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REGULAR ARTICLE

Quantifying the population exposure to employer branding strategies using a Maxwell–Boltzmann distribution-based evolutionary model

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Abstract
Employer branding involves strategies to create a positive corporate image, attracting and retaining high-quality workers. However, the impact of these policies on employees remains unclear in existing literature. In this study, drawing from econophysics literature—particularly the Maxwell–Boltzmann distribution—we use an evolutionary game theory model to investigate the population exposure to these strategies. Through agent-based methods, we analyze two-player populations seeking an optimal equilibrium, exploring the influence of wage offers and employee consumption levels. Additionally, we consider external sponsors, like relatives or universal income providers, who can subsidize wages. Our findings indicate the significant role of external sponsors in game dynamics, prompting their consideration in human resources management.

Keywords Agent-based model · Employer branding · Evolutionary game theory · Human resource management · Labor market · Sponsorship

1 Introduction

Employer branding (EB) strategies are an important tool that organizations use to position themselves as attractive places to work. By doing so, they aim to attract and retain talented employees. While a strong EB can make an organization stand out

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and improve recruitment effectiveness, it also positively affects employee satisfaction and loyalty.

However, there is an aspect of EB that often goes unnoticed: It does not equally impact everyone in the labor market. Some workers, due to economic constraints, might find an organization appealing but cannot consider joining it. For instance, if a firm's wage w does not meet an individual's essential consumption needs c, then the organization's EB becomes less relevant for that person's job choice.

This article investigates the reach and implications of EB to develop a concept of exposure to EB with managerial implications. We employ methods from econophysics and evolutionary game theory to study how different combinations of employees and employers experience EB differently. In Sect. 2, we review the existing literature on EB and introduce our static model. Within this model, we highlight the occupational choice's economic feasibility ratio φ as an exposure condition. In simple terms, φ represents whether an individual can realistically consider a particular employer based on economic factors.

In Sect. 3, we describe the methods employed to explore the dynamics of our static model. We present an evolutionary game theory model informed by econophysics. In particular, we draw inspiration from the Maxwell–Boltzmann distribution to model the diffusion of employees in the labor market as a consequence of exposure to EB. After defining a base game on these premises, we use agent-based methods to investigate the population dynamics arising from the interactions of employees and employers. To provide robust statistics, we rely on a simulation process with these agent-based experiments. We detail the model's dynamics by introducing an exposure measure based on the latter statistics. Several other analyses concerning the existence, nature, and cardinality of Nash equilibria, possible presence of dominant strategies, and additional game-theoretic properties complement the section. These analyses corroborate the necessity of employing computational methods, since analytical solutions are often not viable.

Our findings are presented in Sect. 4, complete with data visualizations. Section 5 offers a discussion on the implications of our results for EB and its limitations, providing insights valuable for businesses, especially practitioners in human resource management. An exogenous variable denoted as s emerges as influential. This variable represents an external sponsorship for employees, often manifested as financial support from relatives or universal income providers. Such sponsors can effectively subsidize wages, amplifying the exposure to EB in a nonlinear way. They play an important role in the game dynamics, a factor that managerial practitioners should carefully consider. Finally, we conclude with Sect. 6, suggesting future research avenues based on our current work.

2 Employer branding strategies and the exposure problem

EB has emerged as an important strategy for organizations to attract and retain talented employees. In this section, we examine its core concepts and introduce our primary thesis regarding the economic constraints that limit EB effectiveness.

2.1 Relevant literature in human resource management

The concept of Employer Branding is relatively new (Ambler and Barrow 1996; Backhaus and Tikoo 2004; Gilani and Cunningham 2017) but has gained popularity in recent years. As Backhaus and Tikoo (2004) define it, "employer branding represents a firm's efforts to promote, both within and outside the firm, a clear view of what makes it different and desirable as an employer" (p. 501). Organizations implement EB by showcasing their distinctive attributes as workplaces (Sullivan 1999) and highlighting superior work experiences compared to competitors (Love and Singh 2011). This typically involves offering compelling benefits, competitive wages, career opportunities, and both material and symbolic rewards (Lievens and Highhouse 2003; Jain and Bhatt 2015).

The EB process generally involves three stages (Backhaus and Tikoo 2004): defining a value proposition that exhibits the firm's distinctive values; marketing this proposition externally to potential employees; and promoting it internally to integrate it into organizational culture. This approach is grounded in the Resource-Based View theory (Barney 1991), which holds that rare and valuable resources—including human capital—contribute to sustainable competitive advantage. The ability to attract and retain talented employees through effective EB strategies is considered crucial for organizational success (Sutherland et al. 2002; Collins and Stevens 2002).

Researchers and practitioners traditionally evaluate EB effectiveness by measuring attraction and retention outcomes (Biswas and Suar 2014; Bussin and Mouton 2019). While this approach seems sensible, we argue that it overlooks a critical aspect: EB strategies do not reach the entire labor market uniformly, as economic factors may prevent potential employees from considering employment offers regardless of how attractive they find the organization.

2.2 Exposure to employer branding strategies: theoretical background and key features

The traditional perspective views EB as a "bridge" connecting an organization to its labor market. However, this metaphor can be misleading in two important ways. First, the bridge does not connect the organization with all actors in the labor market, but only with those who can economically afford to consider the employment offer. Second, EB effectiveness is often evaluated solely in terms of the strategy itself, without considering whether it reaches people who can actually respond to it.

We contend that the current theoretical approach to EB is limited by two flawed assumptions. The first is the assumption of economic homogeneity. In reality, the labor market exhibits considerable economic diversity, and job offers will only be considered by individuals who can economically sustain that employment arrangement. Consider an organization seeking a data scientist. Its job announcements will not be evaluated by all qualified data scientists, but only by those who can financially support accepting such a position.

The second flawed assumption is that of economic reachability. While a job announcement may technically reach all qualified candidates, it only functionally reaches those who can realistically consider it. An EB strategy effectively reaches a worker only when the proposal conveyed by the firm is economically viable for that worker.

These considerations highlight an overlooked factor in the managerial literature: EB strategies' effectiveness depends not only on organizational actions but also on economic compatibility between employer offers and worker needs. Even the most sophisticated EB initiatives can be rendered ineffective in an unfavorable employment environment where economic constraints prevent potential employees from considering otherwise attractive opportunities.

The economic feasibility of employment choices depends on three key variables: c (the worker's consumption needs), w (the wage offered by the employer), and s (external sponsorship or non-labor income available to the worker). In the absence of non-labor income, an individual cannot accept jobs where $w < c$. When organizations encounter candidates who cannot economically support a particular employment choice, their EB strategy is neither effective nor ineffective; it simply cannot be activated.

This dynamic introduces a critical third actor into the traditional employer–employee relationship: the sponsor. When comparing situations where $(w + s) < c$ versus $(w + s) > c$, we see that the variable s can make the difference between an EB strategy being inactivated or becoming effective. The sponsorship s represents any form of non-labor income–family support, investment returns, secondary employment, or universal basic income. From an organization's perspective, s is an exogenous environmental factor that nonetheless significantly impacts EB effectiveness. Figures 1 and 2 illustrate the contrast between traditional and more complex views of EB that account for these external factors.

As shown in Fig. 2, the variables c, w, and s shape the labor market in which an organization operates and indirectly affect the reach of its EB strategies. Figure 3 summarizes the relationship between the total labor market and the portion that is actually reachable through EB initiatives.

2.3 Dichotomies in the labor market: exploring the intersection of worker intent and job offer viability

The conventional understanding of the labor market focuses primarily on willingness to work. Standard definitions—such as those found in the Collins, Merriam-Webster, and Cambridge dictionaries—describe the labor market as comprising people who are able and willing to work.[1] This perspective leads to two implicit

[1] For the Collins English Dictionary (2025), "*when you talk about the labor market, you are referring to all the people who are able to work and want jobs in a country or area, in relation to the number of jobs there are available in that country or area*". In the Merriam-Webster Dictionary (2025), we read that labor market means "*the number of workers who are available to be hired*". The Cambridge Dictionary (2025) instead states that labor market is "*the supply of people in a particular country or area who are able and willing to work.*"

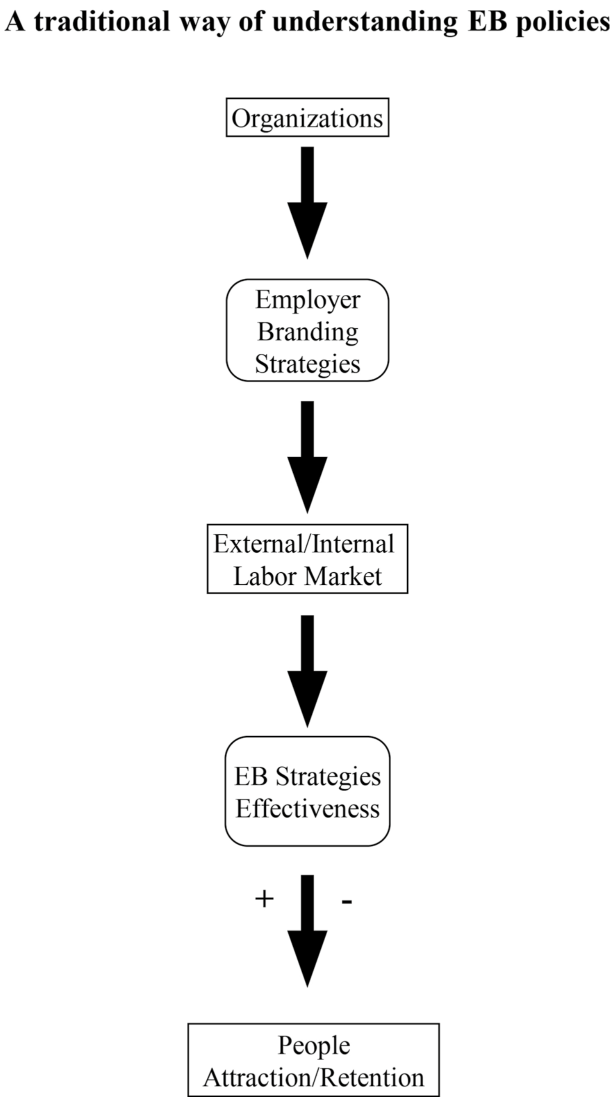


Fig. 1 A "simple" EB strategy. In this traditional view of EB policies, organizations are presumed to have full control over attraction and retention outcomes

assumptions: first, that workers who reject job offers lack sufficient motivation; and second, that rejected offers failed to adequately motivate potential employees.

This focus on willingness overshadows an equally important factor: economic feasibility. Many situations exist where workers are eager to accept positions and employers offer compelling value propositions, yet objective economic circumstances prevent employment relationships from forming. In terms of EB, this means specific economic barriers can render strategies ineffective regardless of their quality or appeal.

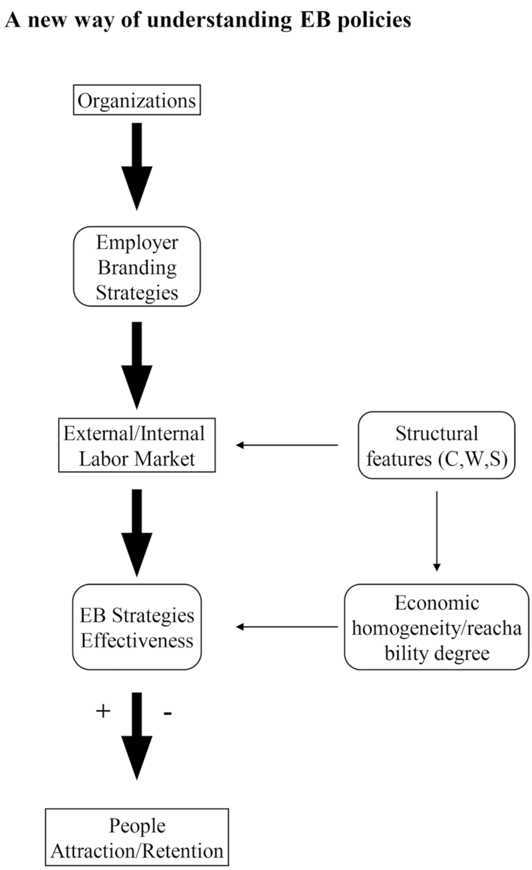


Fig. 2 A "complex" EB strategy. This revised perspective acknowledges that the effectiveness of personnel attraction and retention strategies is significantly affected by variables external to the organization

Based on these considerations, we conceptualize the labor market as divided into four quadrants determined by two dimensions: willingness to work and economic feasibility, as shown in Fig. 4.

We define the occupational choice's economic feasibility (OCEF) ratio $\varphi := c/(w + s)$ as the ratio of consumption needs to available income.[2] This formulation ensures that when $\varphi \leq 1$ consumption needs are met or exceeded by available income, making the employment choice economically feasible (Quadrant II), while $\varphi > 1$ indicates unfeasibility (Quadrant I). Our research focuses on Quadrants I and II, where workers are willing to work but differ in their ability to accept job offers based on economic circumstances.

[2] To avoid discontinuities in certain parts of the payoff functions introduced in the game described in the next section, we define the ratio in this functional form rather than as its mathematical reciprocal, which could have been, in principle, another viable approach.

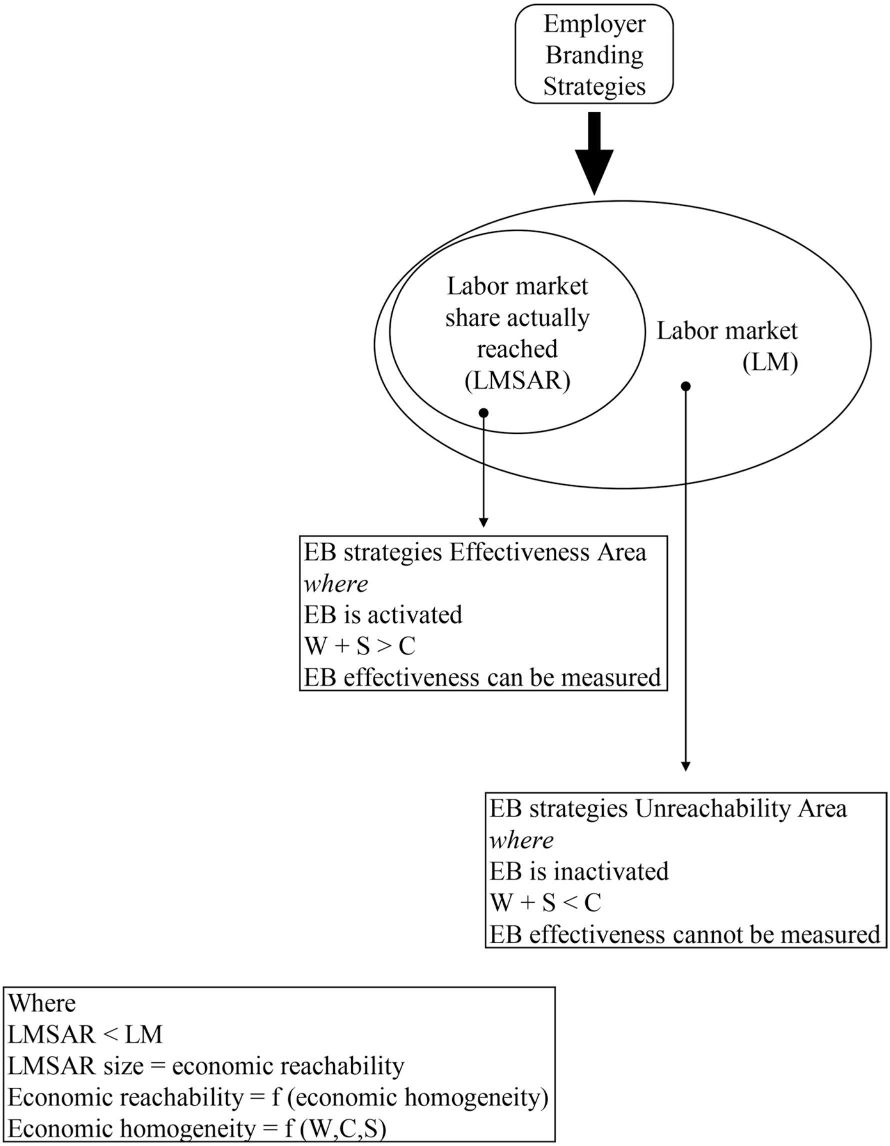


Fig. 3 The relationship between Labor Market Share Actually Reached (LMSAR) and Labor Market (LM). EB strategies reach only part of the labor market because not all workers can realistically evaluate a company's job offerings

In Quadrant I (willing but unable), EB strategies have minimal effect since economic constraints prevent workers from considering offers. This creates both inefficiencies—employers miss talented workers who cannot afford to accept their offers—and inequities, where only those with access to supplementary income can consider certain positions.

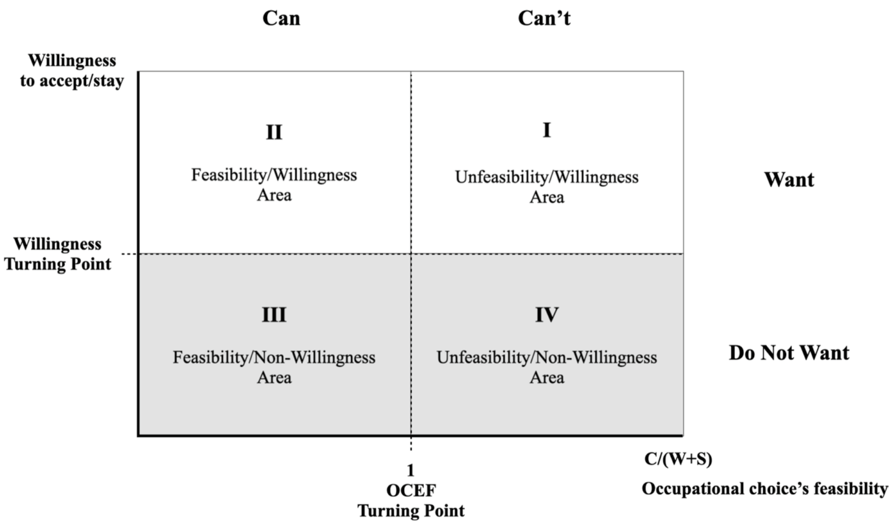


Fig. 4 The labor market quadrants. This framework illustrates that employment choices depend not only on subjective preferences but also on objective economic circumstances, challenging the notion that employment outcomes are determined solely by talent rather than also by fortune (Pluchino et al. 2018)

In Quadrant II (willing and able), traditional EB strategies can function effectively since workers can financially sustain the employment relationship. Here, worker decisions are based more on the qualitative aspects of the employer's value proposition.

This framework suggests different approaches to EB depending on the target quadrant. For Quadrant II, organizations can focus on enhancing their value proposition. For Quadrant I, organizations must first address economic feasibility barriers before their value proposition becomes relevant. The real challenge in many labor markets is attracting not only workers but also sponsors, as non-labor income can facilitate movement from Quadrant I to Quadrant II.

In the next section, we develop a quantitative model to analyze population exposure to EB strategies based on this understanding of economic feasibility as a prerequisite for effectiveness.

3 Methods

Building upon the previously introduced static model, this section delineates the methodologies utilized to construct our dynamic model. To critically assess the impact of EB strategies, we craft an evolutionary game theory model (Sandholm 2015). Comprising two components, this model first lays out a foundational game delineating the interactions between employers and employees, whose payoff functions are shaped to mimic the diffusion of employees in the labor market as a consequence of exposure to EB. The second component offers a simulation, tailored to forecast the protracted behavior of two-player populations and gauge the exposure of these groups to EB initiatives. This model is anchored on the cardinal tenets of EB, previously denoted as w, c, and s. It postulates that for exposure to EB strategies to materialize, both employers and potential employees must meet a specific criterion, the feasibility condition $\varphi \leq 1$.

The methods section is organized into four distinct subsections. In subSect. 3.1, we introduce the Maxwell–Boltzmann distribution and explain how we use it as inspiration for modeling employee diffusion in the labor market. SubSection 3.2 elucidates the foundational static game that encapsulates the interactions between employees and employers. In subSect. 3.3, an Agent-Based Model is detailed, emphasizing its role in simulating the dynamics of employee–employer interactions within dual populations of agents. Finally, in subSect. 3.4, we provide an exposition of the simulation process steered by the Agent-Based Model, describing the design to yield robust statistics pertinent to the aforementioned interactions and all associated measurements, notably those relating to the exposure to EB.

3.1 Modeling employee exposure using the Maxwell–Boltzmann distribution

To model the exposure of employees to EB strategies, we draw inspiration from statistical physics, specifically the Maxwell–Boltzmann distribution. This approach allows us to quantify how employees transition from being unable to consider job offers (Quadrant I) to being able to consider them (Quadrant II).

The Maxwell–Boltzmann distribution describes the probability distribution of particle speeds in a gas at thermal equilibrium (Mantegna and Stanley 1999; Blundell and Blundell 2009; Scalas et al. 2015). For our purposes, we use its mathematical form to model how workers distribute across different levels of exposure to EB strategies. The distribution function $f(v)$ is given by

$$f(v) = \sqrt{\frac{2}{\pi}} \frac{v^2}{a^3} \exp\left(-\frac{v^2}{2a^2}\right) \tag{1}$$

where v represents speed and a is a scaling parameter.

Our adoption of the Maxwell–Boltzmann framework follows established econophysics methodology for modeling equilibrium behavior among interacting economic agents, where this distribution produces the characteristic unimodal, right-skewed shape often employed for mathematically representing diffusion or equilibrium phenomena. For instance, as shown by Patriarca et al. (2004) and Chakrabarti et al. (2013), models of wealth exchange in closed economies can be mapped to kinetic gas models, where the Maxwell–Boltzmann distribution of particle velocities corresponds to the stationary distribution of agents' resources. In particular, Patriarca et al. (2004) demonstrate that starting from the Maxwell–Boltzmann velocity distribution in D dimensions (with some constraints), the associated kinetic-energy distribution follows a Γ-law with shape parameter $n = D/2$, and that the effective dimension D can be interpreted in behavioral terms—specifically, as a function of agents' saving propensity—thus providing a microfounded explanation for an empirically observed regularity in wealth distributions.

In our EB exposure model, we define the velocity $v = w + s - c$; in this setting, higher values of v represent employees more strongly exposed to EB strategies. We treat w (wage offered by the employer) and c (employee's consumption level) as endogenous model parameters, while s (sponsorship amount) is treated as an exogenous parameter. All three parameters are modeled as real numbers in the interval [0, 1].

In doing so, v captures the employee's net economic advantage and thus serves as a one-dimensional measure of exposure. Conceptually, as shown in the previous section, it is true that we represent employees as agents occupying positions in a two-dimensional feasibility space—defined by willingness and economic feasibility—but the actual dynamics of exposure, captured by v, are one-dimensional. Hence, while the diffusion process occurs along a single economic dimension, the analytic form of the Maxwell–Boltzmann speed distribution in three dimensions is adopted as a functional analogue.

In line with this interpretation, we adapt Equation (1) to model the distribution of employees in Quadrant II as:

$$f(w+s-c) = \frac{\sqrt{\frac{2}{\pi}}(w+s-c)^2 \exp\left(-\frac{(w+s-c)^2}{2(2-s)^2}\right)}{(2-s)^3} \tag{2}$$

This function holds only when $w + s \geq c$ (i.e., $\varphi \leq 1$), meaning employees have been exposed to EB strategies. We constrain $v = w + s - c$ to be non-negative, specifically $0 \leq v \leq 1 + s$. We set the scale parameter $a = 2 - s$, modeling how sponsorship affects the distribution: As s increases, the shape of the distribution changes, reflecting how external financial support impacts the exposure process.

Figure 5 illustrates these distributions for different values of s. As sponsorship increases, the curves shift, representing changing patterns of exposure to EB strategies.

This mathematical approach provides a foundation for our utility functions, which we will develop in the next subsection within a game-theoretic framework. Using the Maxwell–Boltzmann distribution allows us to capture the nonlinear effects of sponsorship on EB exposure in a tractable way. Our utility functions will reflect this relationship, particularly for employees in Quadrant II where EB strategies can be effective.

3.2 The employee–employer game

Building on our static model, we now develop a game-theoretic framework that captures the strategic interactions between employees and employers. We construct a two-player 10×10 normal form game representing a one-shot interaction between an employee W (worker) and an employer E.

The employee chooses from a set of consumption levels $C = \{c_1, \dots, c_i, \dots, c_{10}\}$ with $c_i \neq c_j$ for all pairs $c_i, c_j \in C$. The employer selects from a set $R := W \times \{d\}$, where $W = \{w_1, \dots, w_m, \dots, w_{10}\}$ is a set of wage offers, and $d : C \times W \rightarrow \{\alpha, \tilde{\alpha}\}$ is a response function assigning, to each consumption-wage pair, either "successfully exposing to EB strategies" (α) if $\varphi \leq 1$ or "failing to expose to EB strategies" ($\tilde{\alpha}$) if $\varphi > 1$.

We incorporate sponsorship through a set $S = \{s_0, \ldots, s_v, \ldots, s_{20}\}$, where each s_v represents a level of external financial support. These values affect both players' utility functions, though in different ways.

For the employee, we define the payoff function $\pi_W : C \times R \to \mathbb{R}$ as follows,

$$\pi_W(c_i, r_k) = \begin{cases} \frac{\sqrt{2/\pi}\,(w_m+s_v-c_i)^2}{(2-s_v)^3} \exp\left(-\frac{(w_m+s_v-c_i)^2}{2(2-s_v)^2}\right), & \text{when } w_m + s_v \geq c_i, \text{ i.e., } \varphi \leq 1, \\ \frac{s_v}{c_i}, & \text{otherwise.} \end{cases} \tag{3}$$

This function has two distinct parts based on whether the economic feasibility condition is met. When $\varphi \leq 1$ (the employee can consider the job offer), their utility follows our Maxwell–Boltzmann inspired model, reflecting how exposure to EB strategies creates value for the employee. The shape of this function increases with $w + s - c$ up to a point before declining, capturing the idea that there is an optimal level of "excess income" beyond which additional resources provide diminishing returns. When $\varphi > 1$ (the employee cannot consider the job offer), their utility is simply the ratio of sponsorship to consumption needs. This represents the value they derive solely from non-labor income, which is inversely proportional to their consumption needs and directly proportional to available sponsorship.

For the employer, we define the payoff function $\pi_E : C \times R \to \mathbb{R}$ as

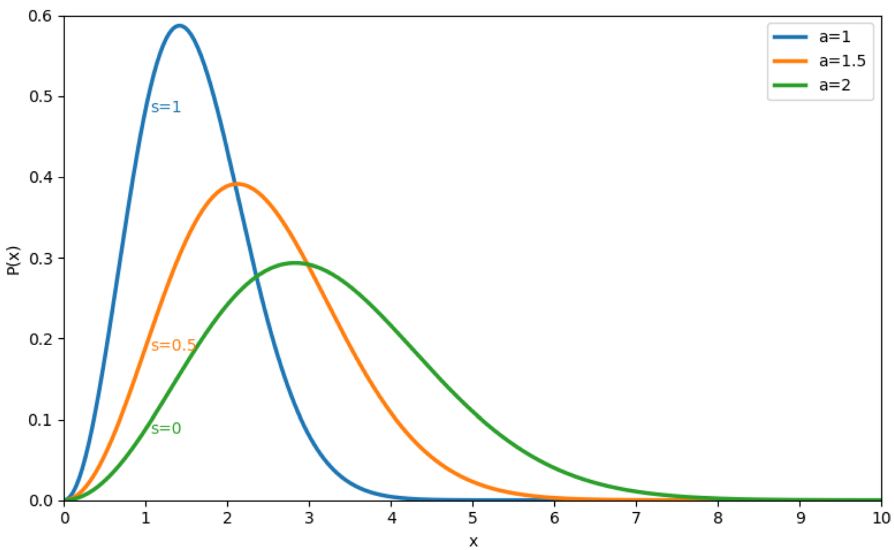


Fig. 5 Maxwell–Boltzmann distributions. The x-axis shows the exposure level (represented by $w + s - c$), and the y-axis shows the probability density. No exposure occurs at $s = 0$ (green curve), while maximum exposure is reached at $s = 1$ (blue curve)

$$\pi_E(c_i, r_k) = \begin{cases} (1+s_v)(1-w_m)w_m, & \text{when } w_m + s_v \geq c_i, \text{ i.e., } \varphi \leq 1, \\ (1-s_v)w_m, & \text{otherwise.} \end{cases} \tag{4}$$

The employer's utility function also has two components. When $\varphi \leq 1$, their utility is a quadratic function of wages $(1 - w_m)w_m$, which reaches its maximum at $w_m = 0.5$. This reflects the trade-off employers face: Paying higher wages attracts better talent but reduces profit margins, while paying too little reduces attractiveness to workers. This function is scaled by $(1 + s_v)$, indicating that employers benefit more when employees have additional financial support because it allows them to accept lower wages while maintaining the same standard of living. When $\varphi > 1$, the employer's payoff is $(1 - s_v)w_m$, which decreases with sponsorship. This may seem counterintuitive, but it represents the opportunity cost to employers when they fail to secure employees despite the presence of external financial support. Higher sponsorship increases this opportunity cost, creating pressure on employers to adjust their wage offers to successfully attract employees.

These utility functions capture the complex interactions between wages, consumption needs, and sponsorship. While some aspects may appear counterintuitive (such as employer utility increasing with sponsorship in one case and decreasing in another), they model real tensions in the labor market where external financial support can both help and challenge employer strategies.

3.3 The agent-based model

When selecting EB strategies, employers must consider both their own experiences and those of their peers. To model this dynamic, we employ an agent-based model that allows employers and employees to update their strategies over time. Agent-based models simulate the actions and interactions of autonomous entities to assess their system-wide effects (Biondi and Righi 2019; Gomes 2015). Such models have been successfully applied to study wealth distribution and economic exchanges (De Pretis et al. 2025), making them particularly relevant for our analysis of economic factors affecting EB strategies.

Our agent-based model represents interactions between a population of employees and a population of employers who repeatedly play the employee–employer game. Each employee is initially assigned one of 10 consumption levels from the set $C := \{0.1, \ldots, 1\}$, and each employer is assigned one of 10 wage offers from the set $W := \{0.1, \ldots, 1\}$. We explore 21 levels of sponsorship $S := \{0, \ldots, 1\}$, including the possibility of no sponsorship. A match between an employee and employer occurs when $w_m + s_v \geq c_i$, i.e., $\varphi \leq 1$.

This setup ensures each employee can be exposed to multiple types of employer strategies, reflecting uncertainty about what employers will offer. We set both populations to contain 1800 agents: This population size is well within the range where finite-population stochastic effects become negligible while maintaining computational tractability (Adami et al. 2016; Hindersin et al. 2019), ensuring our results provide robust statistical estimates of long-run evolutionary dynamics. Moreover, this calibration follows established practice in similar evolutionary game simulations and agent-based models (Radzvilas et al. 2023; Izquierdo et al. 2024; Fagiolo et al. 2007; Lux 2018).

We use discrete rather than continuous strategy sets for several reasons. While continuous sets might offer theoretical advantages for analyzing Nash equilibria, discrete sets enhance practicality and realism. It is more reasonable to assume that strategy imitation involves distinct options rather than infinitely subtle distinctions. Additionally, we assume bounded rationality for both employers and employees, implementing logit choice procedures for strategy selection.

Our model prioritizes computational simulations over analytic solutions due to the system's inherent complexities. Simulations provide unique insights into convergence processes and evolutionary dynamics that analytic methods often struggle to capture, especially for finite-population games with multiple stable states or without evolutionarily stable states.

In the initial state, we uniformly assign 180 agents to each strategy to avoid artificial bias. We implement a probabilistic strategy revision procedure: Each round, agents have a 0.122 probability of receiving a strategy revision opportunity. This means approximately 219 agents revise strategies per round.

The revision process is imitative. When revising, an agent compiles a list of n candidates including themselves and $n-1$ randomly selected others. This captures the tendency to follow popular strategies, as those used by more agents are more likely to appear among the candidates.

Each candidate plays the employee–employer game against every opponent type, and the revising agent observes their average payoffs. This creates a candidate record $(\sigma, \overline{\pi})$, a $n \times 2$ matrix where the first column contains strategies and the second contains average payoffs.

For a revising agent choosing from strategies $\zeta = (1, \dots, z, \dots, Z)$, the logit choice protocol defines the probability of switching to strategy $z \in \zeta$ as:

$$P(\sigma, \overline{\pi}) = \frac{\sum_{h:\sigma_h = z} \exp\left(\eta^{-1}\overline{\pi}_h\right)}{\sum_{\omega \in \zeta} \sum_{h:\sigma_h = \omega} \exp\left(\eta^{-1}\overline{\pi}_h\right)} \tag{5}$$

The parameter η represents noise or bounded rationality, where agents occasionally make mistakes in identifying optimal strategies. We set $n = 30$ candidates (a realistic number of peers an employee might assess) and use a low noise level $\eta = 0.044$, approximating rational behavior while allowing occasional deviations.

3.4 The simulation process and related measurements, including exposure

For simulations, we used the ABED simulation package for NetLogo developed by Izquierdo et al. (2019). To provide robust statistics, we carried out 10^3 simulations with each simulation running for 10^4 iterations. This large number of iterations helps to identify the long-run tendencies of each game variant. Prior to simulations, we calculated the payoff matrices using a Python 3 script, according to the equations for employees and employers shown in subSect. 3.2. A total of 21 matrices—all with dimensions 10×10—were computed for different sponsorship values.

For each payoff matrix, we initially verified if it showed potential game features using a criterion provided by Sandholm (2010). Let $N = 10$ denote the number of strategies available to each player. We checked if

$$a_{ij} + \frac{1}{N^2}\sum_{i,j} a_{ij} - \frac{1}{N}\left(\sum_{i} a_{ij} + \sum_{j} a_{ij}\right) = b_{ij} + \frac{1}{N^2}\sum_{i,j} b_{ij} - \frac{1}{N}\left(\sum_{i} b_{ij} + \sum_{j} b_{ij}\right), \tag{6}$$

where a_{ij} and b_{ij} represent players' payoffs. According to numerical checks (see Sect. 4, **Table** 1), none of the matrices satisfied this property, nor did they meet the minimum conditions to be considered near-potential games (Lahkar and Riedel 2015).

Additionally, using the Python library Nashpy (Knight and Campbell 2018) enhanced with methods from Avis et al. (2009), we investigated the existence and number of Nash equilibria in pure and mixed strategies in every payoff matrix, as well as the existence of dominant strategies, to exclude the presence of trivial results that could be computed analytically (see Sect. 4, Tables 1 and 2, panel (c)).

After these preliminary checks, we ran 10^3 simulations for each payoff matrix and assessed a discrete probability distribution P for the 10 strategies that both employees and employers considered. This led to the computation of $\Delta_{s_i} = (\delta_{ij}, i,j = 1,\dots,10)$, a matrix containing the average probability distribution for strategies given a sponsorship level s_i, paired with $\Theta_{s_i} = (\theta_{ij}, i,j = 1,\dots,10)$, the matrix computing φ values for each possible combination of strategies.

For $s_i = 0$ and a uniform distribution for Δ_{s_i}, we would theoretically observe a 50% exposure between employees exposed and unexposed to EB strategies (equally distributed between Quadrants I and II).

Based on the simulations, the computation of matrices Δ_{s_i} and Θ_{s_i} is crucial for determining population exposure to EB strategies. To assess this exposure, we extract the positive elements of the matrix $(\mathbf{1} - \Theta_{s_i})$ and weight them by Δ_{s_i}. The process involves:

1. Applying a rectifier linear unit function $f(x) = x^+$ and a Legendre ceiling function $f(x) = \lceil x \rceil$ to $(\mathbf{1} - \Theta_{s_i})$:

$$\mathbf{A}_{s_i} = \left\lceil (\mathbf{1} - \Theta_{s_i})^+ \right\rceil \tag{7}$$

2. Weighting via a Hadamard product $\odot$:

$$\mathbf{B}_{s_i} = \sqrt{\Delta_{s_i}} \odot \mathbf{A}_{s_i} \tag{8}$$

3. Calculating the exposure estimate as

$$e(s_i) = \mathrm{tr}(\mathbf{B}_{s_i}^\top \mathbf{B}_{s_i}). \tag{9}$$

Table 1 Relationship between the level of Sponsorship (S) and its impact on Nash Equilibria in the payoff matrices and other measures

S	P-NE	M-NE	DS	PG	MPS
0	1	0	×	×	(S1;S6)
0.05	6	48	×	×	(S7;S7)
0.1	4	7	×	×	(S8;S8)
0.15	5	14	×	×	(S8;S7)
0.2	3	4	×	×	(S9;S8)
0.25	3	4	×	×	(S9;S7)
0.3	1	0	×	×	(S9;S7)
0.35	2	1	×	×	(S10;S7)
0.4	0	1	×	×	(S1;S7)
0.45	0	1	×	×	(S1;S7)
0.5	1	0	×	×	(S1;S6)
0.55	1	0	✓	×	(S1;S6)
0.6	1	0	✓	×	(S1;S6)
0.65	1	0	✓	×	(S1;S6)
0.7	1	0	✓	×	(S1;S6)
0.75	1	0	✓	×	(S1;S6)
0.8	1	0	✓	×	(S1;S6)
0.85	1	0	✓	×	(S1;S6)
0.9	1	0	✓	×	(S1;S6)
0.95	1	0	✓	×	(S1;S6)
1	1	0	✓	×	(S1;S6)

Detailed comparisons include the existence and cardinality of Pure strategy Nash Equilibria (P-NE) and of Mixed strategy Nash Equilibria (M-NE), the existence of Dominant Strategies (DS) and if payoff matrices showed Potential Games (PG) features, and eventually, the identification of the Most Probable Strategies (MPS) arising from simulations

Simulations for the full spectrum of values for C, W, and S sets indicate that as $s_i \in S$ increases, the population exposure to EB strategies tends to increase in a non-linear fashion. This outcome and further results will be presented in the next section.

4 Results

In this section, we present a series of results that offer a detailed analysis of how sponsorship levels influence strategic interactions between employers and employees. **Table** 1 reveals the relationship between varying levels of sponsorship and the dynamics of Nash Equilibria, including the presence of pure and mixed strategy Nash Equilibria, dominant strategies, and the check of possible features of potential games for the payoff matrices. This table also identifies the combination of the most probable strategies for players as emerging from our simulations, shedding light on the strategic tendencies in employer–employee interactions.

When the sponsorship level exceeds 0.5 (i.e., $s > 0.5$), a dominant strategy emerges, leading to the combination (S1;S6) as the most probable strategic outcome. This scenario aligns with the unique pure strategy Nash equilibrium, as shown in Table 3 for $s = 0.5$. Conversely, for a sponsorship level of 0.2 (i.e., $s = 0.2$ as shown in Table 2), the situation is markedly different. Here, we observe multiple pure and mixed strategy Nash equilibria. Their interplay creates complex dynamics, making it challenging to predict the most probable strategy through analytical methods alone. As a result, simulations become essential. Notably, the strategy combination of (S9;S8), which simulations identified as probable, was not anticipated by analytical computations.

In Tables 2 and 3, we present examples of the payoff matrices for both employers and employees that underlie our simulations. These matrices show how the payoffs vary across different strategy combinations and sponsorship levels.

We also conducted graphical analyses, as shown in Figs. 6 through 9. Figure 6 displays the average strategy distributions for both players at sponsorship levels of $s = 0.35$ and $s = 0.4$, highlighting the variability in strategic preferences with changing sponsorship levels. Notably, a shift in the most probable strategy for employees was observed across these two sponsorship levels.

Figure 7 offers a trajectory of the most probable strategies over varying sponsorship levels, illustrating the nonlinear evolution of equilibrium outcomes and the sensitivity of the system to strategic shifts. This trajectory was informed by our simulation data and the insights from Table 1.

Figure 8 compares the model-simulated outcomes with benchmark values, focusing on the maximum probability attained by Maxwell–Boltzmann distributions, and underscores the divergence in outcomes, particularly in scenarios with no EB exposure.

Lastly, Fig. 9 depicts the exposure to EB strategies across different sponsorship levels, providing a comparative analysis against a benchmark uniform distribution exposure. This comparison highlights three different areas where (1) model-simulated exposure is much stronger than the benchmark for small values of s; (2) the opposite holds true (red area) for medium values of s; and (3) model-simulated exposure and the benchmark become nearly equal, with the former slightly dominating for large values of s.

Collectively, these results underscore the complexity of EB strategies and their impact on the strategic decisions within the employer–employee dynamic. They demonstrate the intricate interplay between various levels of sponsorship and the resultant strategic choices made by players, using our Maxwell–Boltzmann distribution-based evolutionary model to capture the multifaceted nature of EB in a competitive labor market.

5 Discussion

Our Maxwell–Boltzmann distribution-based evolutionary model, employed to quantify the population exposure to EB strategies, has provided insights that bridge the gap between the effects of sponsorship s and the nuances of EB effectiveness. The

Table 2 Example of payoff matrix for $s = 0.2$, whose entries are computed according to the equations (4) and (5) of subSect. 3.2. In panel (a) and (b), payoff values for employee (row-player) and employer (column-player) are reported, respectively. Pure strategy Nash Equilibria are marked by bold text. Panel (c) shows detailed information on the probability distribution associated to the three pure and four mixed strategy Nash equilibria

$$\begin{bmatrix} 0.009 & 0.023 & 0.018 & 0.029 & 0.043 & 0.059 & 0.076 & 0.096 & 0.116 & 0.137 \\ 0.003 & 0.013 & 0.009 & 0.018 & 0.029 & 0.043 & 0.059 & 0.076 & 0.096 & 0.116 \\ 0. & 0.006 & 0.003 & 0.009 & 0.018 & 0.029 & 0.043 & 0.059 & 0.076 & 0.096 \\ 0.625 & 0.001 & 0. & 0.003 & 0.009 & 0.018 & 0.029 & 0.043 & 0.059 & 0.076 \\ 0.5 & 0. & 0.5 & 0. & 0.003 & 0.009 & 0.018 & 0.029 & 0.043 & 0.059 \\ 0.417 & 0.417 & 0.417 & 0.417 & 0. & 0.003 & 0.009 & 0.018 & 0.029 & 0.043 \\ 0.357 & 0.357 & 0.357 & 0.357 & 0.357 & 0. & 0.003 & 0.009 & 0.018 & 0.029 \\ 0.312 & 0.312 & 0.312 & 0.312 & 0.312 & \mathbf{0.312} & 0. & 0.003 & 0.009 & 0.018 \\ 0.278 & 0.278 & 0.278 & 0.278 & 0.278 & 0.278 & \mathbf{0.278} & 0. & 0.003 & 0.009 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0.25 & 0.25 & 0.25 & \mathbf{0.25} & 0. & 0.003 \end{bmatrix}$$

(a) Payoff matrix for Employee

$$\begin{bmatrix} 0.113 & 0.234 & 0.200 & 0.263 & 0.300 & 0.312 & 0.300 & 0.263 & 0.200 & 0.112 \\ 0.113 & 0.234 & 0.2 & 0.263 & 0.3 & 0.312 & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.113 & 0.234 & 0.2 & 0.263 & 0.3 & 0.312 & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.234 & 0.2 & 0.263 & 0.3 & 0.312 & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.234 & 0.15 & 0.263 & 0.3 & 0.312 & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.188 & 0.15 & 0.225 & 0.3 & 0.312 & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.188 & 0.15 & 0.225 & 0.3 & 0.312 & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.188 & 0.15 & 0.225 & 0.3 & \mathbf{0.375} & 0.3 & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.188 & 0.15 & 0.225 & 0.3 & 0.375 & \mathbf{0.45} & 0.263 & 0.2 & 0.112 \\ 0.075 & 0.188 & 0.15 & 0.225 & 0.3 & 0.375 & 0.45 & \mathbf{0.525} & 0.2 & 0.112 \end{bmatrix}$$

(b) Payoff matrix for Employer

	Employee										Employer									
# NE	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
4	0	0	0	0	0	0	0	1/2	1/2	0	0	0	0	0	0	139/156	17/156	0	0	0
5	0	0	0	0	0	0	0	1/2	19/262	56/131	0	0	0	0	0	8677/10842	1073/10842	14/139	0	0
6	0	0	0	0	0	0	0	75/131	0	56/131	0	0	0	0	0	247/309	0	62/309	0	0
7	0	0	0	0	0	0	0	0	75/262	187/262	0	0	0	0	0	0	125/139	14/139	0	0

(c) Probability distribution for pure and mixed strategy Nash Equilibria (NE): according to the first column, the latter are numbered from 4 to 7, whereas the former from 1 to 3.

model has successfully illustrated the intricate dynamics between these elements, which is especially evident in the tri-zonal impact of sponsorship on exposure (see Fig. 9).

The delineation into three distinct zones concerning sponsorship's impact represents the complex dynamics in the realm of EB. In the first zone, where s is relatively small, the exposure to EB strategies is significantly heightened, outperforming the exposure from a benchmark uniform distribution where s is assumed to be ineffective. This observation indicates the potential potency of minimal sponsorship. However, in the central zone, an escalation in sponsorship becomes counterproductive, with exposure receding below benchmark levels. This trend is an important reminder of the delicate balance required in EB strategies and can be perceived as a manifestation of employees' possible inclination to opt out of work, thereby distancing themselves from exposure to EB strategies. The third zone, characterized by

Table 3 Example of payoff matrix for $s = 0.5$. Similar to Table 2, in panel (a) and (b) payoff values for employee (row-player) and employer (column-player) are reported, respectively. Since the matrix admits a unique pure strategy Nash Equilibrium, this is simply marked by bold text

0.074	0.114	0.1	0.129	0.159	**0.191**	0.222	0.253	0.282	0.309
0.051	0.086	0.074	0.1	0.129	0.159	0.191	0.222	0.253	0.282
0.031	0.062	0.051	0.074	0.1	0.129	0.159	0.191	0.222	0.253
0.016	0.04	0.031	0.051	0.074	0.1	0.129	0.159	0.191	0.222
0.006	0.023	0.016	0.031	0.051	0.074	0.1	0.129	0.159	0.191
0.001	0.01	0.006	0.016	0.031	0.051	0.074	0.1	0.129	0.159
0.786	0.003	0.001	0.006	0.016	0.031	0.051	0.074	0.1	0.129
0.688	0.	0.688	0.001	0.006	0.016	0.031	0.051	0.074	0.1
0.611	0.611	0.611	0.611	0.001	0.006	0.016	0.031	0.051	0.074
0.55	0.55	0.55	0.55	0.55	0.001	0.006	0.016	0.031	0.051

(a) Payoff matrix for Employee

0.14	0.291	0.248	0.325	0.372	**0.388**	0.372	0.326	0.248	0.139
0.14	0.291	0.248	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.14	0.291	0.248	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.14	0.291	0.248	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.14	0.291	0.248	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.14	0.291	0.248	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.045	0.291	0.248	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.045	0.291	0.09	0.325	0.372	0.388	0.372	0.326	0.248	0.139
0.045	0.112	0.09	0.135	0.372	0.388	0.372	0.326	0.248	0.139
0.045	0.112	0.09	0.135	0.18	0.388	0.372	0.326	0.248	0.139

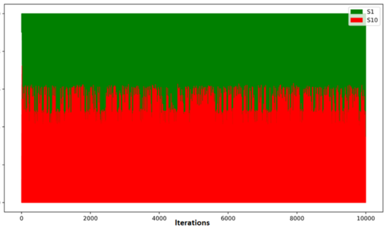
(b) Payoff matrix for Employer

larger values of s, echoes the merits of the first zone but in a more subdued manner, with exposure enhanced moderately above the benchmark.

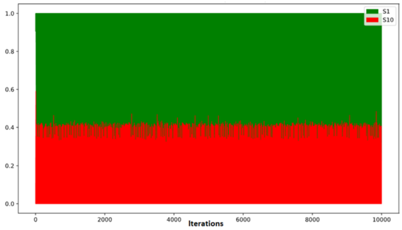
Such zones have important implications for practitioners in human resource management. The high-exposure realm characterized by small values of s highlights the merits of minimal sponsorship in the form of wage support. While the benefits in this zone are evident, human resource managers must proceed cautiously. Venturing too far with ambitious initiatives to stimulate sponsorship might inadvertently lead them into the less favorable central zone. This understanding allows HR managers to exploit the benefits of sponsorship while avoiding potential pitfalls.

Our computational approach in this study warrants special mention. The decision to rely on simulations for an Agent-based Model, rather than a direct analytical approach, was influenced by the presence of multiple Nash Equilibria and the absence of dominant strategies, especially in the range of $0 \leq s < 0.5$. The challenges posed by these elements rendered a straightforward analytical resolution unfeasible, leading to our reliance on simulations.

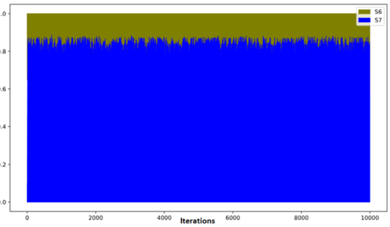
From a theoretical standpoint, our model offers the opportunity to broaden the perspective on EB strategy effectiveness by including the role played by sponsors, a factor previously overlooked in the literature. From a managerial perspective, our model highlights the need to rethink EB strategies within organizations. Before implementing these initiatives, HR managers should measure the degree to which people are exposed to attraction and retention strategies. To achieve this, it is crucial to consider the concepts of "economic homogeneity" and "economic reachability."



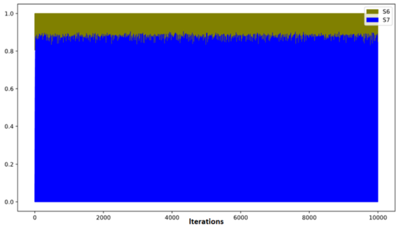
(a) Employees: $s = 0.35$



(b) Employees: $s = 0.4$



(c) Employers: $s = 0.35$



(d) Employers: $s = 0.4$

Fig. 6 Example of average strategy distributions for employees and employers with $s = 0.35$ and $s = 0.4$. Average strategy percentages over 10^3 simulations (Y-axis) are plotted against iterations (X-axis). Left panels show simulations for $s = 0.35$, while right panels show examples for $s = 0.4$. From top to bottom, we display employees' average strategy distributions (Fig. 6a and b, showing primarily S1 and S10, as other strategy percentages are minimal), and employers' average strategy distributions (Fig. 6c and d, showing mainly S6 and S7 for similar reasons). While employers' results show stability (S7 remains the most probable strategy), note that for employees there is a change in the most probable strategy from S10 (red) at $s = 0.35$ to S1 (green) at $s = 0.4$

This approach can ensure that EB strategies are "activated" for the widest possible segment of the target market population.

From an economic viewpoint, our model can stimulate ethical reflections about fairness in the labor market. To the extent that employment opportunities depend on the availability of sponsorship s, the labor market may be considered unfair. To address this issue, companies, labor unions, and institutions should find appropriate solutions to restore employment justice.

In essence, the integration of our Maxwell–Boltzmann distribution-based evolutionary model into the study of EB and sponsorship offers a comprehensive understanding of previously unexplored areas. This model provides clarity on complex dynamics and offers practical insights valuable to researchers, managers, and policymakers.

However, our work has several limitations. While our model provides valuable insights, it is built upon assumptions that may limit its applicability in real-world scenarios. One significant assumption is that of rational behavior from both employers and employees. In reality, decision-making processes are often influenced by bounded rationality, where individuals rely on heuristics and are subject to cognitive biases. Incorporating behavioral economic principles could provide a more realistic depiction of labor market dynamics and EB strategies.

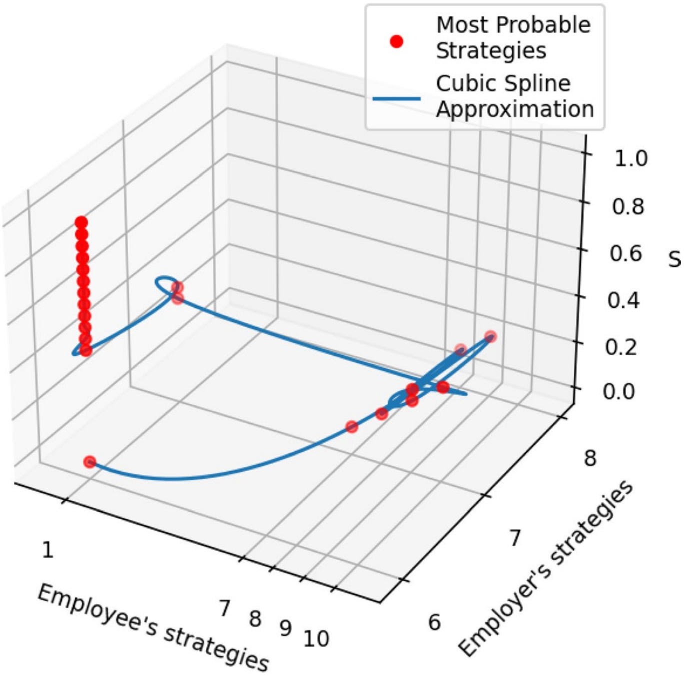


Fig. 7 Trajectory of the most probable strategies for varying level of Sponsorship (S). This figure illustrates the trajectory of the most probable strategies as the parameter s_i within the sponsorship set S varies, combining the information from the first and last columns of Table 1. The X-axis corresponds to the most probable employee strategies, the Y-axis represents the employer strategies, and the Z-axis shows the values of s_i. Each point on the trajectory (approximated by a cubic Hermite spline interpolation) represents the most likely combination of strategies for both players at a specific value of s_i, as obtained via simulations. Most points align with the Nash equilibria computed for the payoff matrices, but some do not, especially when there are multiple pure and mixed strategy Nash equilibria whose complex interactions make it difficult to predict the most probable strategies analytically. The trajectory provides valuable insights into how changes in the sponsorship parameter influence equilibrium outcomes, revealing the complex dynamics of the system and highlighting regions where equilibrium behavior undergoes significant shifts

Additionally, the model assumes economic homogeneity within the labor market, which simplifies the complexities of diverse economic backgrounds and conditions among employees. Real-world labor markets are characterized by varying economic factors that influence individuals' employment decisions. Future research could benefit from accounting for economic heterogeneity, potentially through segmented modeling approaches that consider different economic strata within the workforce.

Furthermore, the model's static parameters may not fully capture the dynamic nature of the labor market. Factors such as changes in economic conditions, shifts in industry demands, and evolving social norms can significantly impact EB strategies and their effectiveness. Incorporating dynamic elements into the model could provide a more robust and adaptable framework for understanding the long-term impacts of EB strategies.

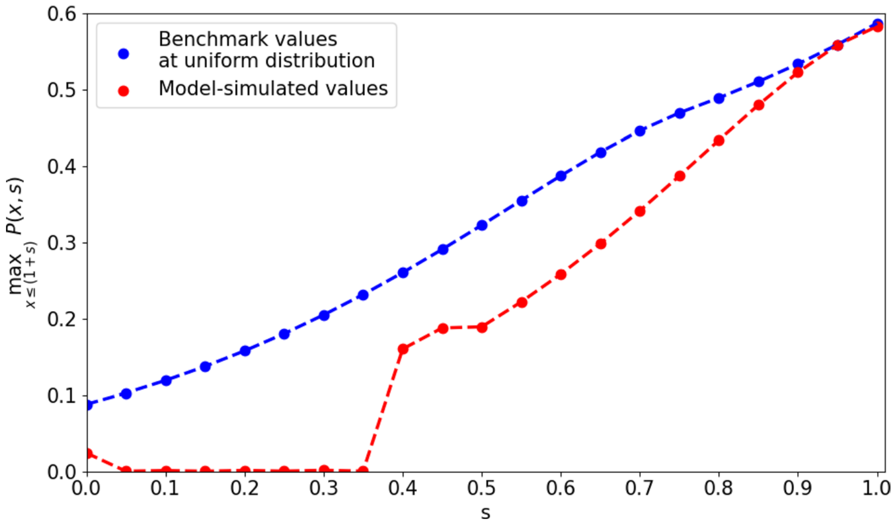


Fig. 8 Model-simulated versus benchmark values at uniform distribution of strategies for maximum probability attained by Maxwell–Boltzmann distributions. The X-axis represents the values of s_i within the sponsorship set S, while the Y-axis plots max $_{0\leq x\leq(1+s)} P(x, s)$, the maximum probability attained by Maxwell–Boltzmann distributions over bounded x values (see Fig. 5) with varying s. Note that at $s = 0$, despite both benchmark and simulated models having $e(s) = 0$ (see Fig. 9), the simulated model shows a lower value for max $_{0\leq x\leq 1} P(x, 0)$, suggesting a higher velocity for no exposure cases (as lower probability corresponds to higher speed in this model)

While we have approached this study primarily from a human resource management perspective, our results show some alignment with economic theories such as the efficiency wages hypothesis developed by Shapiro and Stiglitz (1984), which discusses employers voluntarily paying above-market wages to increase worker productivity. In our model, sponsors and their resources may play a role similar to what employers have in paying wages in excess of market-clearing rates. This connection could be a fruitful topic for further investigation.

6 Conclusions

In this concluding section, we summarize our findings in three key areas: managerial implications, economic considerations, and directions for further research.

From a managerial viewpoint, it is important to understand the concept of "exposure." Before planning and executing an EB strategy, organizations should estimate the weight that sponsorship s has in their target labor market. Two alternatives arise to make an EB strategy effective: Either s is not determinative of the target population's employment choices, or s is crucial to those choices.

In the first case, EB policy takes on a conventional aspect. People find the job proposal economically viable, and the decision to accept or reject it will be related to purely subjective evaluations. In the second case, EB policy requires a different approach. The challenge is not so much convincing workers to consider the proposal,

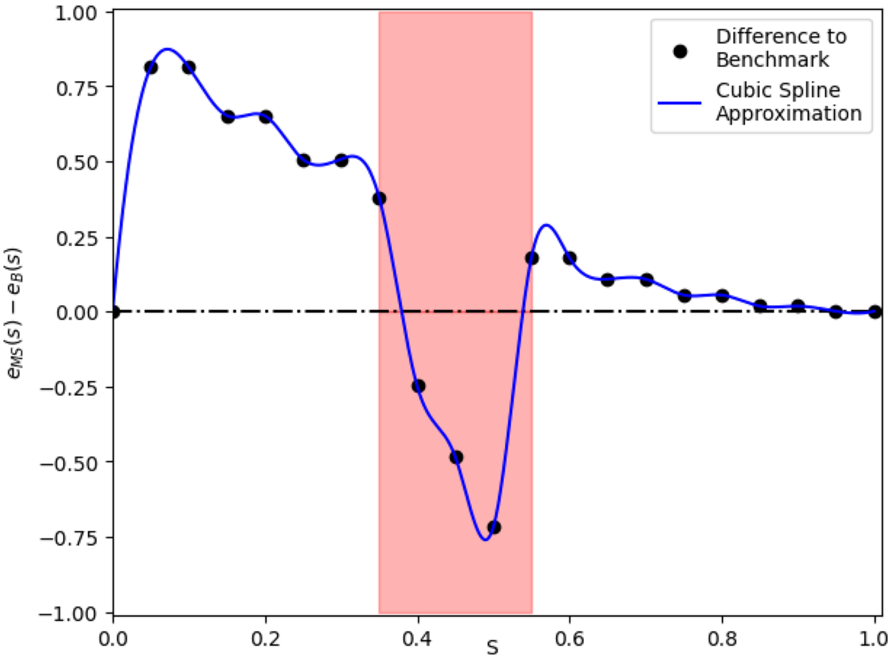


Fig. 9 Exposure to Employer Branding for varying levels of Sponsorship (S). This figure shows model-simulated exposure $e_{MS}(s)$ properly rescaled with respect to the benchmark uniform distribution exposure $e_B(s)$. The X-axis represents the values of s_i within the sponsorship set S, while the Y-axis shows the difference between model-simulated and benchmark exposure, i.e., $e_{MS}(s) - e_B(s)$. Overall exposure is approximated by a cubic Hermite spline interpolation. The red area denotes values of s for which model-simulated exposure is weaker than the benchmark. This clustering pattern can be explained by noting that a separation in exposure occurs around $s = 0.40$ and $s = 0.45$, which have 0 pure and 1 mixed strategy Nash equilibria, acting as a rupture in the game dynamics compared to lower values of s marked by multiple pure and mixed strategy Nash equilibria. In contrast, a stable combination of strategies appears for $s \geq 0.50$, characterized by a unique pure Nash equilibrium and a dominant strategy

but rather putting workers in the economic position to consider it. Therefore, if in the first case workers are already exposed to EB policies and can evaluate their contents without financial concerns, in the second case the organizational challenge is first to "expose" workers to employment proposals.

Evidently, in the second case, the target population includes not only workers but also potential sponsors. EB policies in these cases must be more holistic, as they need to convince potential sponsors to invest personal financial resources on behalf of workers. In other words, the job proposal must be attractive not only to workers but also to those who will provide the necessary support (s) to cover the gap between wages (w) and consumption needs (c).

The EB strategy designed for sponsors must incorporate additional motivating factors beyond those that would influence an employee. For example, if a recent graduate is interested in organizational climate and ethical values, parents providing sponsorship may be more interested in career growth prospects. In this sense, the organization's value proposition should emphasize information regarding career progression. For sponsors, such information acts as a guarantee of the financial return on their investment.

Ultimately, a worker asks questions like: “Does this company express the same values as me?”, “Is the organizational climate satisfying?”, “Is the relationship with colleagues and superiors positive?” A sponsor, on the other hand, asks questions like: “Will the amount of sponsorship I invest today guarantee a return tomorrow?”, “When will my child’s salary progression make my investment unnecessary?”, “Do I like this enterprise enough to supplement the salary it’s offering?”. If organizations want to attract and retain large numbers of talented individuals, EB policy makers must know how to answer all these questions. A shrewd manager must motivate the talents already exposed and economically bring to light the talents hiding in the shadows of quadrant I.

From an economic perspective, we must emphasize that we have dealt with the labor market in general terms. It would be useful in future research to focus on individual industries to understand whether sponsorship consistently conditions the choices of workers and employers in the same way across sectors. After assessing the possible role of sponsorship, it would be necessary to analyze the reasons that may explain such situations. This approach would allow for creating a map of labor market areas most dependent on the action of sponsors.

In areas of higher dependency, one might reasonably assume that there are workers whose employability depends more closely on the availability of non-labor income. Firms operating in areas of high sponsorship dependence should implement EB strategies strongly devoted to attracting sponsors. Additionally, the economic perspective connects with ethical considerations. Those interested in this aspect might ask not “what is the actual role of sponsorship within a certain industry?” but rather “is it fair/acceptable that employment dynamics are conditioned by the availability of non-labor income?”. There is currently little research focused on this issue, despite the many equity and justice questions it raises. For example, do workers who lack sufficient non-labor income and thus cannot accept certain job offers feel discriminated against? If this form of economic discrimination exists, appropriate solutions must be found. If racial, religious, or gender discrimination is unfair, then economic/wealth discrimination should also be addressed. These issues represent important avenues for future research.

In the course of this research, we have explored the dynamics of EB strategies through our proposed model. While our findings have shed light on several important aspects, there remains ample room for expanding this investigation. Future research could benefit from distinguishing between the cross-sectional labor market and specific labor markets. In this article, we have favored the former to provide a broader view across various industries and positions. However, focusing on specific labor markets would reveal patterns and dynamics unique to particular industries or roles.

Additionally, while our model has shown promise theoretically, empirical validation of its performance would be valuable. A thorough empirical study could determine the goodness of fit of our model and provide insights into the particular values of sponsorship that are effectively employed in real-world scenarios.

In conclusion, understanding the complexities of EB exposure in the labor market is an ongoing journey. Our research has paved a path, but the potential for further exploration and richer insights beckons the scholarly community. The highlighted avenues for further work serve as starting points for researchers keen on exploring this domain more deeply.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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