (4) impose that each locker is visited at most once. Constraints (5) guarantee that no arc enters a locker if it is not used. Constraints (6) impose that, if a locker is used, then one

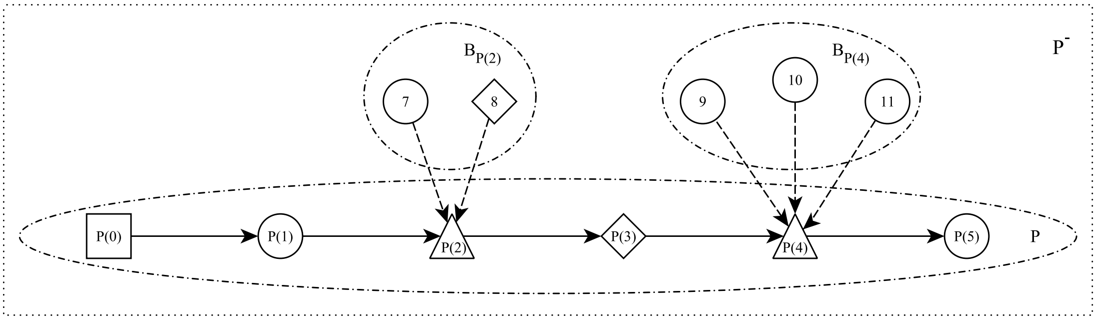


Fig. 2. Example of a partial solution.

arc must enter that locker. Constraints (7) and (8) impose a maximum capacity on the number of pickup and delivery customers, respectively, for each locker. In (9), we adapted the *generalized subtour elimination constraints* (GSEC) for CVRP (see, e.g. Toth and Vigo (2014)) to the studied problem. It is worth noticing that if we ignore the y variables, we obtain the GSEC that avoids subtours on subset S while imposing an upper bound on the number of customers in S visited by the truck starting from another vertex of S. In our case, if a customer $i \in S$ is served via a locker in S, it reduces by one the number of vertices that can be visited by the truck. This is guaranteed by the summation of the y variables on the right hand side of (9). However, the introduction of (9) is not enough to avoid infeasible solutions. Consider a partial solution given by the truck route (0,1,2,0), being 0 the depot, with $q_1 = 5$ and $q_2 = -7$ and $Q = 10$. The truck must start with a load of 7 from the depot to fulfill node 2, but after node 1 the load is $12 > Q$ and thus infeasible. In such a case, constraints (9) do not cut the infeasible route, because the order of visit is crucial; indeed, a route (0,2,1,0) would not be infeasible. This is why we need to introduce a set of constraints that eliminate infeasible partial solutions, as we explain in the following. To take care of this infeasibility, let P^- denote a partial solution composed of an elementary truck path $P = \{P(0), \dots, P(|P|-1)\}$ that originates at the depot and that can visit customers and lockers, and by a set of customers $B_{P(h)}$ served by each locker $P(h) \in P \cap V_L$ visited by the elementary path. An example of such partial solution is given in Fig. 2. For each $P(i) \in P$, let $Q(i) = \sum_{j\in\{1,\dots,|P|-1\}:q_i<0} |q_{P(i)}| + \sum_{P(h)\in P\cap V_L:h\le|P|-1} \sum_{j\in B_{P(h)}:q_j<0} |q_j| + \sum_{j\in\{1,\dots,i\}} q_{P(i)} + \sum_{P(h)\in P\cap V_L:h\le i} \sum_{j\in B_{P(h)}} q_j$ denote the overall truck load up to vertex $P(i)$. The first two terms account for the initial load of the truck to serve the delivery customers in P^-, including the delivery customers served via locker, while the following terms determine the truck load at $P(i)$. If for at least one node $P(i), i \in 1, \dots, |P|-1$, $Q(i) < 0$ or $Q(i) > Q$, the partial solution P^- is infeasible. Let $\mathcal{P}$ be the set of all the partial solutions with the described characteristics. If an infeasible partial solution is found, then we must impose that at least one of the binary variables which define the partial solution does not take value one. Since one part of the partial solution is made of an elementary path, we can thus write the 'tournament' constraint version for the path part in (10). Note that both (9) and (10) are introduced dynamically when violated (see Section 5), and (9) is checked before (10) so that only valid paths are considered by (10), and because its separation procedure is less computationally expensive. Variables x and y are defined as binary in (11) and (12). Constraints (2)–(5), (11), and (12) are derived from Buzzega and Novellani (2022).

Modeling the time windows for home service only. To model the PDVRPLTW *home service* we need to include the constraints that we present in the following.

$$t_j \ge t_i + \tau_{ij} - M(1 - x_{ij}) \quad i \in V_C \cup V_L, j \in V \tag{13}$$

$$t_j \ge \tau_{0j} - M(1 - x_{0j}) \quad j \in V \tag{14}$$

$$e_i \le t_i \le \min(l_i, l_0 - \tau_{i0}) \quad i \in V_C \cup V_L \tag{15}$$

$$t_i \ge 0 \quad i \in V \tag{16}$$

Constraints (13) update the time variables when an arc $(i,j) \in A, i \neq 0$ is traveled, and constraints (14) update the same variables when the arc starts from the depot. In these constraints, M is a sufficiently large number. In constraints (13), for example, M could be calculated as $l_i + \tau_{ij} - e_j$, $(i,j) \in A$. Constraints (15) strengthen the classical time window constraints $e_i \le t_i \le l_i$ for the customers and the lockers, respectively. In (16) the time variables are defined as non-negative. Constraints (13)–(16) are derived from Buzzega and Novellani (2022).

Modeling the time windows for all types of service. To model the PDVRPLTW *all services* we use the same non-negative variable t and add the following constraints to the time window constraints (13)–(16).

$$t_i \ge t_\ell + \tilde{\tau}_{i\ell} - M(1 - y_{i\ell}) \quad i \in V_D, \ell \in V_L \tag{17}$$

$$t_\ell \ge e_i + \tilde{\tau}_{i\ell} - M(1 - y_{i\ell}) \quad i \in V_P, \ell \in V_L \tag{18}$$

Constraints (17) state that the time at which the customer $i \in V_D$ is served by a locker ℓ is the time t_ℓ at which the truck leaves the locker plus the time needed by the customer to go to the locker $\tilde{\tau}_{i\ell}$. Constraints (18) state that the time t_ℓ at which the truck leaves the locker ℓ is the time at which the customer $i \in V_P$ leaves home plus the time needed by the customer to go to the locker $\tilde{\tau}_{i\ell}$.

In Appendices A and B, we show some polynomial inequalities that we introduced to strengthen this formulation; however, we report here the exponential inequalities that we used for the experiments because their separation procedures are included in the branch-and-cut algorithm presented in Section 5.

Exponential inequalities. We include the following inequalities (19) and (20) to avoid infeasible solutions that GSEC is not able to cut, in which the capacity is exceeded because the model uses the pickup parcels for deliveries (see detailed explanation in Section 4.1.1). Inequalities (19), (20) respectively, are a GSEC version in which only delivery, respectively, pickup, customers are considered when computing the lower bound on the number of trucks required in the considered subset S. This allows us to cut infeasible solutions that (9) cannot cut. Consider a set $\{1,2,3,4\}$ with $q_1 = q_3 = q_4 = -1$ and $q_2 = 1$ with $Q = 2$. This set is infeasible, no matter which path is used to visit it, because the truck needs to leave the depot with a load of $3 > Q$ to fulfill the requests of the pickup nodes 1, 3, and 4. Constraints (9) do not cut the infeasible solution, but inequalities (19) do. A similar example could be used to show the validity of constraints (20). These two inequalities thus avoid a more computationally expensive separation of inequalities (10), whose introduction is done dynamically. However, not all infeasible solutions are cut by (19) and (20), and thus the need for inequalities (10) should be verified as explained in Section 5.

$$\sum_{i\in S}\sum_{j\in S} x_{ij} \le |S| - \sum_{i\in S\cap V_C}\sum_{\ell\in S\cap V_L} y_{i\ell} - \left\lceil \frac{\sum_{i\in S\cap V_D}|q_i|}{Q} \right\rceil \quad S \subseteq V_C \cup V_L \tag{19}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ij} \le |S| - \sum_{i\in S\cap V_C}\sum_{\ell\in S\cap V_L} y_{i\ell} - \left\lceil \frac{\sum_{i\in S\cap V_P} q_i}{Q} \right\rceil \quad S \subseteq V_C \cup V_L \tag{20}$$

In Appendix C, one can find the summary of components needed to model the VFF to represent the PDVRPLTWs.

Modeling the customers with multiple requests and simultaneous pickup and delivery. Let us briefly discuss the case in which a customer requires two services, a pickup and a delivery, to be fulfilled simultaneously. We can accommodate this feature in the VFF and in the following formulations by expressing the same customer as two nodes, $i_P \in V_P$ for the pickup and $i_D \in V_D$ for the delivery, with the same location. The distance and the time between these two nodes equal 0. The customer can express several preferences as follows. Note that the previous variable settings, depending on the customer's requests, are still valid and we do not report those cases once more here.

(i) If the customer wants to be served at home for both services within the same time window, we can thus include the constraints $x_{i_D i_P} = 1$ and $x_{i_P i_D} = 0$ in the proposed formulations to impose that the two services are done one after the other. Note that by imposing the delivery to be performed before the pickup, we never incur an infeasible solution.

(ii) If the customer wishes to be served at a locker, and specifically at the same locker for both delivery and pickup, then the following constraint must be imposed $y_{i_D\ell} = y_{i_P\ell}, \ell \in V_L$.

(iii) In case the customer wants to let the model decide between home service and self-service and, in both cases, the two services must be done simultaneously, then the following constraints must be imposed:

$$x_{i_D i_P} = 1 - \sum_{\ell\in V_L} y_{i_D\ell} \tag{21}$$

$$y_{i_D\ell} = y_{i_P\ell} \quad \ell \in V_L \tag{22}$$

$$x_{i_P i_D} = 0 \tag{23}$$

(iv) In the event that a customer allows the pickup and delivery services to be done non-simultaneously and at different locations, even with different time windows, then the node must be doubled, but no additional constraint must be added to formulations. However, the distance and the time between the pickup and the delivery nodes that represent the same customer must be set to 0.

If a customer has multiple requests (multiple deliveries or multiple pickups), to respect our original problem definition that allows at most a single visit to each node, we can sum up all the delivery (pickup) requests and consider them as a single one. However, another possibility is to split the customer into several nodes, one for each request, with the same location. This depends on the needs of the customer who, in this latter case, can express different time windows and preferences for different requests.

Note that all these constraints and modifications can be applied to the following formulations as well.

4.1.2. The two-commodity formulation

In this section, we present another MILP formulation that is based on a two-commodity flow: one for the parcels of the pickup customers and one for those of the delivery ones. We call this formulation the *two-commodity formulation* (TCF). The TCF inherits the variables x, y, and t from the VFF but it also uses the non-negative variables f_{ij} and g_{ij} to represent the load of parcels of pickup, delivery, customers on the truck on arc $(i,j) \in A$, respectively.

The TCF for the PDVRPLTW *home service* is given by (1)–(8), (11)–(16) and by the following constraints (24)–(36), while the TCF for the PDVRPLTW *all services* needs also constraints (17) and (18).

$$\sum_{i\in V_C\cup V_L} f_{i0} = \sum_{i\in V_P} q_i \tag{24}$$

$$\sum_{i\in V_C\cup V_L} f_{0i} = 0 \tag{25}$$

$$\sum_{i\in V_C\cup V_L} g_{0i} = -\sum_{i\in V_D} q_i \tag{26}$$

$$\sum_{i\in V_C\cup V_L} g_{i0} = 0 \tag{27}$$

$$f_{ij} + g_{ij} \le Qx_{ij} \quad (i,j)\in A \tag{28}$$

$$\sum_{j\in V} f_{ji} = \sum_{j\in V} f_{ij} - q_i(1 - \sum_{\ell\in V_L} y_{i\ell}) \quad i\in V_P \tag{29}$$

$$\sum_{j\in V} f_{ji} = \sum_{j\in V} f_{ij} \quad i\in V_D \tag{30}$$

$$\sum_{j\in V} f_{j\ell} = \sum_{j\in V} f_{\ell j} - \sum_{i\in V_P} q_i y_{i\ell} \quad \ell\in V_L \tag{31}$$

$$\sum_{j\in V} g_{ji} = \sum_{j\in V} g_{ij} - q_i(1 - \sum_{\ell\in V_L} y_{i\ell}) \quad i\in V_D \tag{32}$$

$$\sum_{j\in V} g_{ji} = \sum_{j\in V} g_{ij} \quad i\in V_P \tag{33}$$

$$\sum_{j\in V} g_{j\ell} = \sum_{j\in V} g_{\ell} - \sum_{i\in V_D} q_i y_{\ell} \quad \ell\in V_L \tag{34}$$

$$0 \le f_{ij} \le Qx_{ij} \quad i,j\in V \tag{35}$$

$$0 \le g_{ij} \le Qx_{ij} \quad i,j\in V \tag{36}$$

Constraint (24) imposes the pickup flow returning to the depot to equal the sum of all the pickup requests, while the flow leaving the depot is set to zero in (25). Constraint (26) imposes the delivery flow leaving the depot equal to the sum of all the delivery requests, while constraint (27) imposes the delivery flow entering the depot to be null. On each arc used by the truck, the sum of the two flows must not exceed the capacity of the truck as per constraints (28). The same constraints impose both flow variables to be zero if the correspondent arc is not used. Constraints (29) are the flow conservation constraints for pickup customers. Note that if a customer is not served directly at home, these constraints do not impose the flow to fulfill the request. The same happens for delivery customers in constraints (30). Constraints (31) guarantee pickup flow conservation for those customers served via locker. Constraints (32)–(34) are the equivalent constraints for the delivery flow. Non-negativity and a maximum capacity are imposed on variables f and g in (35) and (36), respectively.

Inequalities to strengthen the TCF for the PDVRPLTWs are described in Appendices A and B. The complete TCF for all versions of the problem is summarized in Appendix C.

4.1.3. The one-commodity formulation

The third formulation that we propose, which we call the *one-commodity formulation* (OCF), uses only one set of flow variables. This formulation includes variables x and y, as well as their initial settings, time variables t, and a non-negative variable f_{ij}, which represents the flow carried by the truck on the arc $(i,j)\in A$. The OCF for the PDVRPLTW *home service* is given by (1)–(16), (24), (35) and by the following constraints (37)–(39), while the constraints (17) and (18) must be added to model the PDVRPLTW *all services*. Note that exponentially many constraints (10) are needed to ensure that the parcels picked up along the route are not used to fulfill deliveries, as it could happen by using only one flow variable. Note also that subtours can occur because a set of pickup customers could fulfill the requests of delivery customers in a loop, and thus exponentially many subtour elimination constraints are needed. We use the strengthened version of GSEC (9).

$$\sum_{i\in V_C\cup V_L} f_{0i} = -\sum_{i\in V_D} q_i \tag{37}$$

$$\sum_{j\in V} f_{ji} = \sum_{j\in V} f_{ij} - q_i(1 - \sum_{\ell\in V_L} y_{i\ell}) \quad i\in V_C \tag{38}$$

$$\sum_{j\in V} f_{j\ell} = \sum_{j\in V} f_{\ell j} - \sum_{i\in V_C} q_i y_{i\ell} \quad \ell\in V_L \tag{39}$$

In constraints (37), we impose that the flow exiting the depot, which is also the sum of all the load on the trucks leaving the depot, must equal the sum of all the requests of the delivery customers, with the sign changed, being that the delivery requests are negative by definition, because that is the amount of requests that must be carried from the depot to the customers. Constraints (38) ensure flow conservation for each customer who is served directly at home; in fact, if the customer is served directly by the truck, no variable $y_{i\ell}$ associated with that customer takes value one. Thus the value inside the parenthesis becomes one, and the constraint left states that the difference between the entering and the exiting flows must be the customer requests q_i with sing changed, in order to respect the different signs of the delivery and pickup requests. If, on the other hand, the customer is served with a locker, one $y_{i\ell}$ variable takes value one, the value of the parenthesis is zero, and the sum of the entering and exiting flows at customer node i is zero, indicating that the truck has left no load at that node. In this case, the request of node i must be left at the locker ℓ, for which $y_{i\ell} = 1$. Constraints (39) guarantee the flow conservation for the lockers by imposing that the difference between entering and exiting flows at a locker is either zero if no customer is served from that locker, or the sum of all requests of the nodes served by that locker with changed sign.

Inequalities to strengthen the OCF for the PDVRPLTWs are described in Appendices A and B. A summary of the components needed to adapt the OCF to solve all the versions of the problem is presented in Appendix C.

4.2. Modeling the PDVRPL

The three proposed formulations can be adapted to model the PDVRPL by removing the time variables t and the time windows related constraints. A summary of the formulations for the PDVRPL is reported in Appendix C, and inequalities used to strengthen the formulations are shown in Appendix A.

4.3. Modeling the single vehicle cases

In Appendix D, we report the adaptation of the presented formulations to the single vehicle cases. Among the new constraints required for this adaptation, a GSEC version for the TSP is needed. The new GSEC (40) is justified because, in the single vehicle versions, only one uncapacitated truck is available, and thus we can proceed without considering the requests of the customers. We only need to state that a customer is visited by the truck or served via a locker. We report these constraints explicitly here because we will define their separation in the next section.

$$\sum_{i\in S}\sum_{j\in S} x_{ij} \le |S| - \sum_{i\in S\cap V_C}\sum_{\ell\in S\cap V_L} y_{i\ell} - 1 \quad S \subseteq V_C \cup V_L, S \neq \emptyset \tag{40}$$

5. Branch-and-cut implementation

The OCF and the VFF include an exponential number of constraints, and hence we solve them with a B&C algorithm; on the contrary, the TCF does not need a dynamic introduction of constraints. We use the B&C framework of the solver used (CPLEX 20.10.0), which solves at every node of the enumeration tree the linear relaxation of a MILP model and then invokes user-developed separation procedures to possibly add cuts. We adopted strong branching as the branching rule for all the formulations. At the root node of the B&C procedure, we include an initial solution obtained by using a simple greedy heuristic. Note that it was not always possible to obtain a feasible initial solution for the PDTSPLTWs because of the limitation of having one vehicle and the time window constraints. This is not the case for the PDTSPL, PDVRPL, and PDVRPLTWs, for which it was always possible to build an initial greedy solution.

5.1. Separation procedures

The purpose of this section is to present the procedures that we use to determine whether a given (possibly fractional) solution $(\bar{x}, \bar{y})$ violates the exponentially many constraints included in the formulations and the valid inequalities that we proposed.

5.1.1. Separation S1

To separate GSEC (40), we extend the separation procedure used in Buzzega and Novellani (2022) as follows: we begin by constructing a supporting graph $\overline{G} = (V, \overline{A})$, where $\overline{A} = \{(i,j) \in A : (\bar{x}_{ij} > 0) \vee (\bar{y}_{ij} > 0, i \in V_P, j \in V_L) \vee (\bar{y}_{ji} > 0, j \in V_D, i \in V_L)\}$, and a capacity $\bar{x}_{ij} + \bar{y}_{ij:i\in V_P, j\in V_L} + \bar{y}_{ji:j\in V_D, i\in V_L}$ is assigned to arc $(i,j) \in \overline{A}$.

$O(n)$ max flows are computed using the depot as the source and any vertex $i \in V_C$, in turn, as a sink. If the optimal max flow value obtained is less than one, i is disconnected from the depot, and thus the cut that corresponds to the set S induced by the min cut may be added to the model. Moreover, for the multiple vehicle cases, we obtain a simple algorithmic improvement by checking, for any set S induced by a min cut, if the minimum number of vehicles required to serve S is lower than the max flow value obtained, and, in such a case, we add the corresponding (stronger) (9). In this way, the exact separation of the GSEC (40) is used as a heuristic for separating (9).

5.1.2. Separation S2

To separate (19), we start by adding an auxiliary node a to graph $\overline{G}$, obtaining a new supporting graph G_D, in which a set of arcs (i, a) links the node a to every node $i \in V_D$ with a capacity $|q_i|/Q$. We thus compute the max flow on G_D with the depot as the source and the node a as the sink. The constraint (19) corresponding to the set S induced by the min cut is then checked and added to the model if it is violated.

5.1.3. Separation S3

To separate (20), we begin by adding an auxiliary node a to graph $\overline{G}$, obtaining a new supporting graph G_P, in which a set of arcs (a,i) connects the node a to every node $i \in V_P$ with a capacity q_i/Q. We thus compute the max flow on G_P with node a as the source and the depot as the sink. The constraint (20) corresponding to the set S induced by the min cut is then checked and added to the model if it is violated.

5.1.4. Separation S4

To separate (9), we first add two auxiliary nodes, a_1 and a_2, to graph $\overline{G}$, resulting in a new supporting graph G' in which a set of arcs (a_1,i) links the node a_1 to every node $i \in V_P$ with a capacity q_i/Q and a set of arcs (i,a_2) links every node $i \in V_D$ to the node a_2 with a capacity $|q_i|/Q$. The depot is associated with the sum of all demands $q_0 = -\sum_{i \in V_C} q_i/Q$. If $q_0 < 0$, then an arc $(0,a_2)$ is created having capacity $|q_0|$; if $q_0 > 0$, then an arc $(a_1,0)$ is created with capacity q_0. We thus compute the max flow on G' with node a_1 as the source and a_2 as the sink. The constraint (9) corresponding to the set S induced by the min cut is then checked, and, if violated, is added to the model.

5.1.5. Separation S5

Constraints (10) are separated exactly by generating all possible paths that start from the depot by using a depth-first strategy on graph $\overline{G}$. We initialize path P with $P(0) = 0$, then select the outgoing arc with the largest positive $\overline{x}$ value and extend the path to include the head of the selected arc. Every time we add a vertex to the path, we check if it is infeasible with respect to the load carried on the truck. If so, then we add the cut; otherwise, we continue extending the path. If the new arc's head points to a locker, we add it to the path, as well as all the arcs represented by positive $\overline{y}$s for those customers served by that locker, and then we check whether the path is infeasible, and if so, add the cut. The path extension continues as long as the sum of the involved $\overline{x}$ and $\overline{y}$ is large enough to possibly lead to a violated cut. When this condition is not satisfied, we backtrack to the previous vertex in the path. Anytime we backtrack, we continue the depth-first search by selecting the next arc with the largest positive value of $\overline{x}$, if any, and then we extend the path consequently. The overall time complexity of the procedure when separating integer solutions is $O(|\overline{A}|)$. While separating fractional solutions requires a higher time complexity, since we separate them exactly.

The described separation procedures are invoked at every node of the enumeration tree for the VFF and the OCF when solving the PDVRPLTWs and the PDVRPL. On the basis of computational evidence, we stop the separation process as soon as we find and add a violated cut, if any. The separation procedures are invoked in the order in which they are described for the following reasons: $S1$ is invoked first because it works as a heuristic for separating (9) and because, once invoked $S1$, the following separation procedures work on solutions that do not present subtours; despite separating exactly (19) and (20), respectively, $S2$ and $S3$ are also faster heuristics for separating (9) than $S4$; $S2$ is positioned before $S3$ because the chance of exceeding the capacity with delivery requests ($S2$) is higher than doing so with pickup requests ($S3$), due to their limited number; procedure $S4$ separates exactly (9) and thus allows $S5$ to look for infeasible paths only when strictly needed, being that $S5$ is the computationally most expensive of all the procedures.

The separation procedures are applied to integer and fractional solutions, but in the last case, only for nodes in the enumeration tree with a depth less than or equal to 8.

When solving the PDTSPLTWs and the PDTSPL with the VFF and the OCF, we only need to include separation $S1$.

6. Computational results

In this section, we show the computational results obtained by running the proposed methods on a single thread of an Intel Core i3-2100 @ 3.10 GHz CPU with 8.00 GB of RAM. The time limit was set to two hours for each method and instance. All our methods were implemented in C++ and CPLEX 20.10.0 was used as the MILP solver.

6.1. Benchmark instances

Routing problems with lockers are very recent, and a standard set of benchmark instances has not been established yet. We make use of the instances used by Jiang et al. (2019), who adapted 40 benchmark instances for the TSP with time windows to the TSPLTW by including a set of locker nodes in the graph. In particular, they used two sets: the first set, that here we call *Set1*, with time windows of amplitude 20, that was initially proposed by Dumas et al. (1995); and the second set, with time windows of 100, proposed by Gendreau et al. (1998) and Dumas et al. (1995), that we call *Set2*. Each of these sets is made of four groups of five instances, with the same number of nodes $n = 20, 40, 60, 100$ (note that for the problems with time windows we will only use instances with up to 60 customers). Hence, *Set1* and *Set2* are made up of 20 instances each. The number of added lockers is $m = \lceil n/10 \rceil$. Each instance is modified to include m lockers by adding the first m nodes of the following set of coordinates: $(25.0, 25.0)$, $(12.5, 12.5)$, $(37.5, 37.5)$, $(12.5, 25.0)$, $(37.5, 12.5)$, $(12.5, 25.0)$, $(25.0,\ 37.5)$, $(37.5,\ 25.0)$, $(25.0,\ 12.5)$, and $(0, 0)$. We chose the last $\lceil 0.2 \cdot n \rceil$ nodes from the n available as pickup customers, and the rest as delivery customers. This is justified by the fact that the number of returns is generally smaller than that of deliveries. The distance matrix d_{ij} is computed as by Gendreau et al. (1998) as a Euclidean distance between each couple of nodes $(i,j) \in A$ rounded to the second decimal and corrected so to respect the triangular inequality in the following way: if $d_{ik} > d_{ij} + d_{jk}, i, j, k \in V$ then we set $d_{ik} = d_{ij} + d_{jk}$. The traveling times are set to equal the distance: $\tau_{ij} = d_{ij}, (i,j) \in A$ and $\tilde{\tau}_{i\ell} = d_{i\ell}, i \in V_C, \ell \in V_L$. For all instances, we used the data setting of Jiang et al. (2019): $\alpha_1 = 1$, $\alpha_2 = 0.5$,

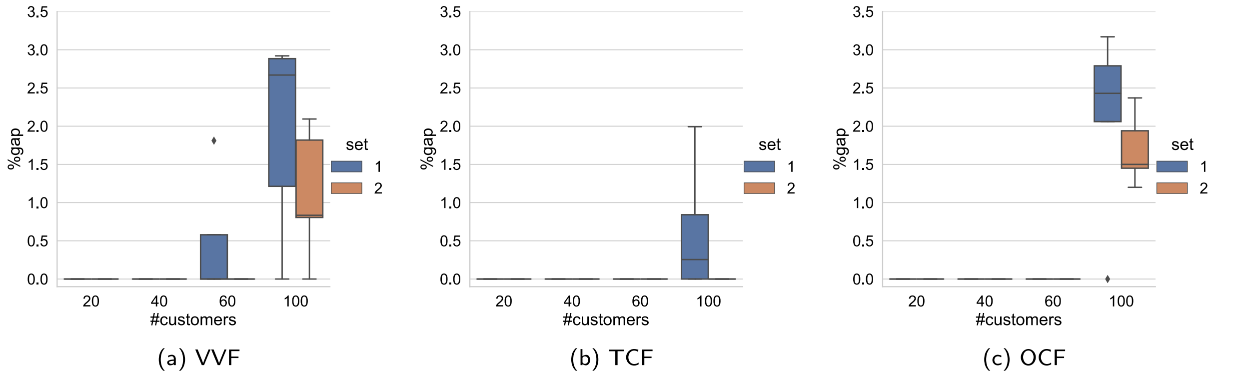


Fig. 3. Boxplots displaying the distributions of the percentage gaps when solving the two sets of the PDTSPL instances with the three formulations proposed, aggregated by number of customers.

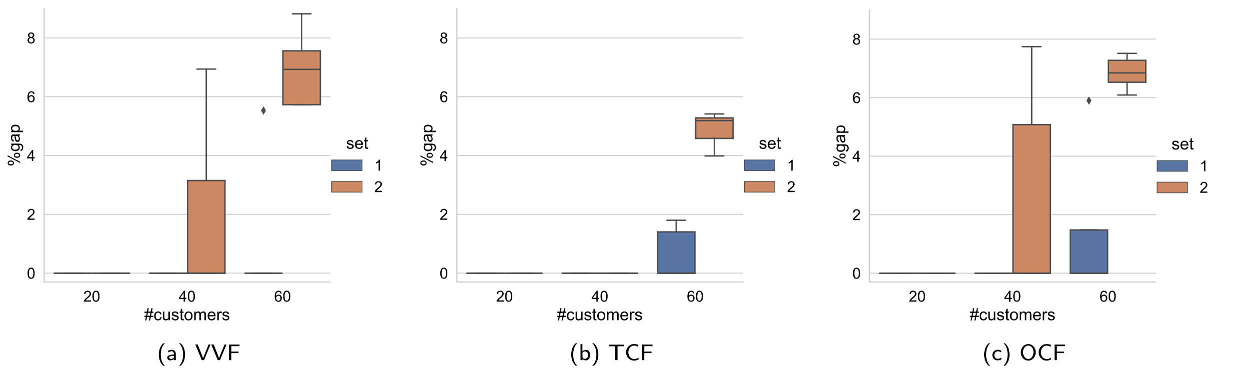


Fig. 4. Boxplots displaying the distributions of the percentage gaps when solving the two sets of the PDTSPLTW *home service* instances with the three formulations proposed, aggregated by number of customers.

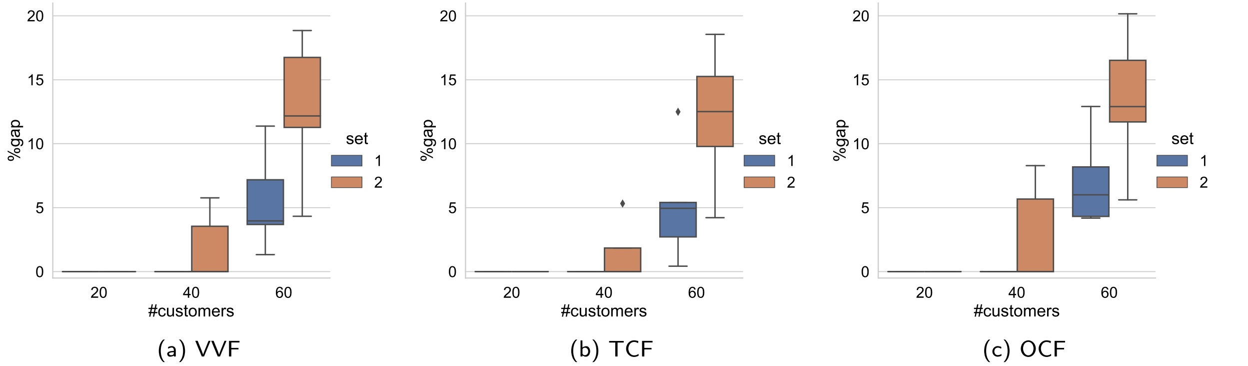
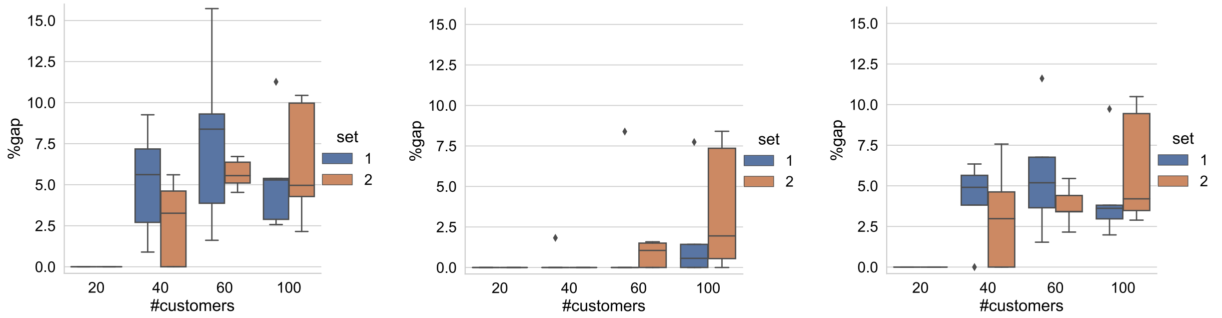


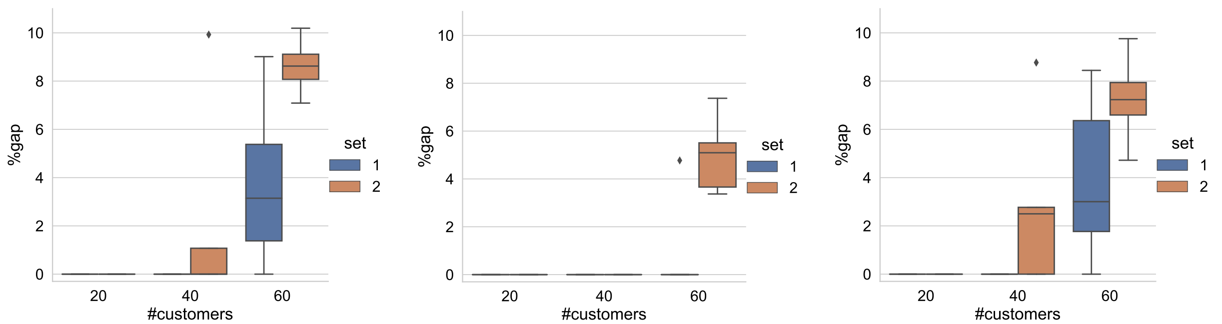
Fig. 5. Boxplots displaying the distributions of the percentage gaps when solving the two sets of the PDTSPLTW *all services* instances with the three formulations proposed, aggregated by number of customers.

and $c_\ell = 5, \ell \in V_L$. We set $k = 10$. Set $V_L(i)$ for customer $i \in V_C$ includes locker $\ell \in V_L$ if $d_{i\ell} \leq 20$. All customers can be served at home. As by Jiang et al. (2019), we set the time windows of the lockers equal to the beginning and the end of the time horizon $[e_\ell, l_\ell] = [e_0, l_0], \ell \in V_L$. We recall that, by imposing a tighter time window, one could model counters instead of lockers. We also recall that by setting a restrictive l_0, a maximum time for each truck route is imposed. For the multiple vehicle case, we considered $q_i = 1, i \in V_P$, $q_i = -1, i \in V_D$ and a capacity $Q = \lceil n/2 \rceil$.



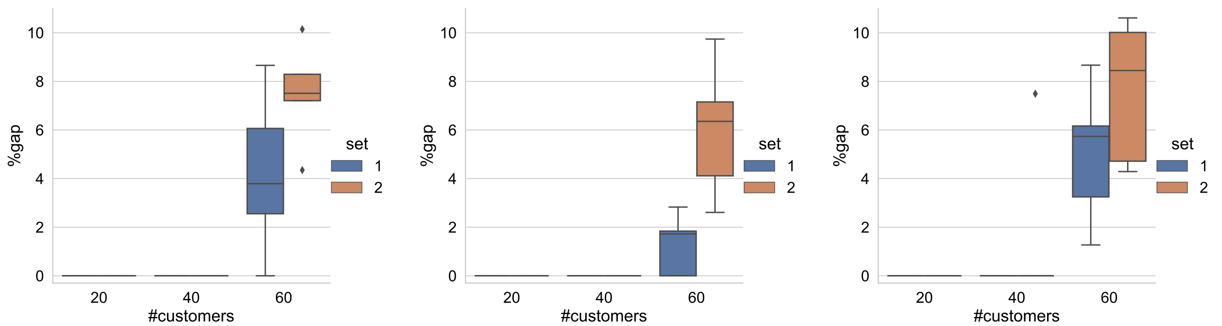
(a) VVF (b) TCF (c) OCF

Fig. 6. Boxplots displaying the distributions of the percentage gaps when solving the two sets of the PDVRPL instances with the three formulations proposed, aggregated by number of customers.



(a) VVF (b) TCF (c) OCF

Fig. 7. Boxplots displaying the distributions of the percentage gaps when solving the two sets of the PDVRPLTW *home service* instances with the three formulations proposed, aggregated by number of customers.



(a) VVF (b) TCF (c) OCF

Fig. 8. Boxplots displaying the distributions of the percentage gaps when solving the two sets of the PDVRPLTW *all services* instances with the three formulations proposed, aggregated by number of customers.

6.2. Results analysis

In this section, we compare the proposed formulations and suggest some managerial insights.

6.2.1. Comparison of the formulations

This section compares the proposed formulations in light of the various problems examined.

We show the percentage gap (%gap) of the proposed formulations on the two sets of instances used with respect to the number of customers for each single vehicle problem in Figs. 3–5, and for each multiple vehicle problem in Figs. 6–8. The %gap is calculated

Table 3
Results for the single vehicle cases: number of optima and average seconds.

	VVF		TCF		OCF	
PDTSPL	#opt/tot	avg sec	#opt/tot	avg sec	#opt/tot	avg sec
20	10/10	0.39	10/10	0.86	10/10	0.94
40	10/10	45.37	10/10	64.31	10/10	112.47
60	8/10	2305.37	10/10	435.64	10/10	1878.50
100	2/10	6480.12	7/10	5013.23	1/10	6888.19
tot/avg	30/40	2207.82	37/40	1378.51	31/40	2220.02
PDTSPLTW home service						
20	10/10	2.58	10/10	8.16	10/10	7.40
40	8/10	1499.01	10/10	1342.46	8/10	1592.60
60	4/10	5072.85	3/10	5096.91	3/10	5104.82
tot/avg	22/30	2191.48	23/30	2149.18	21/30	2234.94
PDTSPLTW all services						
20	10/10	4.44	10/10	12.10	10/10	15.28
40	8/10	1543.86	8/10	1747.74	8/10	2256.44
60	0/10	7200.00	0/10	7200.00	0/10	7200.00
tot/avg	18/30	2916.10	18/30	2986.61	18/30	3157.24

Table 4
Results for the multiple vehicle cases: number of optima and average seconds.

	VVF		TCF		OCF	
PDVRPL	#opt/tot	avg sec	#opt/tot	avg sec	#opt/tot	avg sec
20	10/10	1.89	10/10	7.40	10/10	7.31
40	2/10	5817.60	9/10	1710.93	3/10	5521.75
60	0/10	7200.00	6/10	4539.55	0/10	7200.00
100	0/10	7200.00	3/10	6362.10	0/10	7200.00
tot/avg	12/40	5054.87	28/40	3155.00	13/40	4982.27
PDVRPLTW home service						
20	10/10	0.64	10/10	1.36	10/10	1.05
40	8/10	2617.98	10/10	764.53	7/10	3058.04
60	1/10	6884.04	4/10	5162.54	1/10	6661.71
tot/avg	19/30	3167.55	24/30	1976.14	18/30	3240.26
PDVRPLTW all services						
20	10/10	1.08	10/10	2.97	10/10	1.58
40	10/10	1396.77	10/10	949.27	9/10	1656.79
60	1/10	6531.03	2/10	6493.27	0/10	7200.00
tot/avg	21/30	2642.96	22/30	2481.84	19/30	2952.79

as $100 \cdot (BestUB - LB)/BestUB$, where $BestUB$ is the value of the best solution obtained within the time limit among the three formulations and LB is the lower bound obtained by each formulation at the end of the iterations.

All the data is represented in boxplots with the following characteristics: the ends of the boxes are the 25th and 75th percentiles of the data distribution and the horizontal line in the box shows the median value of the distribution. The lower outlier limit (the lower whisker) is $25\% - 1.5 \cdot IQR$, while the upper outlier limit (the upper whisker) is $75\% + 1.5 \cdot IQR$, where IQR is the interquartile range and is calculated by subtracting the 25th percentile from the 75th. The diamonds represent outlier values. Note that when the same value appears in most occurrences, the relative boxplot degenerates into a straight line.

Moreover, in Tables 3 and 4, we show the number of optima obtained by the three formulations when solving the single and the multiple vehicle problems, respectively, and the average computing times in seconds. The results are shown with respect to the number of customers.

The TCF is the formulation showing the best results, on average, among those that we proposed. In fact, it can solve all instances with up to 60 customers and some with 100 for the single vehicle case with no time windows, and almost all instances with up to 40 customers for the multiple vehicle case without time windows, but also some instances with 60 and 100 customers. The introduction of time windows makes instances harder to solve for the single vehicle case, but this is not always the case for the multiple vehicle case, where the decoupling of time windows and truck times can ease the solution. In fact, when time windows are introduced, the largest instances solved by TCF for the single vehicle case are as large as 40 customers, but in the multiple vehicle case, all instances with up to 40 customers, and some with 60, can be solved to optimality. Moreover, the version with time windows for all services appears to be the most challenging to solve.

The TCF, moreover, often shows the best solving times (see Tables 3 and 4), on average, and even more so in the multiple vehicle cases; however, when the greedy algorithm does not provide a feasible solution, the TCF is not always able to obtain an upper bound for the single vehicle cases with time windows, whereas the other formulations are always able to provide one, with the VVF offering the best upper bounds in those cases. Furthermore, the VFF is always the fastest formulation when solving instances with 20 customers. The OCF, instead, is never the best formulation, but it is not always the worst.

As a result, businesses with up to 60 daily customers (40 if time windows are considered) can use TCF to solve their problems to optimality within two hours, whereas companies with a large number of customers should address pickup and delivery routing problems with lockers on a daily basis by using metaheuristic algorithms. On the other hand, for solving small instances, we advise using the VFF.

6.2.2. Analysis of the results and managerial insights

In this section, we propose an analysis of the results based on the solutions' structures and costs, from which we deduce some managerial insights.

Analysis of the solutions' structures. In Fig. 9, we show the number of lockers used and the number of delivery and pickup customers served via lockers, with respect to the instance size, in the solutions of problems with a single vehicle. In Fig. 10, we depict the same values for the multiple vehicle problems, and we also add the number of vehicles used. Data is displayed in violin plots, which contain the information of a boxplot but also show the distribution of quantitative data across different values, in this case, a kernel density estimation of the underlying distribution.

When analyzing the structure of the solution, one can see that both the number of lockers used and the number of customers served via lockers increase with the introduction of time windows, because their use helps to decouple the time at which the trucks serve the customers from their time windows. This is even more evident for the *home service* version, where services via lockers do not require respecting time windows, and thus using a larger number of lockers may be profitable. These numbers are higher in the single vehicle case than in the multiple vehicle case because it can be more difficult and expensive to adhere to each time window with a single truck. As a result, decoupling the time windows from the truck time, as done by the lockers, can offer better solutions. However, for instances with 20 customers, the solutions of the PDVRPL always use all the lockers, while they are never used in their totality when increasing the instance size. In fact, the introduction of lockers can aid in the pursuit of less expensive solutions, but not all lockers can offer a valuable trade-off if used because reaching a locker may necessitate a truck detour. Hence the location of the locker is a crucial, even if strategical, decision for the delivery companies.

The number of vehicles used is always two for the multiple vehicle case without time windows (PDVRPL), while it increases slightly with their introduction. In fact, more trucks might be required to respect the time windows or find less costly routes while upholding all the constraints. A larger number of trucks, on the other hand, may imply higher operational costs, which we did not include in the tackled problems but could easily be included in the objective function.

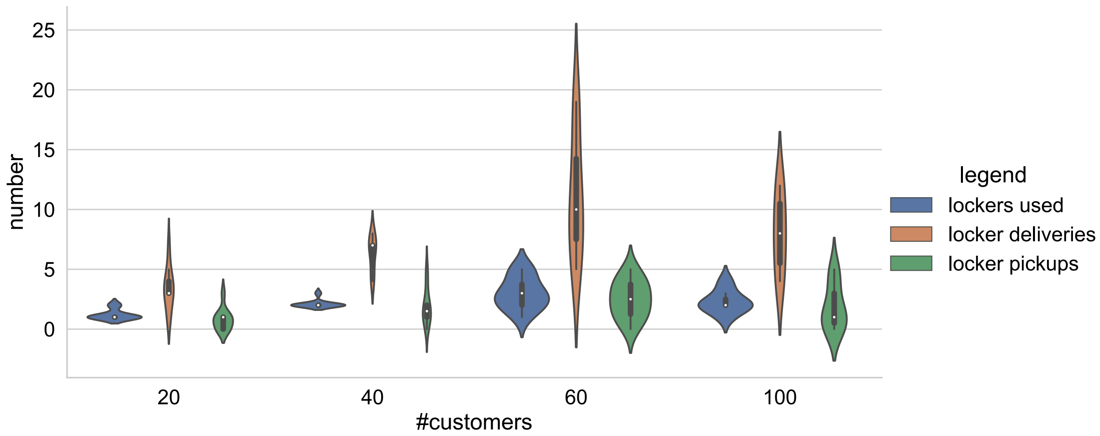
An example of different solutions for the same instance ($n = 20$, *Set1*) for each of the problem versions is shown in Fig. 11. One can see that by inserting more time window constraints, the routes become more and more complex. The problems with time windows exclusively for home service allow more customers to be served with lockers than those in which time windows are imposed on all services.

Analysis of the solutions' costs. In Fig. 12, a boxplot depicts the %gap between the cost of the solution of the instances of each problem version studied and, when available, the cost of the solution of the correspondent PDTSPL instances, averaged with respect to the instance size. The %gap is calculated as $100 \cdot (z_{(i)} - z_{TSPL})/z_{(i)}$, where z_{TSPL} is the value of the PDTSPL solution and $z_{(i)}$ is, in turn, the value of the solution of the PDTSPLTW *home service*, PDTSPLTW *all services*, PDVRPL, PDVRPLTW *home service*, and PDVRPLTW *all services*.

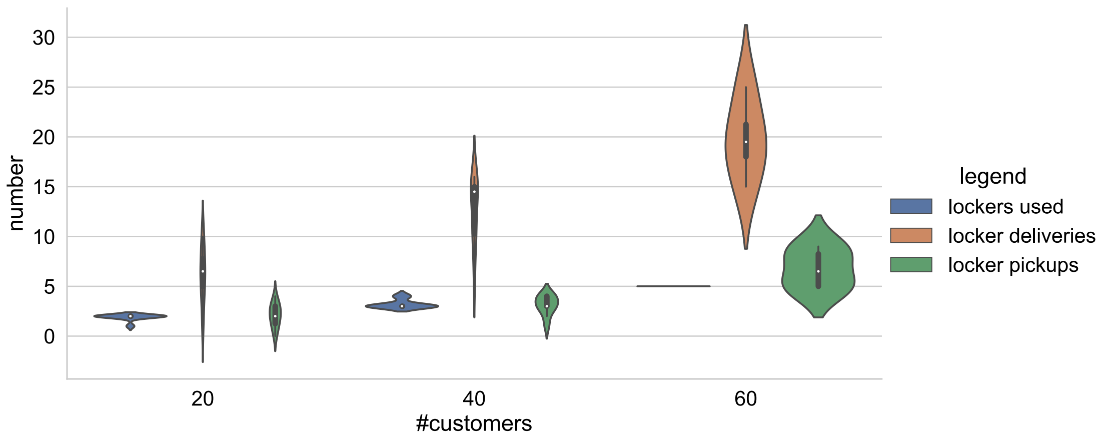
When introducing time windows for home service in the single vehicle case, the cost increases by more than 20% on average for the 20-customer instances and by more than 15% for instances with 40 and 60 customers. The gap increases by up to about 30% for instances in which the time windows apply to all services. The introduction of multiple capacitated vehicles for the problem without time windows makes the solution cost increase with respect to the single vehicle case by about 10%, on average, for instances with 20 customers, and slightly less for instances with more than 20 customers. This is due to the capacity constraints of vehicles. The gaps are higher than 20%, on average, for the PDVRPLTW *home service* and even higher for the PDVRPLTW *all services*, but are lower with respect to the single vehicle cases with time windows.

Since the introduction of time windows for home service increases the costs, and for all services even more, from the managerial point of view, the companies might consider increasing the price of pickups and deliveries performed in tight time windows to overcome these costs, or incentivize even more the use of lockers. This is shown even better in the following analysis.

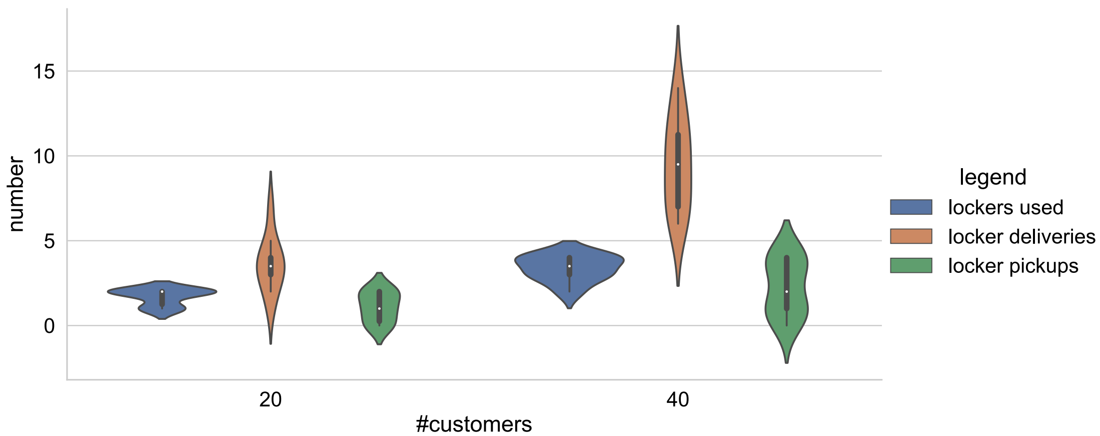
In Fig. 13, we compare the values of the optimal solutions of the PDTSPL and of the PDTSPLTWs with those of the classical *pickup and delivery TSP* (PDTSP) and of the classical *pickup and delivery TSP with time windows* (PDTSPTW) to evaluate if and how much the introduction of lockers is beneficial to the solution value. To do so, we compute %gap on the total costs as follows (note that, for brevity, we shortened the names of the problems by removing PDTSP, so that, for example, z is shortened for z_{PDTSP} and z_L is shortened for z_{PDTSPL}): $100 \cdot (z - z_L)/z$, and $100 \cdot (z_{TW} - z_{LTW-as})/z_{TW}$ and $100 \cdot (z_{TW} - z_{LTW-hs})/z_{TW}$, where z is the optimal solution of the given problem, if available. We also show the %gap computed solely on the traveling costs as follows: $100 \cdot (z - tc_L)/z$, $100 \cdot (z_{TW} - tc_{LTW-as})/z_{TW}$, and $100 \cdot (z_{TW} - tc_{LTW-hs})/z_{TW}$, where tc is the traveling cost in the optimal solution of a given problem, if available. In Fig. 14, we report the same values for the multiple vehicle cases, and thus we compare our results with those of the classical *pickup and delivery VRP* (PDVRP) and *pickup and delivery VRP with time windows* (PDVRPTW). Everything is depicted in boxplots and it is shown with respect to the instance size.



(a) PDTSPL

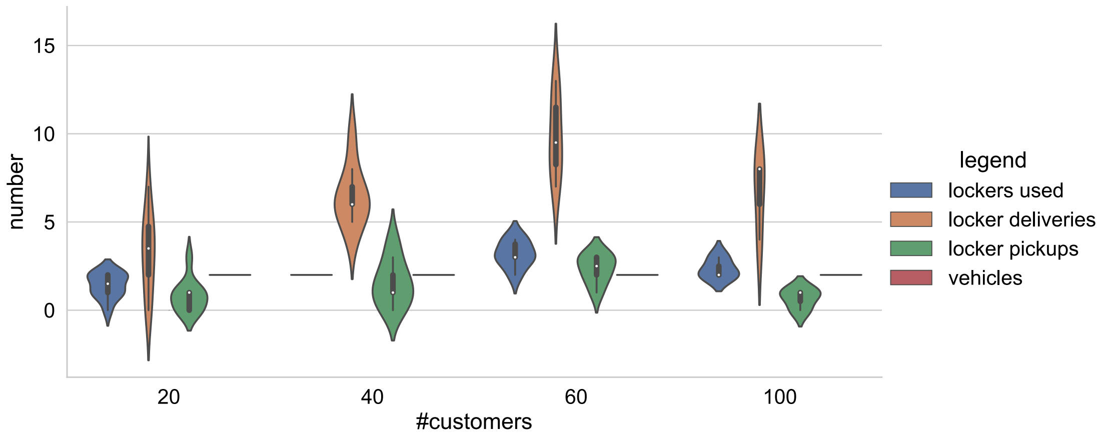


(b) PDTSPLTW *home service*

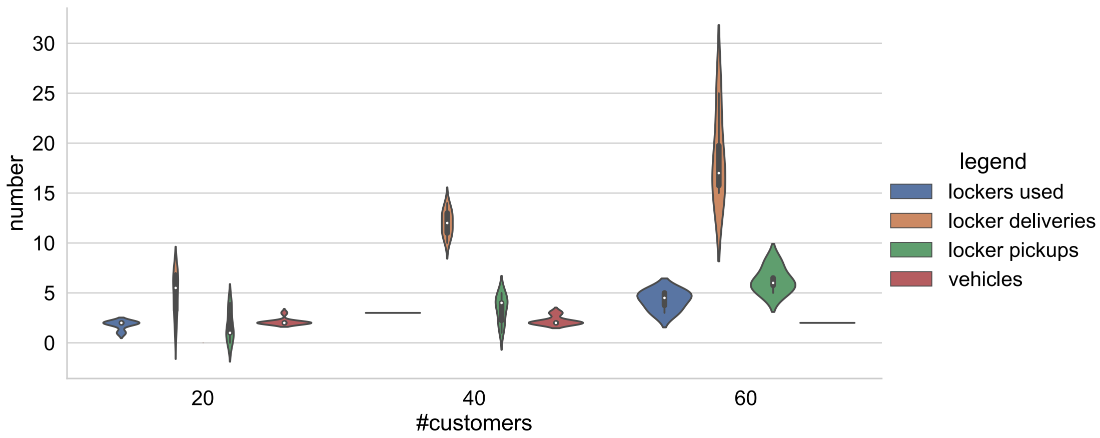


(c) PDTSPLTW *all services*

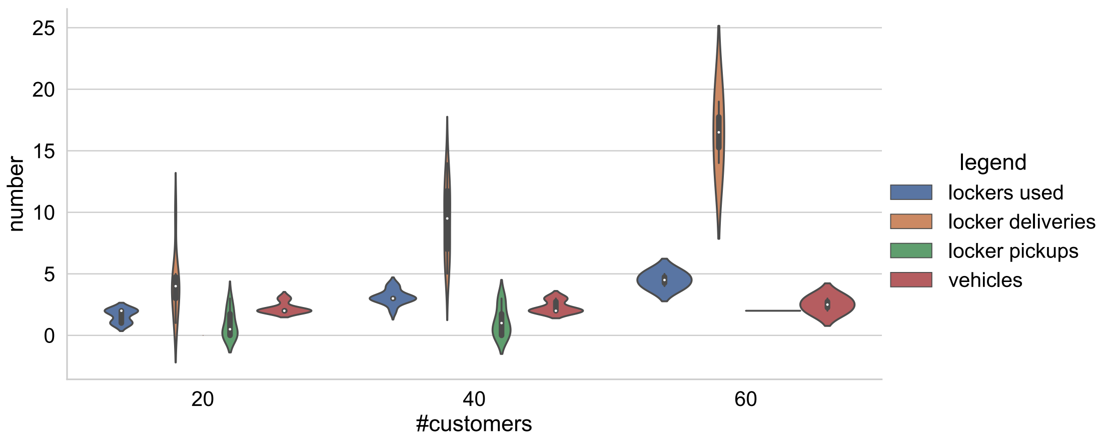
Fig. 9. Violin plots displaying the distribution and the density estimation of the number of lockers used and the number of deliveries and pickups performed via lockers in the optimal solutions of the instances of the PDTSPL (a), the PDTSPLTW *home service* (b), and the PDTSPLTW *all services* (c). All data is aggregated by the number of customers.



(a) PDVRPL



(b) PDVRLTW *home service*



(c) PDVRLTW *all services*

Fig. 10. Violin plots displaying the distribution and the density estimation of the number of lockers used, the number of deliveries and pickups performed via lockers, and the number of vehicles used in the optimal solutions of the instances of the PDVRPL (a), the PDVRPLTW *home service* (b), and the PDVRPLTW *all services* (c). All data is aggregated by the number of customers.

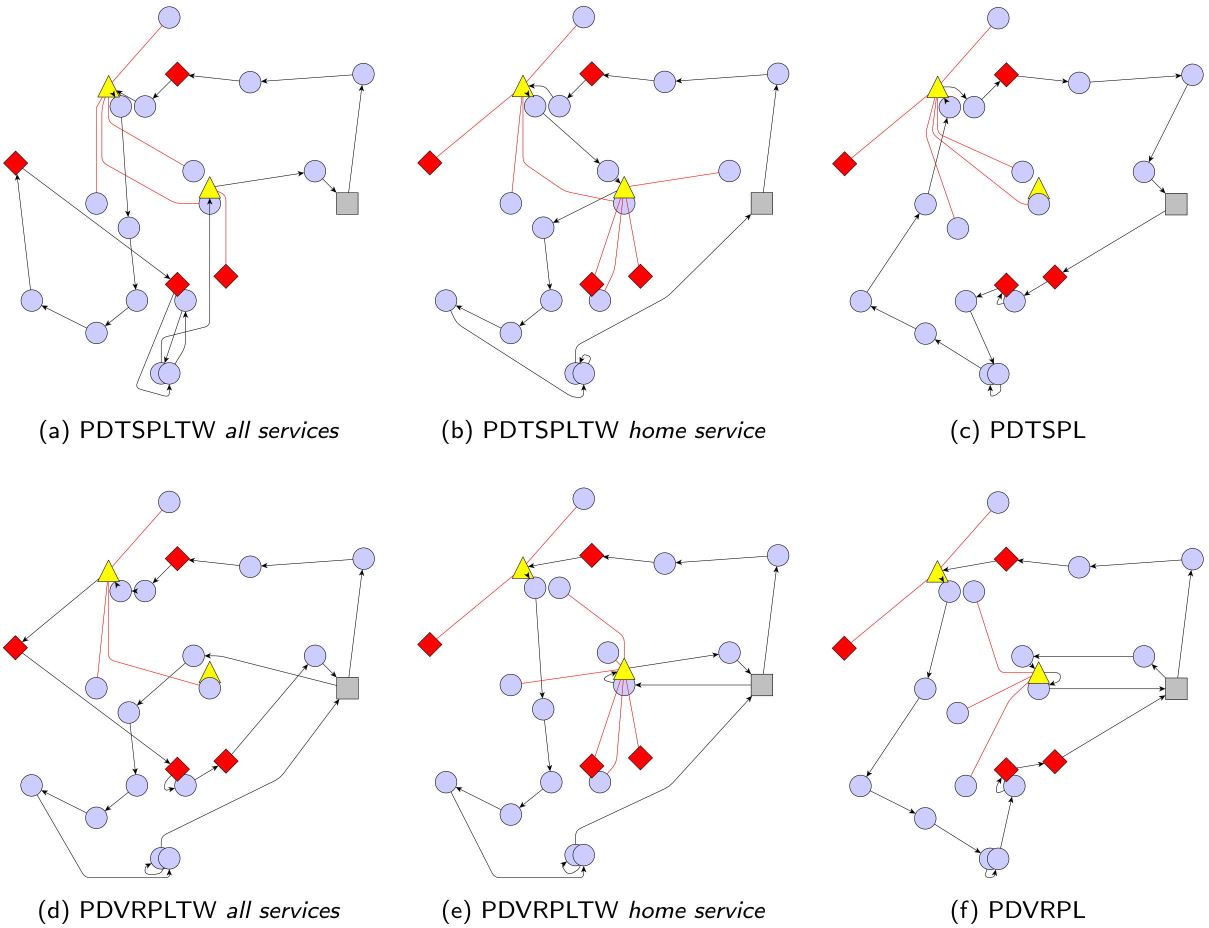


Fig. 11. Graphical representation of the studied problems' solutions on an instance with 20 customers of *Set1* (tw = 20).

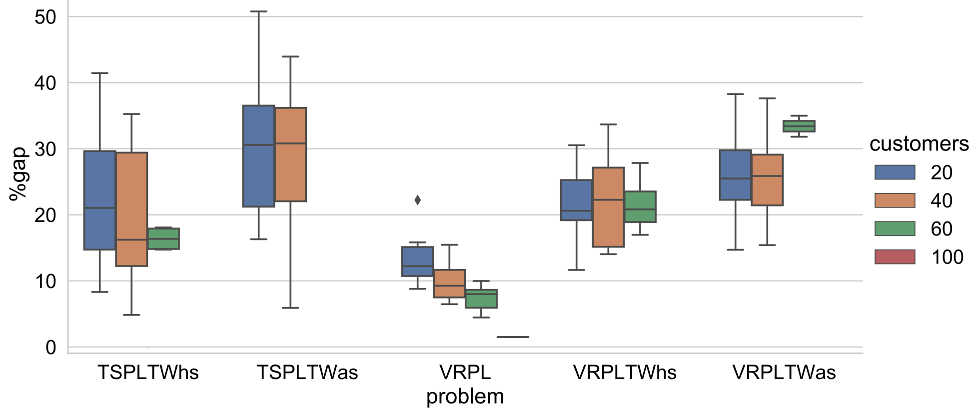
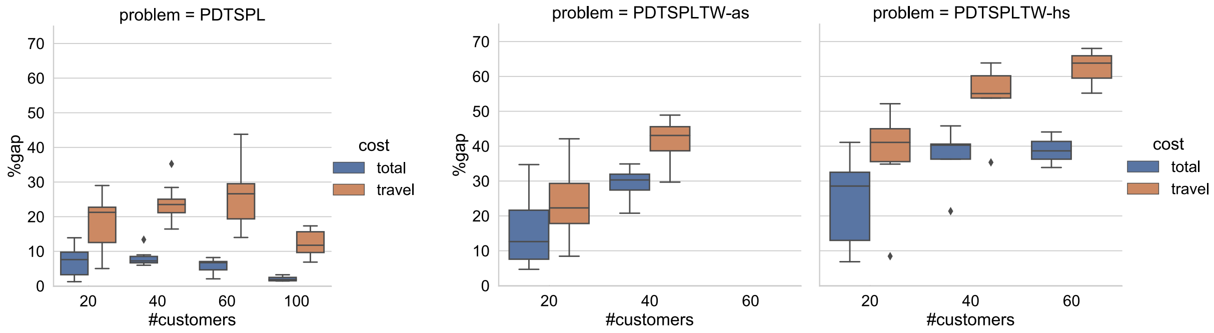


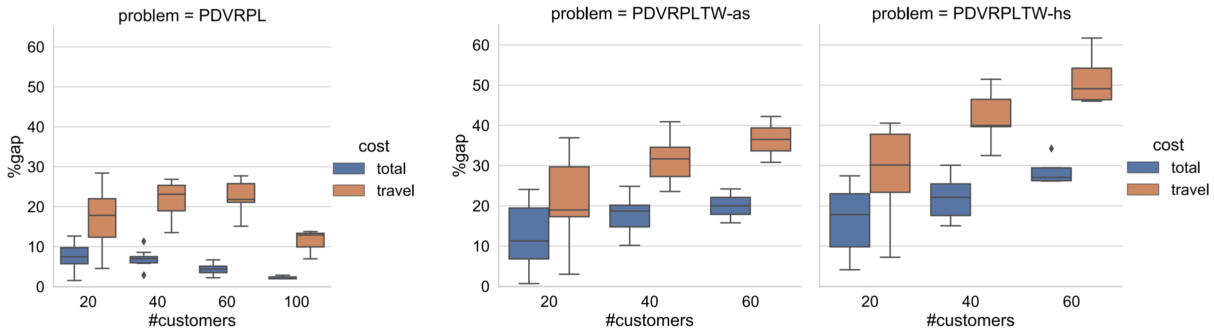
Fig. 12. Boxplots displaying the distribution of the percentage gaps of the difference between the cost of the PDTSPL solution and those of the other problems studied, separated by instance size. Note that we shortened the problem names by removing PD.

Considering the costs in the single vehicle case, the introduction of lockers (as in the PDTSPL) can result in a cost reduction with respect to the classical PDTSP of between 5 and 10% in most cases for instances with up to 40 customers, and slightly less for larger instances. On the other hand, it is clear that the traveling cost is significantly reduced in every instance. When time windows are considered, the introduction of lockers produces a cost decrease of up to 30% in the PDTSPLTW *all services* and 40% in



(a) PDTSPL (b) PDTSPLTWs

Fig. 13. Boxplots displaying the distribution of the percentage gaps of the difference between the total costs and travel costs of the PDTSP solutions (no lockers) and those of the PDTSPL (a). The same is reported between the costs of the PDTSPTW solutions (no lockers) and those of the PDTSPLTWs (b). All are aggregated by instance size.



(a) PDVRPL (b) PDVRPLTWs

Fig. 14. Boxplots displaying the distribution of the percentage gaps of the difference between the total costs and travel costs of the PDVRP solutions (no lockers) and those of the PDVRPL (a). The same is reported between the costs of the PDVRPTW solutions (no lockers) and those of the PDVRPLTWs (b). All are aggregated by instance size.

the PDTSPLTW *home service*. This is due to the fact that routes respecting all the time window constraints can be highly expensive, while the introduction of lockers can help decoupling time windows and truck times, and thus providing a largely less costly solution.

When considering the multiple vehicle case, the cost difference between the PDVRP and the PDVRPL is similar to that shown for the single vehicle case. In case of time windows, one can see that the cost decrease is very high for the PDVRPLTW *all services*: from 7 to 20% in most cases for instances with 20 customers and around 20% for larger instances. The cost decrease is even higher when considering time windows only for home service, with over 20% for larger instances. This is due to the fact that, when time windows are not considered at the lockers, the number of customers that can be served via locker is higher, and this helps in decreasing the costs.

To conclude, we have shown that the introduction of lockers is profitable for the logistics companies because it decreases total costs (traveling costs minus the discount for using lockers) by up to 30% and 40% in the single vehicle case with and without time windows, respectively, and by more than 20% for the multiple vehicle case. For a seamless service, a lower chance of failed deliveries, and, most importantly, shorter and less expensive routes, delivery companies should consider incentivizing the use of lockers and prefer this to penalizing customers who choose strict time windows.

As previously stated, lockers allow for lower last-mile traveling costs; however, the cost of installing and operating them must be considered. If, on the one hand, the cost of purchasing a locker varies from 200 to 3000 USD on the most famous e-commerce platforms, which should be increased by the installation and land rental costs, on the other hand, this is a one-time cost and represents a strategical decision that cannot be compared with the day-to-day reduction of the operational costs (labor, fuel, and vehicle deprecation costs), externalities, and failed deliveries consequent to the introduction of lockers. On the contrary, the cost of operating lockers appears on a more operational basis and includes the costs of utilities (electricity, ICT, etc.) and maintenance. The prices to cover the operational costs of the main companies that offer packages for installing lockers in apartment blocks go from 1.50 to 3 USD per month per cell (see, e.g., 2ndKitchen (2022)), and thus we can consider those costs negligible on a daily basis if compared to the savings due to the introduction of lockers in terms of labor time and fuel reduction only.

Table 5
Additional symbols used to perform comparisons with other works from the literature.

Parameters	
π_ℓ	Penalty for each customer served via locker $\ell \in V_L$
α_3	Weight associated with the lockers setup costs
α_4	Weight associated with the cost of violating time windows
α_5	Weight associated with the trucks setup costs
T	Maximum number of trucks available
Variables	
z'	Objective function value for the problem tackled by Jiang et al. (2019)
z''	Objective function value for the problem tackled by Mancini and Gansterer (2021)
s_i	Non-negative continuous variable that represents the amount of violation of the time windows at node $i \in V$

Table 6
Comparison with Jiang et al. (2019).

n	tw	%gap$_{MTZ}$	sec$_{MTZ}$	#opt$_{MTZ}$/tot	%gap$_{TCF}$	sec$_{TCF}$	#opt$_{TCF}$/tot
20	20	0.00	13.40	5/5	0.00	9.05	5/5
	100	0.00	99.73	5/5	0.00	83.35	5/5
tot/avg		0.00	56.56	10/10	0.00	46.20	10/10
40	20	1.59	3756.76	4/5	1.46	3053.54	4/5
	100	1.82	3400.90	3/5	1.32	3935.60	4/5
tot/avg		1.70	3578.83	7/10	1.39	3494.57	8/10
60	20	6.95	7200.00	0/5	6.40	7200.00	0/5
	100	8.89	7200.00	0/5	9.20	7200.00	0/5
tot/avg		7.92	7200.00	0/10	7.80	7200.00	0/10
100	20	29.94	7200.00	0/5	30.19	7200.00	0/5
	100	29.88	7200.00	0/5	29.83	7200.00	0/5
tot/avg		29.91	7200.00	0/10	30.01	7200.00	0/10
		9.88	4508.85	17/40	9.80	4485.19	18/40

7. Comparison with other works from the literature

In this section, we compare the method that provides the best results among those that we proposed, the TCF, with three works from the literature that solve similar problems; two of them consider only delivery services, and the last one considers simultaneous pickup and delivery services. In Table 5, we report the additional symbols needed to adapt the TCF to those similar problems.

7.1. Comparison with Jiang et al.

Jiang et al. (2019) propose a Miller–Tucker–Zemlin (MTZ) formulation for a problem that they call the *TSP with time windows for the last-mile delivery in online shopping*, a simplification of the PDTSPLTW in which only delivery is considered and where time windows are considered for all types of service. Their objective function differes from the one presented in this paper in that it includes locker setup costs that are proportional to the number of customers served by a locker: a penalty π_ℓ must be paid for each customer served via locker $\ell \in V_L$. The authors solve exactly only the relaxation of the problem that they propose, by including soft time windows constraints; i.e., allowing the latest time window to be exceeded at a cost of a penalty. To compare the TCF to Jiang et al.'s method we need to keep track of the time windows violation, so we introduced a new set of non-negative variables s_i, $i \in V$. The new objective function is thus (41), and constraints (42) and (43) must be imposed to guarantee that s_i represents the exceeded time, if any, for each node $i \in V$. Note that the two new components of the objective function are multiplied by the two coefficients, $\alpha_3 = 5$ and $\alpha_4 = 1$. For all $\ell \in V_L$, the setup cost $\pi_b = 1$.

$$min\ z' = \alpha_1 \sum_{i\in V}\sum_{j\in V} d_{ij}x_{ij} + \alpha_2 \sum_{i\in V_C}\sum_{\ell\in V_L} c_{i\ell}y_{i\ell} + \alpha_3 \sum_{i\in V_C}\sum_{\ell\in V_L} \pi_\ell y_{i\ell} + \alpha_4 \sum_{i\in V} s_i \tag{41}$$

$$s_i \geq t_i - l_i \quad i \in V \tag{42}$$

$$s_i \geq 0 \quad i \in V \tag{43}$$

Moreover, some time window constraints must be modified to allow time variables t_i to exceed the latest time window of node $i \in V$.

In Table 6, we compare the results of our TCF modified to include the characteristics of the problem studied by Jiang et al. (2019) and their MTZ model implemented by us and run on our computer (see Section 5). The table shows the number of customers (n), the size of the of time windows (tw), the percentage gap between the best known upper bound and the lower bound provided by each of the two methods (%gap$_{MTZ}$ and %gap$_{TCF}$), the running time in seconds having set a time limit of two hours (sec$_{MTZ}$ and

Table 7
Comparison with Mancini and Gansterer (2021).

n	m	%gap$_{MG}$	sec$_{MG}$	#opt$_{MG}$/tot	%gap$_{BN}$	sec$_{BN}$	#opt$_{BN}$/tot	%gap$_{TCF}$	sec$_{TCF}$	#opt$_{TCF}$/tot
25	5	0.00	26.19	10/10	0.00	20.73	10/10	0.00	26.53	10/10
50	5	1.21	2201.70	8/10	0.00	179.71	10/10	0.00	330.59	10/10
75	5	10.01	3600.00	0/10	0.46	1338.01	8/10	0.95	2305.54	6/10
tot/avg		3.74	1942.63	18/30	0.15	512.82	28/30	0.32	887.55	26/30

sec$_{TCF}$), and the number of optima obtained for each subset (opt$_{MTZ}$ and opt$_{TCF}$). On average, our model is faster and provides better results than MTZ. However, both models struggle to find the optimal solution for larger instances. We can thus conclude that the TCF, a method designed for more complex problems that include both pickup and delivery (and more variables and constraints), can provide better results than a method specifically designed for delivery-only problems such as the one by Jiang et al.

7.2. Comparison with Mancini and Gansterer

Mancini and Gansterer (2021) propose the *vehicle routing with private and shared delivery locations* that is also a delivery-only version of the PDVRPLTW. They solve it both exactly and heuristically. We are interested in their exact results to compare with the TCF. To do so, we need to accommodate the modifications required in the TCF. First of all, the objective function must include a term aimed at minimizing the number of trucks used (multiplied by coefficient $\alpha_5 = 1$). The new objective function can thus be rewritten as in (44).

$$min\ z'' = \alpha_1 \sum_{i\in V}\sum_{j\in V} d_{ij}x_{ij} + \alpha_2 \sum_{i\in V_C}\sum_{\ell\in V_L} c_{i\ell}y_{i\ell} + \alpha_5 \sum_{j\in V_C\cup V_L} x_{0j} \tag{44}$$

The parameters used are as follows: $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_5 = 1$, $c_{i\ell} = 5, i \in V_C, \ell \in V_L$.

Mancini and Gansterer consider a fixed service time for serving a customer at home and a service time when visiting a locker: we include these times into the traveling time of each arc in our model. The authors use an unlimited set of uncapacitated trucks as well as lockers with a capacity slightly grater than the number of customers, and thus we set $Q = n$, $q_i = 1, i \in V_C$ and $v_\ell = n$ for all lockers.

Mancini and Gansterer propose three sets of 10 instances (available online) with five lockers and 25, 50, and 75 customers, respectively, randomly generated in a 10 × 10 km area. The maximum travel time for each truck is 12 h ($l_0 = 720$). The time window of a customer is one of the 12 h. The travel time is equal to the travel distance, which is considered three times the Euclidean distance between two points. In our adaptation, the travel times also include the service times as explained previously. All customers can be served either with a truck or via a nearby locker: only lockers within a travel time radius of 15 can be served by a locker; otherwise the correspondent y variable is set to 0.

In Table 7, we report the results taken from the paper by Mancini and Gansterer, averaged per instance size. They are compared to the results obtained by our best method, the TCF, and the best method proposed by Buzzega and Novellani (2022) (BN), which shows the best results for these instances. In this case, we report the results of the algorithms run by the authors of the other papers. We show the average gap computed by using the best known solution, the seconds needed to obtain it, or the time limit of one hour, and the number of optima for each set of 10 instances. One can see that the TCF could obtain better results than the MILP proposed by Mancini and Gansterer. However, it cannot improve the results of Buzzega and Novellani (that used an Intel 678 i7-6850K @ 3.60 GHz with 64 GB RAM CPU), being that their method is clearly dedicated to delivery-only problems.

7.3. Comparison with Yu et al.

Yu et al. (2022) propose the *VRP with simultaneous pickup and delivery and parcel lockers*, a version of the PDVRPLTW in which all customers have both a pickup and a delivery request that must be satisfied within a given time window, if the service is done at home, by using a limited set of homogeneous vehicles. The authors solve the problem exactly and heuristically, but we are interested in comparing our best method, the TCF, with their exact method. To do so, we accommodate the needed modifications in the TCF. The objective function aims to minimize only the distance traveled, so we can use the original objective function (1) and impose $\alpha_1 = 1$ and $\alpha_2 = 0$.

Let us call T the maximum number of trucks available. To impose the model to use at most T trucks, we must include the following constraint: $\sum_{i\in V} x_{0i} \leq T$.

Because each customer is both a delivery and a pickup customer, the number of customer is doubled, being $i \in V_D$ the node representing the delivery action and $i + n_d \in V_P$ the one representing the pickup action at the same customer. Note that the time windows of these two nodes are set to the same value. The distance $d_{i,i+n_d}$ and the time $\tau_{i,i+n_d}$ are set to 0.

In the setting of Yu et al., each customer is visited at most once, and both delivery and pickup actions are done at the same moment if the customer is served at home. Moreover, if the customer is served at a locker, both pickup and delivery are done at that node. Thus, if the customer preference is to be served at home, we can impose $x_{i,i+n_d} = 1, i \in V_D$, while $x_{i+n_d,i} = 0, i \in V_D$, because it is better to unload the truck first and then perform the pickup. We can impose $x_{i,i+n_d} = x_{i+n_d,i} = 0, i \notin V_H$ if the customer prefers to not be served at home. On the other hand, for all preferences, the locker used for the pickup and for the delivery must

Table 8
Comparison with Yu et al. (2022).

n_d	m	T	BKS_{YU}	sol_{YU}	sec_{YU}	sol_{TCF}	sec_{TCF}	#vu
10	1	3	161.96	*161.96*	0.52	*161.96*	0.06	1
10	1	3	206.07	*206.07*	0.73	*206.07*	0.58	2
10	1	3	98.15	*98.15*	0.20	*98.15*	0.03	1
25	2	10	323.85	*323.85*	145.59	*323.85*	0.48	4
25	2	10	311.49	311.49	18 000.00	*311.49*	7.36	3
25	2	10	341.22	341.22	18 000.00	*341.22*	59.69	3
50	3	15	817.39	817.39	18 000.00	*817.39*	3.30	9
50	3	15	514.18	539.94	18 000.00	***512.78***	11 053.73	5
50	3	15	650.43	690.02	18 000.00	**632.79**	18 000.00	6

be the same, if that locker is used, and thus we can impose: $y_{i\ell} = y_{i+n_d,\ell}, \ell \in V_L, i \in V_D$. For those customers whose services can be performed either at home or at lockers, we can impose the following constraint: $x_{i,i+n_d} = 1 - \sum_{\ell V_L(i)} y_{i\ell}, i \in V_D$. Note that the setting on the variables x and y imposed in Section 4 is still valid.

A service time is considered each time a customer is visited at home (both pickup and delivery are included in that service time), and thus we include this service time in the times of the arcs leaving the pickup node of each customer, being that the pickup is performed after the delivery. A service time occurs also when a locker is visited, and thus we add the service time to the times of the arcs leaving the locker.

The instances used by Yu et al. are derived from the well-known Solomon's instances (see, e.g., Solomon (1987)). In particular, they use the $RC1$ set, where some customers are randomly placed and others are clustered. They select three 10-customer, three 25-customer, and three 50-customer instances, to which they add one, two, and three lockers each, respectively, and the number of trucks allowed is three, 10, and 15, respectively.

In Table 8, we compare the results of Yu et al. with those obtained with the TCF for these instances. We report the results shown in Yu et al.'s paper, whose algorithm was run for at most 18 000 s on an Intel(R) Core(TM) i7-4770 CPU @ 3.46 GHz with Gurobi as the MILP solver. Our algorithm was run on an Intel(R) Core(TM) i7-3770 CPU @ 3.40 GHz with CPLEX 20.10.0. Table 8 shows the number of delivery customers n_d (that is the number of customers for Yu et al., which is actually doubled in our model), the number of lockers m, the number of trucks available T. Then we report the best known solution shown in the paper by Yu et al. (BKS_{YU}), computed with both their exact and heuristic methods, the best solution obtained with the exact algorithm (sol_{YU}), and the seconds required to do so (sec_{YU}). Afterwards, we show the results of the TCF, namely the best solution (sol_{TCF}) and the required seconds to obtain optimality or the time limit (sec_{TCF}). Moreover, we also show the number of vehicles used in the best solution (#vu). Note that we improve the results for all the instances, obtaining eight optimal solutions out of nine instances, and improving the best known solution for the remaining one. In italic we report the certified optimal solutions and in bold the new best known ones. We show that our TCF method is good also for instances with simultaneous pickup and delivery; however, we believe new algorithms designed especially for these problems could provide improved results.

8. Conclusions

In this paper, we have studied the introduction of lockers in pickup and delivery problems and in pickup and delivery problems with time windows, considering single and multiple vehicle cases. We have proposed three mixed integer linear programming formulations, several valid inequalities, and two branch-and-cut algorithms designed to iteratively insert the exponentially many constraints of two of the three formulations. We show that the two-commodity-based formulation performs best on the studied instances, whereas the vehicle flow formulation is the fastest on smaller instances. We demonstrated that the introduction of lockers and their use to deliver parcels and pickup returns can provide the delivery companies with less costly solutions. Moreover, we showed that our methods can be competitive with respect to methods designed for delivery-only problems with lockers and for simultaneous pickup and delivery problems with lockers.

Possible future research could be focused on the improvements of the newly proposed formulations, on the development of metaheuristic and branch-and-price algorithms, and could consider problems with different features, such as a more detailed delivery preference for customers and lockers modeled with cells of different capacities. Other gaps in the literature deserving future studies are pickup and delivery problems involving multiple echelons, moving lockers, simultaneous pickup and delivery, and cases in which the information is stochastic, dynamic, and/or online.

CRediT authorship contribution statement

M. Dell'Amico: Conceptualization, Funding acquisition, Resources, Writing – review & editing, Supervision, Validation. **R. Montemanni:** Conceptualization, Writing – review & editing, Supervision, Validation. **S. Novellani:** Conceptualization, Methodology, Software, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Inequalities for the PDVRPLTWs and the PDVRPL

In the following, we introduce some polynomial inequalities to strengthen the proposed formulations for solving the PDVRPLTWs and the PDVRPL. A valid inequality imposing a lower bound on the number of vehicles that must leave the depot to serve all customers, as in (45), is added to strengthen the VFF. As previously stated, GSECs (9) are exponentially many and are included only when violated in a branch-and-cut fashion; however, we statically include the GSEC for $|S| = 2$, as in (46).

$$\sum_{j\in V} x_{0j} \geq \max\left\{ \left\lceil \frac{|\sum_{i\in V_D} q_i|}{Q} \right\rceil, \left\lceil \frac{\sum_{i\in V_P} q_i}{Q} \right\rceil \right\} \tag{45}$$

$$x_{ij} + x_{ji} \leq 1 - \sum_{\ell\in V_L} y_{i\ell} \quad (i,j) \in A \tag{46}$$

Inequalities (45) and (46) are valid for solving the PDVRPLTWs and the PDVRPL with both the TCF and the OCF.

Appendix B. Inequalities for the PDVRPLTWs

In the following, we write some valid inequalities based on time windows for the PDVRPLTWs.

$$x_{ij} = 0 \quad i,j \in V_C \cup \{0\} : e_i + \tau_{ij} > l_j \tag{47}$$

$$x_{ij} + \sum_{k\in H^+_{ij}} x_{jk} \leq 1 \quad i \in V, j \in V \setminus \{0\}, i \neq j \tag{48}$$

$$\sum_{k\in H^-_{ij}} x_{ki} + x_{ij} \leq 1 \quad i \in V \setminus \{0\}, j \in V, i \neq j \tag{49}$$

Constraints (47) set to zero all the variables representing the arcs starting from i and arriving in j such that the minimum arrival time in j (given by the earliest starting time in i plus the time needed to travel from i to j, $e_i+\tau_{ij}$) is greater than the latest possible arrival in j: l_j. A similar rationale can be extended to three-node paths. Let us define, for each pair of nodes $(i,j), i \in V, j \in V \setminus \{0\}, i \neq j$, the set $H^+_{ij} = \{k \in V : \max(e_i + \tau_{ij}, e_j) + \tau_{jk} > l_k\}$ that includes all the nodes k for which a path (i,j,k) is infeasible because the earliest time at which the truck can visit node k, after i and j, in this order, exceeds its latest time window l_k. Constraints (48) avoid these infeasible three-node paths by imposing to select at most one arc variable of the path (i,j,k); additionally, it includes all possible $x_{jk}, k \in H^+_{ij}$, because of constraints (3). Let us also define, for each pair of nodes $(i,j), i \in V \setminus \{0\}, j \in V, i \neq j$, the set $H^-_{ij} = \{k \in V : \max(e_k + \tau_{ki}, e_i) + \tau_{ij} > l_j\}$, that includes all the nodes k for which a path (k,i,j) is infeasible because the earliest time at which the truck can visit node j, after visiting k and i in this order, exceeds its latest time window l_j. Constraints (49) avoid these infeasible three-node paths (k,i,j) including all possible $x_{ki}, k \in H^-_{ij}$, similarly to (48). Inequalities (47)–(49) derive from those presented by Buzzega and Novellani (2022) and are valid for both the PDVRPLTW *home service* and the PDVRPLTW *all services* (see Section 3).

$$x_{i\ell} + y_{j\ell} \leq 1 \quad i \in V, \ell \in V_L, j \in H_{i\ell} \tag{50}$$

$$t_\ell \leq \sum_{i\in V_D} (l_i - \tilde{\tau}_{i\ell}) y_{i\ell} + \sum_{i\in V_P} l_\ell y_{i\ell} + e_\ell \quad \ell \in V_L \tag{51}$$

$$t_\ell \leq (l_i - \tilde{\tau}_{i\ell} - M) y_{i\ell} + M \quad i \in V_D, \ell \in V_L \tag{52}$$

$$t_\ell \geq (e_i + \tilde{\tau}_{i\ell}) y_{i\ell} \quad i \in V_P, \ell \in V_L \tag{53}$$

$$t_\ell \leq (l_i - \tau_{\ell i} - M) x_{\ell i} + M \quad i \in V_C, \ell \in V_L \tag{54}$$

$$t_\ell \geq (e_i + \tau_{i\ell}) x_{i\ell} \quad i \in V_C, \ell \in V_L \tag{55}$$

Consider the set $H_{i\ell} = \{j \in V_D : \max(e_i + \tilde{\tau}_{i\ell}, e_\ell) + \tilde{\tau}_{j\ell} > l_j \vee j \in V_P : \max(e_i + \tau_{i\ell}, e_\ell) - \tilde{\tau}_{j\ell} < e_j\}$ of customers j whose time windows will be violated if served by a locker ℓ visited by the truck after node i. In such a case, (50) impose that at most one of the following events occur: the truck travels the arc (i,ℓ) or the customer j is served by the locker ℓ. Constraints (51) restricts the time variable t_ℓ to its earliest time window, e_ℓ, if locker ℓ does not serve any customer. On the other hand, if locker ℓ serves customers, then these constraints are not binding. The multipliers of the y variables could be large numbers that we set to $(l_i - \tau_{i\ell})$ and l_ℓ, due to the characteristics of the problem. Constraints (52) impose that, if customer $i \in V_D$ is served from locker ℓ, the time variable at the locker is limited so that the customer's latest time window is respected, which means that t_ℓ must be early enough to allow the delivery customer i to travel the arc (i,ℓ) within its latest time window l_i. Otherwise, t_ℓ is not limited. Similarly, constraints (53) impose that if customer $i \in V_P$ is served from locker ℓ, the time variable at the locker is limited so as to respect the customers' earliest time window. Note that inequalities (50)–(53) are only valid for the PDVRPLTW *all services*. Inequalities (54) and (55) are derived from Buzzega and Novellani (2022). If i is visited just after locker ℓ on a route, inequalities (54) impose the time variable t_ℓ at the locker ℓ to be early enough to allow the truck to arrive at customer i before its latest time window l_i. If $x_{i\ell} = 0$ then t_ℓ is not limited. According to inequalities (55), if a locker ℓ is visited after node i, the time variable in ℓ cannot be less than the time required to travel from i to ℓ plus the earliest departure time in i, e_i. Otherwise, no restriction is imposed on t_ℓ. M is a sufficiently large number that in this case could be calculated as $M = \min(l_\ell, l_0 - \tau_{\ell 0}), \ell \in V_L$.

Appendix C. Summary of the formulations for the PDVRPLTWs and the PDVRPL

C.1. The vehicle flow formulation

The VFF for the PDVRPLTW *home service* (see Section 3) includes (1)–(16). Inequalities (19), (20), (45)–(49), (54), and (55) are included to strengthen the formulation.

The VFF for the PDVRPLTW *all services* (see Section 3) is obtained by (1)–(18). Inequalities (19), (20) and (45)–(55) are included to strengthen the formulation.

The VFF for the PDVRPL is made up of (1)–(12). Inequalities (19), (20), (45) and (46) also apply.

C.2. The two-commodity formulation

The TCF for the PDVRPLTW *home service* is given by (1)–(8), (11), (12), (24)–(36), and the time window constraints (13)–(16). Inequalities (45)–(49), (54) and (55) also apply.

The TCF for the PDVRPLTW for *all services* is obtained with (1)–(8), (11), (12), (24)–(36), and the time window constraints (13)–(18). Inequalities (45)–(55) also apply.

The TCF for the PDVRPL is made up of (1)–(8), (11), (12), (24)–(36). Inequalities (45) and (46) also apply.

C.3. The one-commodity formulation

The OCF for the PDVRPLTW *home service* is made up of (1)–(12), (24), (35), (37)–(39) and the time window constraints (13)–(16). Inequalities (19), (20), (45)–(49), (54), and (55) also apply.

The OCF for the PDVRPLTW *all services* is made up of (1)–(12), (24), (35), (37)–(39) and the time window constraints (13)–(18). Inequalities (19), and (20), (45)–(55) also apply.

The OCF for the PDVRPL is given by (1)–(12), (24), (35) and by the time constraints (37)–(39). Inequalities (19), and (20), (45) and (46) also apply.

Appendix D. Adaptation of the formulations to the single vehicle cases

The formulations that we presented can be adapted to solve the PDTSPL and the PDTSPLTW. Let us define constraints (56) that impose the number of vehicles exiting and entering the depot to one, and recall that the GSEC version for the TSP (40) is also required.

$$\sum_{j \in V} x_{0j} = \sum_{j \in V} x_{j0} = 1 \tag{56}$$

The VFF for the PDTSPL can then be written as (1)–(8), (11), (12), (56), and (40).

Because all nodes in the PDTSPL must be visited by the truck or served with lockers regardless of the number of requests, we can simply update the TCF and the OCF to include these new considerations when solving a PDTSPL. The TCF can be adapted to solve the PDTSPL by using (1)–(8), (11), (12), (25), (27), (30), (33), (56), and the following constraints:

$$\sum_{i \in V_C \cup V_L} f_{i0} = |V_P| \tag{57}$$

$$\sum_{i \in V_C \cup V_L} g_{0i} = -|V_D| \tag{58}$$

$$f_{ij} + g_{ij} \leq n x_{ij} \quad (i,j) \in A \tag{59}$$

$$\sum_{j \in V} f_{ji} = \sum_{j \in V} f_{ij} - (1 - \sum_{\ell \in V_L} y_{i\ell}) \quad i \in V_P \tag{60}$$

$$\sum_{j \in V} f_{j\ell} = \sum_{j \in V} f_{\ell j} - \sum_{i \in V_P} y_{i\ell} \quad \ell \in V_L \tag{61}$$

$$\sum_{j \in V} g_{ji} = \sum_{j \in V} g_{ij} - (1 - \sum_{\ell \in V_L} y_{i\ell}) \quad i \in V_D \tag{62}$$

$$\sum_{j \in V} g_{j\ell} = \sum_{j \in V} g_{\ell j} - \sum_{i \in V_D} y_{i\ell} \quad \ell \in V_L \tag{63}$$

$$0 \leq f_{ij} \leq |V_P| x_{ij} \quad i, j \in V \tag{64}$$

$$0 \leq g_{ij} \leq |V_D| x_{ij} \quad i, j \in V \tag{65}$$

The OCF for the PDTSPL must thus include (1)–(8), (11), (12), (25), (56), (40), and the following constraints:

$$\sum_{i \in V_C \cup V_L} f_{0i} = n \tag{66}$$

$$\sum_{j \in V} f_{ji} = \sum_{j \in V} f_{ij} + (1 - \sum_{\ell \in V_L} y_{i\ell}) \quad i \in V_C \tag{67}$$

$$\sum_{j \in V} f_{j\ell} = \sum_{j \in V} f_{\ell j} + \sum_{i \in V_C} y_{i\ell} \quad \ell \in V_L \tag{68}$$

$$0 \leq f_{ij} \leq n x_{ij} \quad i, j \in V \tag{69}$$

Note that constraints (56) and (66)–(69) are derived from Buzzega and Novellani (2022).

Inequalities (46) are included to strengthen all the formulations.

All formulations for the PDTSPL can be adapted to solve the PDTSPLTW *home service* by including constraints (13)–(16). Inequalities (46)–(49), (54), and (55) also apply.

All formulations for the PDTSPL can be adapted to solve the PDTSPLTW *all services* by including constraints (13)–(18). Inequalities (46)–(55) also apply.

References

2ndKitchen, 2022. https://2ndkitchen.com/apartments/package-lockers/. (Accessed: 2022-10-06).

Amazon, 2022. Amazon hub US. https://www.amazon.com/primeinsider/tips/amazon-locker-qa.html. (Accessed: 2022-03-22).

Bug, P., Gordon, N., Staudenmaier, A.-S., 2018. Fashion business case study on the german click & collect situation. In: Chow, P.-S., Chiu, C.-H., C. Y. Yip, A., K. Y. Tang, A. (Eds.), Contemporary Case Studies on Fashion Production, Marketing and Operations. Springer Singapore, Singapore, pp. 139–163. http://dx.doi.org/10.1007/978-981-10-7007-5_9.

Buzzega, G., Novellani, S., 2022. Last mile deliveries with lockers: formulations and algorithms. Soft Comput. 1–19. http://dx.doi.org/10.1007/s00500-021-06592-6.

Clarke, G., Wright, J.W., 1964. Scheduling of vehicles from a central depot to a number of delivery points. Oper. Res. 12 (4), 568–581. http://dx.doi.org/10.1287/opre.12.4.568.

Dell'Amico, M., Deloof, W., Hadjidimitriou, S., Vernet, G., Schoenewolf, W., 2011. Citylog—sustainability and efficiency of city logistics: The m-bbx (modular bentobox system). In: 2011 IEEE Forum on Integrated and Sustainable Transportation Systems. IEEE, pp. 132–135. http://dx.doi.org/10.1109/FISTS.2011.5973626.

Dumas, Y., Desrosiers, J., Gelinas, E., Solomon, M.M., 1995. An optimal algorithm for the traveling salesman problem with time windows. Oper. Res. 43 (2), 367–371. http://dx.doi.org/10.1287/opre.43.2.367.

Dumez, D., Lehuédé, F., Péton, O., 2021. A large neighborhood search approach to the vehicle routing problem with delivery options. Transp. Res. B 144, 103–132. http://dx.doi.org/10.1016/j.trb.2020.11.012.

Foster, B.A., Ryan, D.M., 1976. An integer programming approach to the vehicle scheduling problem. J. Oper. Res. Soc. 27 (2), 367–384. http://dx.doi.org/10.1057/jors.1976.63.

Gendreau, M., Hertz, A., Laporte, G., Stan, M., 1998. A generalized insertion heuristic for the traveling salesman problem with time windows. Oper. Res. 46 (3), 330–335. http://dx.doi.org/10.1287/opre.46.3.330.

Ghaderi, H., Zhang, L., Tsai, P.-W., Woo, J., 2022. Crowdsourced last-mile delivery with parcel lockers. Int. J. Prod. Econ. 108549. http://dx.doi.org/10.1016/j.ijpe.2022.108549.

Grabenschweiger, J., Doerner, K.F., Hartl, R.F., Savelsbergh, M.W.P., 2021. The vehicle routing problem with heterogeneous locker boxes. CEJOR Cent. Eur. J. Oper. Res. 29, 113–142. http://dx.doi.org/10.1007/s10100-020-00725-2.

Jena, S.K., Meena, P., 2022. Shopping in the omnichannel supply chain under price competition and product return. J. Retail. Consum. Serv. 65, 102848. http://dx.doi.org/10.1016/j.jretconser.2021.102848.

Jiang, L., Dhiaf, M., Dong, J., Liang, C., Zhao, S., 2019. A traveling salesman problem with time windows for the last mile delivery in online shopping. Int. J. Prod. Res. 1–12. http://dx.doi.org/10.1080/00207543.2019.1656842.

JLL, 2022. How can parcel lockers improve last-mile delivery? https://www.jll.fi/fi/trendit-ja-tutkimukset/kaupungit/how-can-parcel-lockers-improve-last-mile-delivery. (Accessed: 2022-03-22).

Kerr, I., Różycki, M., 2022. In-store parcel lockers get up and running. https://www.parcelandpostaltechnologyinternational.com/analysis/in-store-parcel-lockers-get-up-and-running.html. (Accessed: 2022-03-22).

Kim, W., Wang, X.C., 2022. The adoption of alternative delivery locations in New York city: Who and how far? Transp. Res. A 158, 127–140. http://dx.doi.org/10.1016/j.tra.2022.02.006.

Lin, Y., Wang, Y., Lee, L.H., Chew, E.P., 2022. Profit-maximizing parcel locker location problem under threshold luce model. Transp. Res. E 157, 102541. http://dx.doi.org/10.1016/j.tre.2021.102541.

Logmore, 2022. Last mile and parcel lockers as logistics customer satisfaction factors. https://www.logmore.com/post/last-mile-customer-experience. (Accessed: 2022-03-22).

Mancini, S., Gansterer, M., 2021. Vehicle routing with private and shared delivery locations. Comput. Oper. Res. 113, 105361. http://dx.doi.org/10.1016/j.cor.2021.105361.

Mitrea, I.A., Zenezini, G., De Marco, A., Ottaviani, F.M., Delmastro, T., Botta, C., 2020. Estimating e-consumers attitude towards parcel locker usage. In: 2020 IEEE 44th Annual Computers, Software, and Applications Conference. COMPSAC, IEEE, pp. 1731–1736. http://dx.doi.org/10.1109/COMPSAC48688.2020.000-5.

Morganti, E., Seidel, S., Blanquart, C., Dablanc, L., Lenz, B., 2014. The impact of e-commerce on final deliveries: alternative parcel delivery services in France and Germany. Transp. Res. Procedia 4, 178–190. http://dx.doi.org/10.1016/j.trpro.2014.11.014.

Oliveira, W.J.P., dos Santos, A.G., 2020. Last mile delivery with lockers: Formulation and heuristic. In: ICEIS (1). pp. 460–467.

Orenstein, I., Raviv, T., Sadan, E., 2019. Flexible parcel delivery to automated parcel lockers: models, solution methods and analysis. EURO J. Transp. Logist. 8 (5), 683–711. http://dx.doi.org/10.1007/s13676-019-00144-7.

Perboli, G., Rosano, M., 2019. Parcel delivery in urban areas: Opportunities and threats for the mix of traditional and green business models. Transp. Res. C 99, 19–36. http://dx.doi.org/10.1016/j.trc.2019.01.006.

Rabe, M., Chicaiza-Vaca, J., González-Feliu, J., 2020. System behavior for automated parcel lockers as an urban logistics delivery scheme.

rebound, 2022. The returning conundrum. https://www.reboundreturns.com/returning-conundrum-report-download/. (Accessed: 2022-06-12).

Risberg, A., 2022. A systematic literature review on e-commerce logistics: towards an e-commerce and omni-channel decision framework. In: The International Review of Retail, Distribution and Consumer Research. Taylor & Francis, pp. 1–25. http://dx.doi.org/10.1080/09593969.2022.2089903.

Rohmer, S., Gendron, B., 2020. A guide to parcel lockers in last mile distribution–highlighting challenges and opportunities from an OR perspective.

Sitek, P., Wikarek, J., 2019. Capacitated vehicle routing problem with pick-up and alternative delivery (CVRPPAD): model and implementation using hybrid approach. Ann. Oper. Res. 273 (1–2), 257–277. http://dx.doi.org/10.1007/s10479-017-2722-x.

Sitek, P., Wikarek, J., Rutczyńska-Wdowiak, K., Bocewicz, G., Banaszak, Z., 2021. Optimization of capacitated vehicle routing problem with alternative delivery, pick-up and time windows: a modified hybrid approach. Neurocomputing 423, 670–678. http://dx.doi.org/10.1016/j.neucom.2020.02.126.

Solomon, M.M., 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. Oper. Res. 35 (2), 254–265. http://dx.doi.org/10.1287/opre.35.2.254.

Statista, 2022a. How common are online returns? https://www.statista.com/chart/27584/how-common-are-online-returns-gcs/. (Accessed: 2022-06-12).

Statista, 2022b. Retail e-commerce sales worldwide from 2014 to 2025 (in billion U.S. dollars). https://www.statista.com/statistics/379046/worldwide-retail-e-commerce-sales/. (Accessed: 2022-06-12).

Symonds, D., 2022. DHL expands parcel locker network in Germany. https://www.parcelandpostaltechnologyinternational.com/news/lockers-pudo/dhl-expands-parcel-locker-network-in-germany.html. (Accessed: 2022-03-22).

Tilk, C., Olkis, K., Irnich, S., 2021. The Last-mile Vehicle Routing Problem with Delivery Options. OR Spectrum http://dx.doi.org/10.1007/s00291-021-00633-0.

Toth, P., Vigo, D., 2014. Vehicle Routing: Problems, Methods, and Applications. SIAM, http://dx.doi.org/10.1137/1.9781611973594.

Vakulenko, Y., Hellström, D., Hjort, K., 2018. What's in the parcel locker? Exploring customer value in e-commerce last mile delivery. J. Bus. Res. 88, 421–427. http://dx.doi.org/10.1016/j.jbusres.2017.11.033.

Wen, J., Li, Y., 2016. Vehicle routing optimization of urban distribution with self-pick-up lockers. In: 2016 International Conference on Logistics, Informatics and Service Sciences. LISS, IEEE, pp. 1–6. http://dx.doi.org/10.1109/LISS.2016.7854384.

Yang, G., Ji, G., 2022. The impact of cross-selling on managing consumer returns in omnichannel operations. Omega 111, 102665. http://dx.doi.org/10.1016/j.omega.2022.102665.

Yu, V.F., Susanto, H., Yeh, Y.-H., Lin, S.-W., Huang, Y.-T., 2022. The vehicle routing problem with simultaneous pickup and delivery and parcel lockers. Mathematics 10 (6), 920. http://dx.doi.org/10.3390/math10060920.