STRESS SHIELDING EFFECT IN CYLINDRICAL JOINTS WITH CLEARANCE USING FGM

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ABSTRACT

Analytical solutions are presented for the quasi-conformal contact problem with clearance between a loaded rigid pin and a circular ring made of functionally graded material (FGM), which displays a power-law radial variation of the elastic modulus within the thickness. The stress and displacement fields in the FGM ring are taken from the general solution for plane elastic problems involving FGMs in polar coordinates, analogous to Mitchell's solution for elastic isotropic materials. The frictionless contact conditions yield a set of dual trigonometric series equations that can be reduced to a linear system of infinite equations, then solved by truncation. Due to the nonlinear nature of the advancing contact problem, an inverse method of solution is employed, namely, the contact stress distribution is derived for a set of contact angle values. The corresponding pin load is then calculated, and a nonlinear pointwise relationship is obtained between the contact angle and the pin load. We show that the contact angle can be increased by properly choosing the inhomogeneity factor of the FGM ring. Consequently, more uniform distributions can be achieved for the contact pressure and the hoop stress, thereby preventing mechanical failure in many fasteners and bolt connections. The present findings can assist mechanical engineers in designing innovative pinned connections and improving their load-bearing capacity by exploiting the FGM capabilities.

Introduction

Contact problems of Functionally Graded Materials (FGMs) structures have recently attracted considerable attention in mechanical engineering. These materials have the potential to replace or supplement the surface-hardening technique or the insertion of a harder collar in some applications. Indeed, their composition and properties change gradually over the volume (typically from surface to core). They may display a gradual transition from a hard to a soft phases, from a metallic to a ceramic phase, or even a combination of both. In the field of bearing design, this approach entails the customization of material properties, including hardness, wear resistance, and toughness, across the entire component rather than merely its surface The gradual variation of material properties inside the bearing surface leads

to a smooth distribution of the stress and strain fields, which could reduce the risk of surface crack initiation and propagation compared to traditional surface-hardened methods.
In this work, we provide the analytical solution to the frictionless contact problem between a rigid circular pin and a circular FGM lug, under clearance-fit conditions (Fig. 1). The Young modulus of the FGM ring is assumed to vary in the radial direction according to a power-law and a constant Poisson ratio is assumed. The stress and displacement fields in the elastic plane and the FGM ring are represented using a Michell-type series, which is taken from the solution provided in [1]. Using the frictionless contact boundary conditions, the problem is reduced to the solution of a set of dual trigonometric series equations. Following the approach developed in [2, 3], a linear system of infinite equations for the unknown coefficients of the Michell-type series representations is obtained and then solved by truncation. Results are then presented for the contact pressure and von Mises equivalent stress, for various loading levels, and geometrical and material parameters.

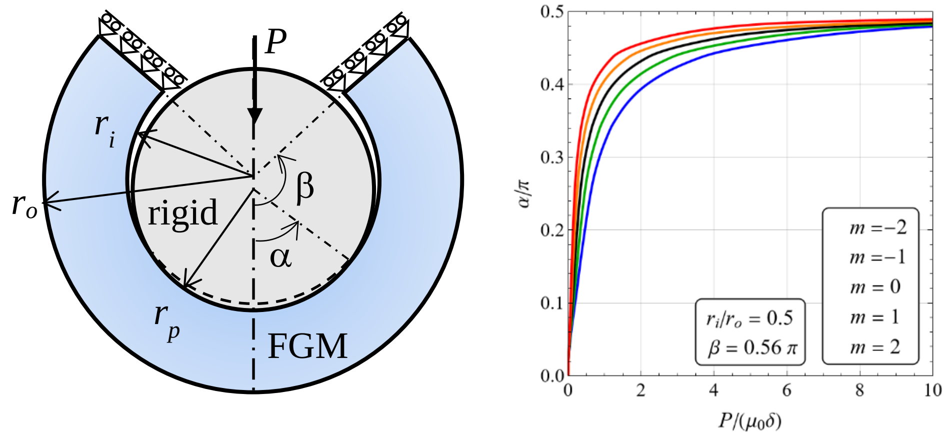

Figure 1. Pin-lug connection and relation between the contact angle α and the pin load.

Problem formulation

Let r_i and r_o denote the inner and outer radii of the FGM lug, respectively. The elastic shear modulus μ of the lug varies along the radial direction according to the relation $\mu = \mu_0 (r/r_i)^m$, where μ_0 is the shear modulus at the inner surface and m is the grading index. It is assumed that the Poisson coefficient ν of the lug is constant. To perform a fully analytical investigation under plane stress or plane strain conditions, we adopt a simplified geometrical model of the pinned connection, by considering an incomplete FGM ring of angular extent 2β (Fig. 1).
The stress-free conditions at the outer surface of the ring require

$$\sigma_{rr}(r_o, \theta) = \sigma_{r\theta}(r_o, \theta) = 0, \text{ for } 0 \le |\theta| \le \beta, \quad (1)$$

and the frictionless contact conditions between the rigid pin and the inner surface of the incomplete ring, at $r = r_i$ require

$$\begin{aligned} &\sigma_{r\theta}(r_i, \theta) = 0, && \text{for } 0 \le |\theta| \le \beta, \\ &\sigma_{rr}(r_i, \theta) = 0, && \text{for } \alpha \le |\theta| \le \beta, \\ &u_{rr}(r_i, \theta) = v_0 \cos\theta - \delta\,(1-\cos\theta), && \text{for } 0 \le |\theta| \le \alpha, \end{aligned} \quad (2)$$

where α is the unknown contact half-angle, v_0 denotes a rigid body motion of the pin along the loading direction, and δ is the clearance. Moreover, the constraint at the edges of the incomplete ring, namely at $\theta = \pm\beta$, requires $u_\theta(r, \pm\beta) = \sigma_{r\theta}(r, \pm\beta) = 0$, for $r_i < r < r_o$.
The cylindrical components of the stress and displacement fields within the FGM ring are represented using a Michell-type series containing only even terms in θ. They follow from the solution presented in [1] for a power-law radial variation of the elastic modulus, by replacing n with $n\pi/\beta$, for $n = 1, 2, 3\ldots$ to meet the boundary conditions at $\theta = \pm\beta$:

$$\begin{aligned}
\frac{\sigma_{rr}}{2\mu} &= \frac{\delta}{r_o}\left[\sum_{i=1}^{2} B_{0i}\left(\frac{r}{r_o}\right)^{a_i} + \sum_{n=1}^{\infty}\sum_{i=1}^{4} B_{ni}\left(\frac{r}{r_o}\right)^{b_{ni}} \cos\frac{n\pi\theta}{\beta}\right],\\
\frac{\sigma_{\theta\theta}}{2\mu} &= \frac{\delta}{r_o}\left[\sum_{i=1}^{2} B_{0i}(a_i+1)\left(\frac{r}{r_o}\right)^{a_i} + \sum_{n=1}^{\infty}\sum_{i=1}^{4} B_{ni}\left(\frac{r}{r_o}\right)^{b_{ni}} (b_{ni}+2)\,\Delta_{ni} \cos\frac{n\pi\theta}{\beta}\right],\\
\frac{\sigma_{r\theta}}{2\mu} &= \frac{\delta}{r_o}\sum_{n=1}^{\infty}\sum_{i=1}^{4} B_{ni}\left(\frac{r}{r_o}\right)^{b_{ni}} \frac{n\pi}{\beta}\,\Delta_{ni}\sin\frac{n\pi\theta}{\beta},\\
\frac{u_r}{\delta} &= \sum_{i=1}^{2} B_{0i}\,\kappa_i\left(\frac{r}{r_o}\right)^{a_i+1} + \sum_{n=1}^{\infty}\sum_{i=1}^{4} B_{ni}\left(\frac{r}{r_o}\right)^{b_{ni}+1} \frac{\chi_{1ni}}{2(1-\rho^2)}\cos\frac{n\pi\theta}{\beta},\\
\frac{u_\theta}{\delta} &= \frac{\beta}{2(1-\rho^2)\pi}\sum_{n=1}^{\infty}\sum_{i=1}^{4} B_{ni}\left(\frac{r}{r_o}\right)^{b_{ni}+1} \frac{\chi_{2ni}}{n}\sin\frac{n\pi\theta}{\beta},
\end{aligned} \tag{3}$$

where $\mu = \mu_0\,(r/r_i)^m$ is the elastic shear modulus of the FGM ring. The contact conditions (1) and (2) yield a mixed boundary value problem with a moving boundary, which can be formulated in terms of dual trigonometric series equations. Following the approach developed in [2, 3] a linear system of infinite equations for the unknown coefficients B_{ni} of the Michell-type series representations is obtained and then solved by truncation.

Results

The normalized distributions of the contact pressure with the angular coordinate θ are plotted in Fig. 2 for a lug with aspect ratio $r_i/r_o = 0.5$ and for various contact angles, both for $m = 1$ $m = -1$. In general, the same pin load causes larger contact angle for negative grading index than for positive ones (Fig. 1). The results show that the Hertzian distribution of the contact pressure is valid only for very low load levels and a small contact zone (black lines). As the load magnitude increases, two pressure bumps take place near the contact endpoints, and the contact pressure becomes almost uniform in the central part of the contact region. The position of the peaks depends on the ring thickness and grading index. It approaches the edges of the contact region for a very thin ring as well as for negative m, whereas it moves toward the center for thicker rings and positive m. The contour plots of the normalized von Mises equivalent stress $\sigma_{eq}\, r_i/P$ in the right-side region ($0 \leq \theta \leq \beta$) of a lug with $r_i/r_o = 0.5$ are illustrated in Fig. 3 for the same contact angle $\alpha = 0.45\,\pi$, for $m = 1$, 0, and -1. These plots show that the maximum von Mises equivalent stress occurs at about $\theta \approx 73°$ on the inner surface of the lug, independently of the grading index. The equivalent stress on the outer lug surface is much lower than on the inner surface, especially for the negative grading index m. This occurrence suggests that the load-bearing capacity of the pin-lug connection can be enhanced by realizing a FGM lug with a negative grading index, namely by increasing both the elastic modulus and the strength of the FGM lug near the surface in contact with the loaded pin.

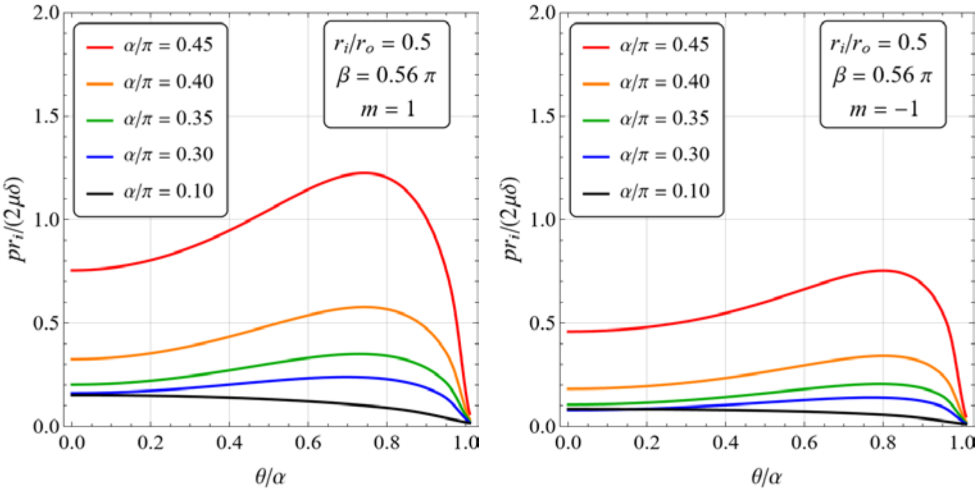

Figure 2. Normalized variations of the contact pressure p with the angular coordinate θ for various contact angles, for $m = 1$ and $m = -1$.

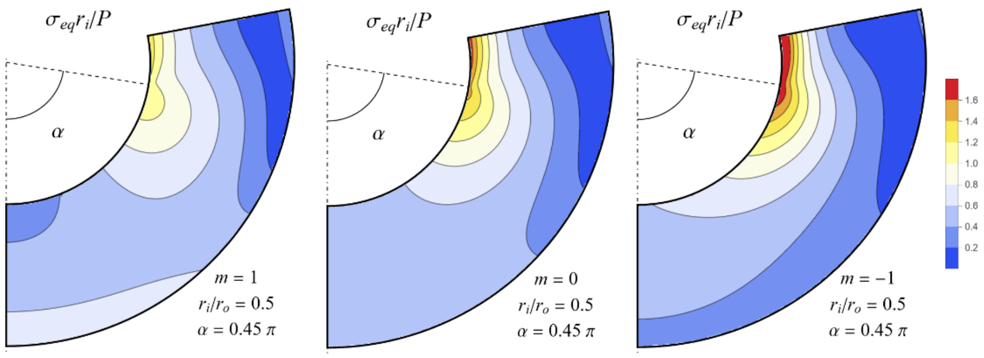

Figure 3. Contour plots of the normalized von Mises equivalent stress $\sigma_{eq}r_i/P$ in the FGM ring with $r_i/r_o = 0$, for $\alpha = 0.45\,\pi$, for $m = 1, 0, -1$.

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