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MATHEMATICS
EDUCATION
How to solve it?

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PSYCHOLOGY OF MATHEMATICS EDUCATION**

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GENDER DIFFERENCES AND DIDACTIC CONTRACT: ANALYSIS OF TWO INVALSI TASKS ON POWERS PROPERTIES

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The results of standardized tests such as PISA and the Italian INVALSI, point out the existence of a gender gap in mathematics. This gap is deeply studied in mathematics educations literature. In this paper we analyse two INVALSI items of grade 10 in which male and female answers have distinctly different behaviour. Our aim is to observe if this different trend of male and female answers is influenced in particular by effects of didactic contract. In this analysis we integrate quantitative and qualitative methods. The quantitative analysis is based on IRT models and it allow us to highlight the trend of the correct and wrong answers, distinguishing between male and female. The qualitative analysis involves interviews to students and confirm that the choice of a particular response is influenced by didactic contract effects.

INTRODUCTION

The INVALSI tests are national standardized tests administered every year in different grades of primary and secondary schools in order to have systematic checks on students' knowledge and skills in maths and Italian. The increasing importance given to standardize tests such as INVALSI and PISA, provides new opportunities not only in the evaluation of educational systems' performances, but also in the educational field. If 10 years ago the usage of PISA results in mathematics education was still limited (Sfard, 2005), in the recent years, many researchers began to use standardize assessments for their studies. For instance, the results of PISA and INVALSI tests showed the existence of a gender gap in mathematics in favour of male and gave the opportunity of study this issue in large populations using also specific statistic tools. In our analysis, we observe that the gender gap is not uniformly distributed on all the items of a test: only some of the tasks present a marked gender gap (in terms of percentage of correct answer of male and female). Moreover, according to recent studies on INVALSI tests (Casella, 2015), the psychometrical analysis of the item functioning reveals that some items present different performances for male and female.

In this paper, we focus our attention on two INVALSI items of grade 10 in which male and female populations have a strictly different behaviour. We select these two items also because we suppose that two wrong choices (we are dealing with multiple choice tests) are related with didactic contract effects. Our purpose is to investigate if, in this particular case, the gender gap and the different behaviour revealed in the quantitative analysis, can be influenced also by a different response to didactic contract for male and female. The first part of our study is a quantitative analysis of the two items based on Item Response Theory models and evidences the different trends related to gender.

The second part is purely qualitative and consists in interviews with the purpose of understanding the processes that led students to choose a particular distractor.

THEORETICAL LENSES

In the recent years, national (INVALSI) and international (such as PISA) assessments pointed out that in mathematics male and female have different performances: boys outperform girls at all school levels and in almost all the countries. This issue has been debated for several years and a large number of studies has focused on the determinants of gender-gap (Forgasz et al., 2010). Standardized assessment, and in particular PISA studies, have given an increasing importance to investigation on gender differences in academic achievement. Various studies highlight the importance of social factors to explain gender-gap in mathematics, evidencing that in more gender-equal cultures this gap disappears (Guiso et al., 2008). This hypothesis is also supported by the fact that this gap is not present at the early stage of school but it raises during the school years. In this research we endorse this idea and, in particular, we assume that education is the main cause of gender differences. A recent study based on PISA 2009 results confirmed this hypothesis and reveals also that “In addition, gender role attitudes within the family environment [...] is found to affect girls’ performance positively.” (González de San Román & De La Rica, 2012).

This research hence fits into a constructivist perspective in which cognitive functions are formed according to the context, and are described as products of social interactions. The learning process cannot be separated from this interactive context defined on the bases of three components: student, teacher and knowledge (Chevallard, 1985). In this paper, we use the idea of didactic contract defined by Guy Brousseau as

behaviour of the teacher expected from the pupil and the behaviour of the pupil expected from the teacher constitute the didactic contract. (Brousseau, 1980).

The didactic contract imposes rules of behaviour and it is the key to analyse the students response to the items analysed in this paper. The relations established between students and teacher, within the *milieu*, could be also studied in detail through other theoretical constructs, such as the concepts of *coutume didactique* and sociomathematical norms (for a comparison of these concepts and an example of how they can network, see Ferretti et al.,; Ferretti, 2015). Moreover our aim is to study if didactic contract, as a product of social context, have a different influence on male and female and, therefore, on gender gap in maths. At first, it’s interesting to notice that, as we have already seen for gender gap in mathematics, also didactic contract seems to be not present in pre-scholar pupils (Baldisserri et al., 1993) but it origins in primary school.

METHODOLOGY

In this research, we use both quantitative and qualitative analysis. The first part of the analysis is purely quantitative and give us the opportunity to observe the behaviour of the items in a large-scale assessment and to make assumption about that behaviour.

The second part of the analysis has the purpose of validate these assumptions through interviews to a restricted group of students.

Quantitative analysis

The two items investigated in this study belong to 2011 and 2012 INVALSI tests of grade 10. For each test, the data analysed are those of INVALSI's sample. This sample consisted of approximately 40.000 students and it is representative of the population of Italian grade 10 students. The INVALSI team proved the consistency of both tests by using the Classical Test Theory tools and made a first analysis using IRT models and in particular the Rasch Model (INVALSI, 2012b; Rasch, 1960). The Rasch model is a simple logistic model and it is useful to analyse a standardized test such as INVALSI because it allows joint estimation of two kind of parameters: a difficulty parameter for each item and an ability parameter for each student. More specifically, this model express the probability of choosing the correct answer in an item as a function of the item's difficulty and the ability of the students in the whole test and this function is called *Item Characteristic Curve*. In this way, it is possible to use Rasch parameters to represent also the empirical data and, in particular, we can represent the trend of each possible response as a function of the students' ability. Those specific graphs are named *Distractor Plots*.

In the INVALSI National Annual Report 2011, gender differences in math tests are identified on the basis of total medium score observed for male and female (INVALSI, 2011). This gap is perceived in both of the tests analysed and it is statistically significant (INVALSI, 2011; INVALSI, 2012a).

Starting from these INVALSI results and the same dataset, we compare percentages of male and female answers, then we use the Rasch Model to study distractor plots for male and female separately. Distractor plots allow us to study gender differences in relation with the ability level of the students and, in particular, we can observe if there are differences not merely in choosing the right answer but also in trends of the incorrect ones.

Qualitative analysis

The second part of our study is purely qualitative and consists in interviews of a restricted group of students about the items analysed in this paper. For this purpose, we administer in two classes of the same high school a brief questionnaire, consisting in 5 mathematical items including one of the two items studied in this research (Fig. 3). The other items are designed to contextualize the studied task into a mathematical test and to evidence if a student face up the test seriously. Just after the correction of the questionnaire, we select 22 of the 49 students for the interview. We select them on the basis of their response to the task studied and their maths score provided by the teacher. In particular, we choose to interview principally students good at maths (school mark $> 6.5/10$) who didn't answer correctly to the item. The interviews are semi-structured, task based and in couples. We decide to interview together students that had selected different options and ask them to explain to the classmate the reasons of their decision.

At a later stage, we present them the other item and ask to compare it with the first one. Each interview takes about 20 minutes and is audio taped. At last, we transcribe the interviews and analyse the transcriptions.

ITEMS ANALISYS

In this research, we focus our attention on two similar items: the question intent is the same, both concern the same content (powers properties) and the answers are analogous. Both tasks are multiple-choice questions with only one correct answer but one is set into an algebraic context (Fig. 1) and the other into an arithmetical context (Fig. 3). Moreover, in both items we register a remarkable difference in male and female performances.

The expression $a^{37} + a^{38}$ is also equal to

- A. $2a^{75}$
- B. a^{75}
- C. $a^{37}(a+1)$
- D. $a^{37 \cdot 38}$

Figure 1: Item from the grade 10 INVALSI test administered in 2012 [1]

The correct answer is C and it is chosen only by 35% of students. Option A, in which the base is the sum of the basis and the exponent is the sum of exponents, is chosen by 19% of students. Option B and D are similar because the resulting power has the same base of the original ones but the resulting exponent is the sum of two exponent in option B and the product in option D. The 26% of the respondents select answer B, which is the most attractive wrong answer and option D is chosen by 16% of students. In addition, only 3% of students do not respond to this question and this may mean that students are fairly confident about their answers. In the table below, we can also observe that male responded better than female: 38% of male give the right answer compared with 31% of female.

| | Total | Male | Female |
|----------|-------------|-------------|-------------|
| A | 19 % | 19 % | 20 % |
| B | 26 % | 27 % | 26 % |
| C | 35 % | 38 % | 31 % |
| D | 16 % | 14 % | 19 % |
| Missing | 3 % | 3 % | 3 % |

Table 1: Results of item (Fig. 1) from INVALSI test administered in 2012.

Observing the table (Tab. 1) of percentage, we can also see that the response D is more attractive for girls. These and others particularities of these item responses are more visible using the results of Rasch analysis to graph distractor plots.

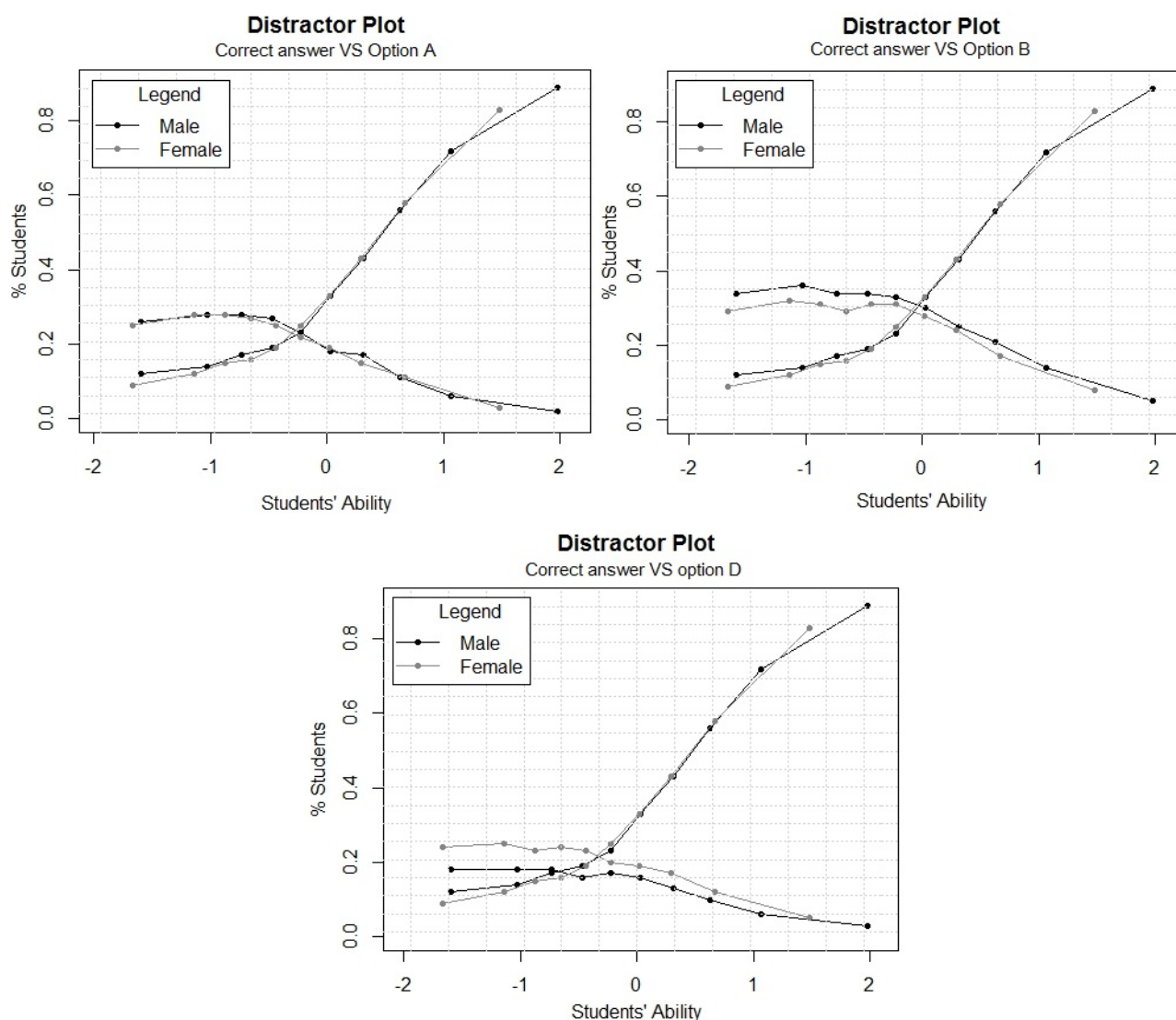


Figure 2: Distractor plots of the first item [1]: comparison between the right answer and the other options (the right answer is the one with increasing trend).

Distractor plots reveal that the trend of the correct answer is almost the same for male and female, although the different percentage seen before. This means that girls and boys with the same ability level choose the correct response with the same percentage, the gender gap observed before in the correct answer (Tab. 1) arises from the fact that female reaching highest ability levels are fewer than male. In Figure 2 we can notice that also the trend of the answer A is almost the same for both male and female but the differences are evident in the behaviour of the other two options. As we observed before, answer D is more attractive for female at all ability levels and, obviously, especially for the lower ones in which it is chosen by a higher percentage of students. Furthermore, boys at all levels of competency prefers the response B compared to girls.

These results become more relevant compared with those of the second item analysed. Indeed, the evidence observed before are all confirmed analysing the second item (Fig. 3) in which the sum of power is analogous but given in an arithmetical context.

- The expression $10^{37} + 10^{38}$ is also equal to
- A. 20^{75}
 - B. 10^{75}
 - C. $11 \cdot 10^{37}$
 - D. $10^{37 \cdot 38}$

Figure 3: Item from the grade 10 INVALSI test administered in 2011.

This second item results more difficult than the first one and the percentage of correct answers is only 22%. Furthermore, the gender-gap is still present and more relevant than before: only 18% of female choose the correct answer in comparison with the 27% of male. It is very interesting to notice that the trends observed in the distractor plots of the first item, are the same that we can notice in this question, despite the fact that the first one has an higher percentage of correct answers.

STUDENT'S INTERVIEWS

The interviews reveal that most of the students facing with these items are immediately led to identify and apply some kind of rule, in particular those who choose a wrong option. This is a typical didactic contract behaviour. Many students say clearly that, when they see two powers with the same bases they directly think to have to apply powers properties and the reason of this behaviour is inherent into didactical practice:

- 1 S1: I had in mind powers rules, I was confused. I didn't even think to option C.
- 2 I: So when you see powers you immediately think...
- 3 S1: I think to rules. I think: "It will be something with rules".
- 4 I: Why?
- 5 S1: I don't know, it is what we've always done in the exercises.

Students who choose options B and D remember more or less correctly power properties and for this reason, they expect that the result have the same basis. Then they exclude option A because of the base. Moreover, we observe that options B and D are attractive also for students who know properly powers' properties. In the interview below, for example, the student remember the properties for the product and therefore he excludes option B, but even more he is led to find some other rules to solve the exercise.

- 1 I: Why students choose $10^{37:38}$?
- 2 S2: Maybe there was a rule according to which to sum powers with the same basis you have to multiply the exponents.
- 3 I: And this one? Why did they select 10^{75} ?
- 4 S2: Also for this one because there was a rule that says to sum the exponents, but this (rule) is when there is a product! This (answer) is not right because this rule is valid when there is a product!

Students who choose options B and D, often explain their decision on the basis of some kind of rules that derive from their school experience and, in particular, from their relation with the teacher and the milieu habits. Those responses can be related with didactic contract: students choose answers B and D believing that when they solve an exercise that include powers with the same bases they have to apply powers' rules. This behaviour is also observed in students who know powers properties. Moreover, we observed that this attitude belongs to the classroom habits and routine, therefore we can refer this phenomenon to didactic costume seen as the habits picked up in didactical practice during mathematical lessons.

CONCLUSIONS AND FUTURE PERSPECTIVES

This paper presents a study of gender-gap in mathematics from a two-fold point of view: we study the behaviour of male and female facing a mathematical task using specific statistic tool and, in particular, distractor plots. The results of standardized tests analysed using Rasch Model enable us to observe gender differences not only in the whole test, but also focusing on a particular item, objet of our study. Moreover, distractor plots evidence the different behaviour of male and female in choosing each possible answer related their ability in the whole test. The two items analysed in this paper presented the same interesting evidences comparing male and female performances. The gender-gap (in terms of percentage of correct answer) is remarkable in both the tasks. Moreover, distractor plots reveals that, in both the items, Option A have the same trend for male and female, male prefer Option B and female prefer option C at all levels of competency, also for the highest ones. The interviews allow us to interpret these results on the base of students responses and underline that answer B and D can be explained using the construct of the didactic contract. Indeed, the students interviewed always refer to classroom practice, relation with the teacher and the milieu habits. Integrating all these information, we notice that the gender gap, in these particular tasks, is influenced by didactic contract effects: indeed, Option A and the missing percentages are the same for male and female in both the items and therefore the gap in the correct answer percentages, is due to options B and C which are related to didactic contract. We also assume that, in this tasks, male and female are influenced in a different way by didactical contract because, even though both options B and D are related with this construct, the first is preferred by male and the second by female. This different behaviour of male and female could be analyse deeper in future studies, including more interviews.

Finally, the structure of this research could be used also to analyse other items or to study differences not only between male and female but also for other groups of students. Indeed, the Rasch analysis and, in particular, the study of distractor plots shall can provide numerous others evidences that can be interpreted using qualitative analysis and interviews.

References

- Baldisserri, F., D'Amore, B., Fascinelli, E., Fiori, M., Gastaldelli, B., & Golinelli, P. (1993). I palloncini di Greta. *La matematica e la sua didattica*, 4, 444-451.
- Brousseau, G. (1980a). Les échecs électifs dans l'enseignement des mathématiques à l'école élémentaire. *Revue de laringologie otologie rinologie*, 3, 107-131.
- Cascella, C. (2015). Male and Female Performance in Mathematics: Empirical Evidence from Italy. *The International Journal of Interdisciplinary Educational Studies*, 9(3-4), 1-9.
- Chevallard Y. (1985). *La transposition didactique. Du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage.
- Ferretti, F, Gambini, A., & Bolondi, G. (in press). The age of the earth effect: a situation of didactic contract. *13th International Congress on Mathematical Education (ICME-13)*, Hamburg, Germany.
- Ferretti, F. (2015). *L'effetto "età della Terra". Contratto didattico e principi regolativi dell'azione degli studenti in matematica* (Doctoral Dissertation, University of Bologna, Italy). Retrieved Genuary, 2016, from <http://amsdottorato.unibo.it/7213>.
- Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *SCIENCE-NEW YORK THEN WASHINGTON*, 320(5880), 1164.
- González de San Román, A., & De La Rica, S. (2012). Gender gaps in PISA test scores: The impact of social norms and the mother's transmission of role attitudes. *IZA Discussion Paper*, 6338, *Institute for the Study of Labor*.
- INVALSI, (2011). *Rilevazione nazionale degli apprendimenti 2010-2011. Le rilevazioni degli apprendimenti*. Retrieved Genuary, 2016, from http://www.invalsi.it/snv1011/documenti/Rapporto_SNV%202010-11_e_Prova_nazionale_2011.pdf.
- INVALSI, (2012a). *Rilevazione nazionale degli apprendimenti 2011-2012. Quadro di sistema*. Retrieved Genuary, 2016, from http://www.invalsi.it/snv2012/documenti/Rapporti/Rapporto_rilevazione_apprendimenti_2012.pdf.
- INVALSI, (2012b). *Rilevazione nazionale degli apprendimenti 2011-2012. Rapporto tecnico*. Retrieved Genuary, 2016, from http://www.invalsi.it/areadati/concidee/Rapporto_Tecnico_Prove.pdf.
- Rasch, G. (1960). Probabilistic models for some intelligence and attainment tests. Copenhagen: Denmark's Paedagogiske Institut.
- Sfard, A. (2005). What Could be More Practical than Good Research?. *Educational Studies in Mathematics*, 58(3), 393-413.
- Forgasz, H., Becker, J., Lee, K., & Steinhorsdottir, O. (2010). International perspectives on gender and mathematics education. *Charlotte, NC: Information Age Publishing*.