Preliminary steps of a parametric analysis of corrugated fins for compact heat exchangers

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Abstract. Wavy fins are widely employed in compact heat exchangers, especially in those working in harsh environments, where the high resistance to clogging of wavy fins helps to ensure reliable and stable thermal performance. The thermal-hydraulic performance of these extended surfaces heavily depends upon geometric and flow parameters. Hence, accurate predictive tools are required for the design of compact heat exchangers equipped with wavy fins. However, the available correlations and empirical models present a very uneven coverage of the design space, which limits their applicability. This work presents the first steps and preliminary results of a parametric analysis of wavy fins, which is aimed at the derivation of continuous multivariate response surfaces suitable for optimisation procedures. The analysis is carried out by means of a Design Of Experiment procedure, which is performed by means of numerical techniques. Each wavy fin configuration is analysed by developing a dedicated three-dimensional Computational Fluid Dynamics model of a unitary fin geometry under periodic flow conditions. Preliminary numerical results are employed to derive multivariate response surface for the equivalent Darcy friction factor and Colburn factor, which are validated against reference numerical and experimental data.

1 Introduction

Compact heat exchangers working in harsh environments are often equipped with wavy or corrugated extended surfaces, as these present a far higher resistance to clogging and fouling compared to other commonly used fin configurations [1]. Specifically, wavy fins see regular use in the external channels of heat exchangers mounted on agricultural and earth-moving machines, where the presence of dirt particles and pieces of straw severely limits the use of common extended surface arrangements as offset-strip fins, as these fragments can easily get stuck in between the tight rows of staggered ducts, adversely affecting the performance of the coolers and complicating maintenance and cleaning operations.

Wavy fins arrays are constituted by a series of ducts with uniform cross section, usually of rectangular or triangular shape, which is developed along a sinusoidal path defined by amplitude and wavelength. Fin configurations with a triangular or herringbone development are also available [2], although sinusoidal ducts usually allow for a better heat transfer effectiveness to head losses ratio [3]. The corrugated side walls induce the periodic formation and disruption of counter-rotating vortices in the troughs formed by the walls [4]. These vortices cause the local interruption of the boundary layer, favouring heat transfer

in correspondence of the recirculation regions [5]. The ratio between heat transfer enhancement and increase of the pressure losses is heavily affected by the fin pitch [6], the amplitude and wavelength of the corrugated channels, and the flow rate [1]. At the design stage, the influence of geometric and flow parameters on the thermal-hydraulic performance of wavy fins is often evaluated by means of empirical models based on numerical and experimental data available in the literature. However, these models often cover the design space in a very uneven fashion with narrow applicability ranges [7], severely limiting the number of configurations that can be studied and introducing considerable uncertainties when they are employed in extrapolation. In an industrial context, the lack of confidence introduced by these shortcomings often forces the designers to evaluate individual configuration through experimental tests of prototypes, which considerably drive up the design costs and lead times.

One of the first studies on the performance of wavy fins was conducted by Kays and London [8], who presented heat transfer and friction data for three fin cores with equal wave length, similar amplitude, and different hydraulic diameters. Through the comparison of several corrugated channels against plain ducts, the authors determined that wavy fins provide optimal heat transfer enhancement in the range $1750 \leq \mathrm{Re}_{\mathrm{D_h}} \leq 2000$, while at higher $\mathrm{Re}_{\mathrm{D_h}}$-values the heat transfer performance over pressure losses benefits tend to diminish due to increased turbulence. Through the years several authors proposed empirical models for the prediction of the Colburn and friction factors in wavy fins. These are usually provided in the form of polynomials that are functions of the Reynolds number and selected ratios between geometric parameters. In these equations the coefficients are determined through curve fitting procedure based on experimental data as in [9, 6], and numerical results [10, 11].

All these predictive models are very convenient tools to derive preliminary estimates on the thermal-hydraulic performance of wavy fins. However, as it will be shown later, many of these correlations are developed on the basis of sets of configurations which are very unevenly distributed across the design space, which translates into applicability windows that are often narrow and difficult to define, thus limiting the correct use of these empirical models.

This work presents the first steps and preliminary results of a parametric analysis on the thermal-hydraulic performance of wavy fins, aimed at the derivation of continuous multivariate response surfaces for the equivalent Darcy friction factor f and Colburn factor j. This study is performed by following a Design Of Experiment (DOE) procedure, which is carried out using computational methods. A full factorial DOE is performed with respect to the main geometric parameters of wavy fins, namely the non-dimensional fin pitch, wavelength, and amplitude, whose levels are selected with the aim of covering relevant applications of wavy fins for heat exchanges employed in vehicular applications. Each geometric configuration is analysed by developing a CFD model of an elementary unit geometry under periodic flow conditions. Numerical solutions are attained through a validated conjugate heat transfer solver based on the Finite Volume method, and implemented in the computational toolkit OpenFOAM 7. Preliminary numerical results on heat transfer and head losses related to a single air flow rate are employed to derive multivariate response surfaces for the equivalent Darcy friction factor f and Colburn factor j. The dependency of f and j on the Reynolds number is introduced a posteriori by using scaling laws available in the literature, due to the limited amount of numerical results available. The validity of this approach is assessed by comparing the outcomes of the obtained response surfaces with some of the numerical and experimental results on single fins and complete heat exchangers that were presented in [12]. The initial results are promising as the generalised response surfaces can produce numerical results as accurate as those retrieved from dedicated CFD models developed in [12] for the geometric configurations considered.

2 Problem statement

The present parametric analysis is performed on wavy fins presenting a rectangular cross-section with slightly sloped side walls, and a sinusoidal longitudinal development. Figure 1 shows schematics of the front and top views of the wavy fin geometry considered, along with the most relevant geometric parameters. The pitch and height of the fin are denoted by x and y, respectively. The thickness of the walls is assumed uniform and it is represented by t. On the top view, w stands for the wavelength, while $2a$ refers to the peak-to-peak amplitude of the sine wave. The side walls are slightly inclined forming an angle $\alpha/2$ with the vertical axis.

3 Numerical methods

Wavy fins are analysed by developing Computational Fluid Dynamics models of basic unit geometries which are formed by considering the periodicity of the geometry and flow along the streamwise and spanwise directions. The periodic flow solution is attained by using the numerical methodology developed in [13] for the analysis of offset-strip fins. The problem is addressed by means of a dimensionless framework

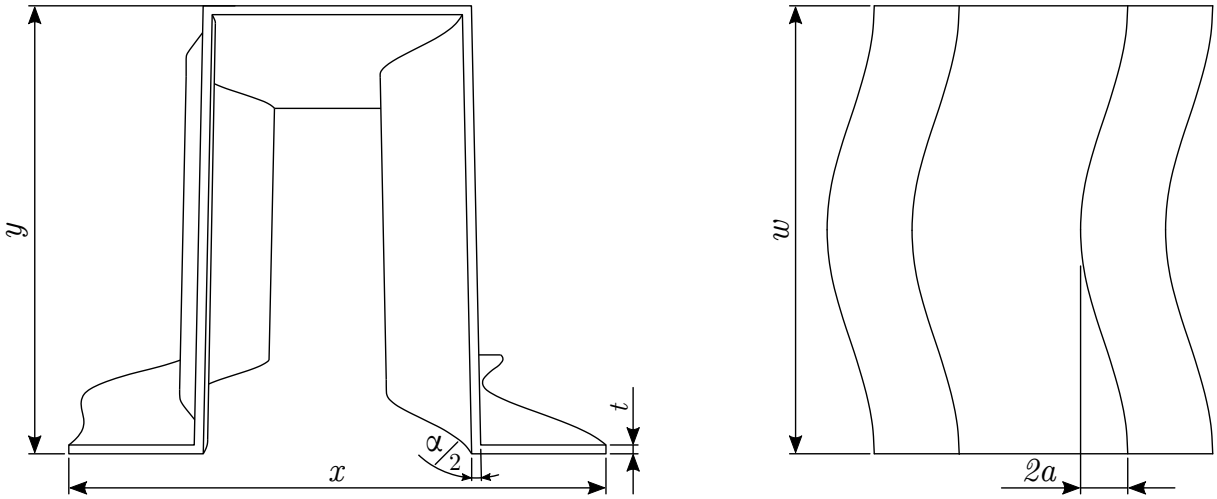


Figure 1: Front (left) and top (right) views of the examined wavy fin geometry, along with the most significant dimensions.

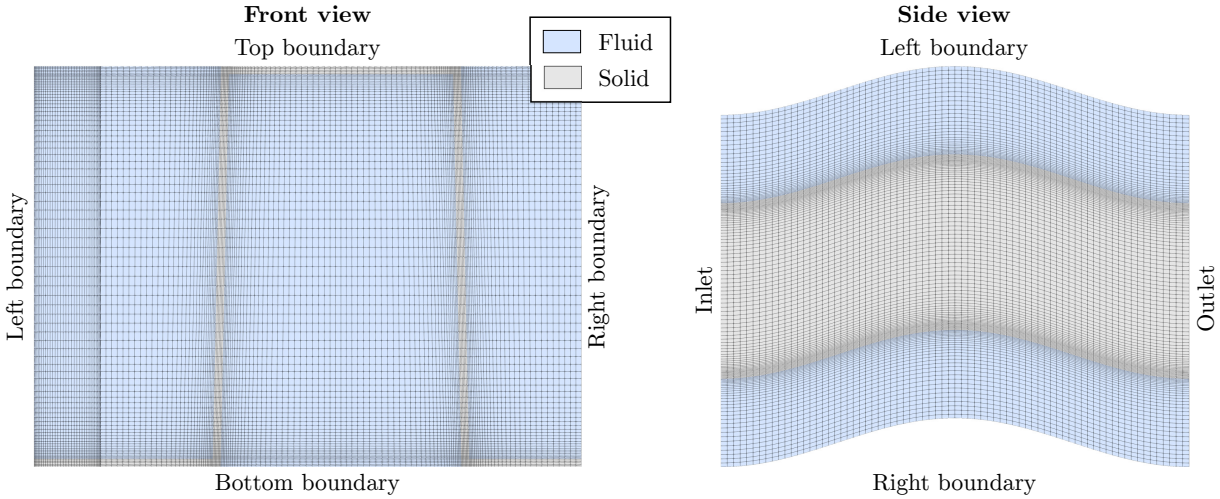


Figure 2: Selected views of one of the generated computational grids, related to the geometrical configuration defined by $x^* = 1.2$, $w^* = 1.6$, and $2a^* = 0.167$.

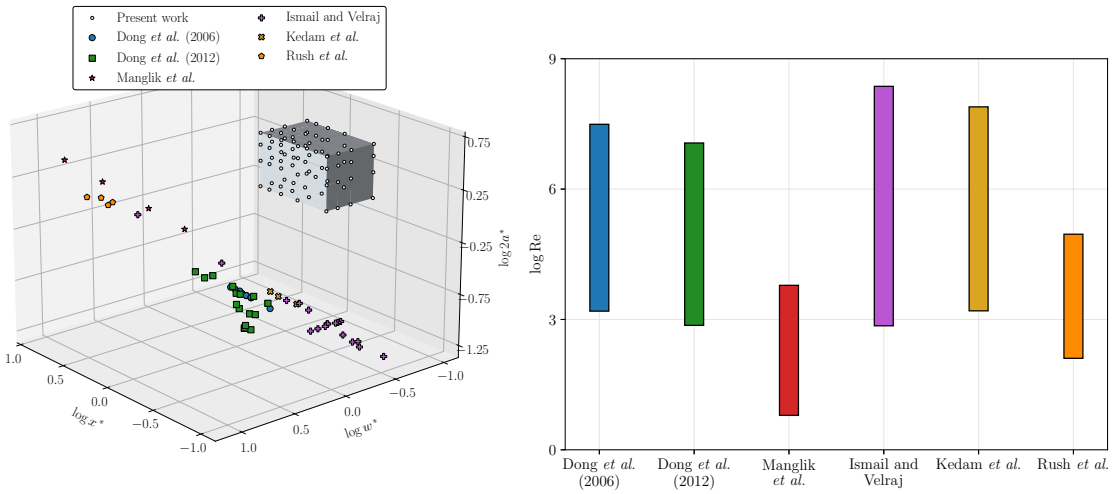
based on the fin height. The latter is used to scale the computational domains resulting in the following set of dimensionless geometric parameters:

$$y^* = \frac{y}{y} = 1\,, \qquad x^* = \frac{x}{y}\,, \qquad w^* = \frac{w}{y}\,, \qquad 2a^* = \frac{2a}{y}\,, \qquad t^* = \frac{t}{y}\,. \tag{1}$$

The flow within the fins is defined on the basis of the Reynolds and Prandtl numbers:

$$\mathrm{Re} = \frac{\bar{u}\,y}{\nu} \qquad \text{and} \qquad \mathrm{Pr} = \frac{c_\mathrm{p}\,\mu}{\lambda}\,, \tag{2}$$

where $\bar{u}$ is the integral mean of fluid velocity over the inlet section. The computational domains are discretised using block-structured meshes obtained through an automatic mesh generation algorithm implemented in the commercial meshing software Pointwise. Figure 2 shows selected views of the computational mesh generated for the configuration defined by $x^* = 1.2$, $w^* = 1.6$, and $2a^* = 0.167$, where the fluid and solid regions are clearly highlighted.



(a) (b)

Figure 3: Comparison between the dimensional (a) and flow (b) parameters considered in this work compared with those analysed in the literature.

4 Design of Experiment

The effects of the geometric and flow parameters of wavy fins on their thermal-hydraulic characteristics are assessed by means of a Design Of Experiment procedure. The non-dimensional pitch, wavelength, and amplitude are identified as the most relevant geometric parameters, while the inclination angle and thickness of the walls are considered of minor importance and are therefore assumed constant, and set to $t^* = 0.02$ and $\alpha = 2.4°$, respectively. This is a valid assumption, as in the design of heat exchangers the choice of fin thickness is often limited by the commercially available raw aluminium sheets, and the need to guarantee the mechanical strength and stability of the coolers.

A full factorial DOE is performed with respect to the dimensionless fin pitch, wavelength, and amplitude and the flow regime, which is defined by couples of Reynolds and Prandtl numbers. Table 1 reports the levels of the geometric parameters x^*, w^*, and $2a^*$, which are selected to cover a range of configurations relevant for the design of compact heat exchangers used in vehicular applications [12].

The geometric configurations considered in this work are shown in the three-dimensional plot of Fig. 3a, where they are compared with those analysed in some of the most relevant studies on wavy fins available in the literature. The latter cover the design space in a very uneven and sparse manner, as most of the data points related to each publication are densely confined in narrow regions associated with small deviations of the geometric parameters. As a result, none of the empirical models developed on the basis of these data sets can be considered universal, and their applicability ranges do not appear as well defined. Furthermore, none of the considered contributions from the literature address the fin configurations examined in this work.

Figure 3b reports the fin-height-based Reynolds number ranges considered in each study from the literature, showing that most of the fin configurations were tested in the range $10^3 \leq \mathrm{Re} \leq 10^6$. In this case the values of the Prandtl number related to each study are not reported in the plot, as all the considered publications focused on air flows at $\mathrm{Pr} = 0.7$. The present study is still at a very early stage of development, therefore only a single Reynolds number value has been analysed thus far. The preliminary numerical results are presented and discussed in the following.

Table 1: Selected levels of the geometric parameters involved in the DOE analysis of wavy fins.

x^*	0.4	0.8	1.2	1.6	2.0
w^*	0.9	1.6	2.3	3.0	
$2a^*$	0.1	0.17	0.23	0.3	

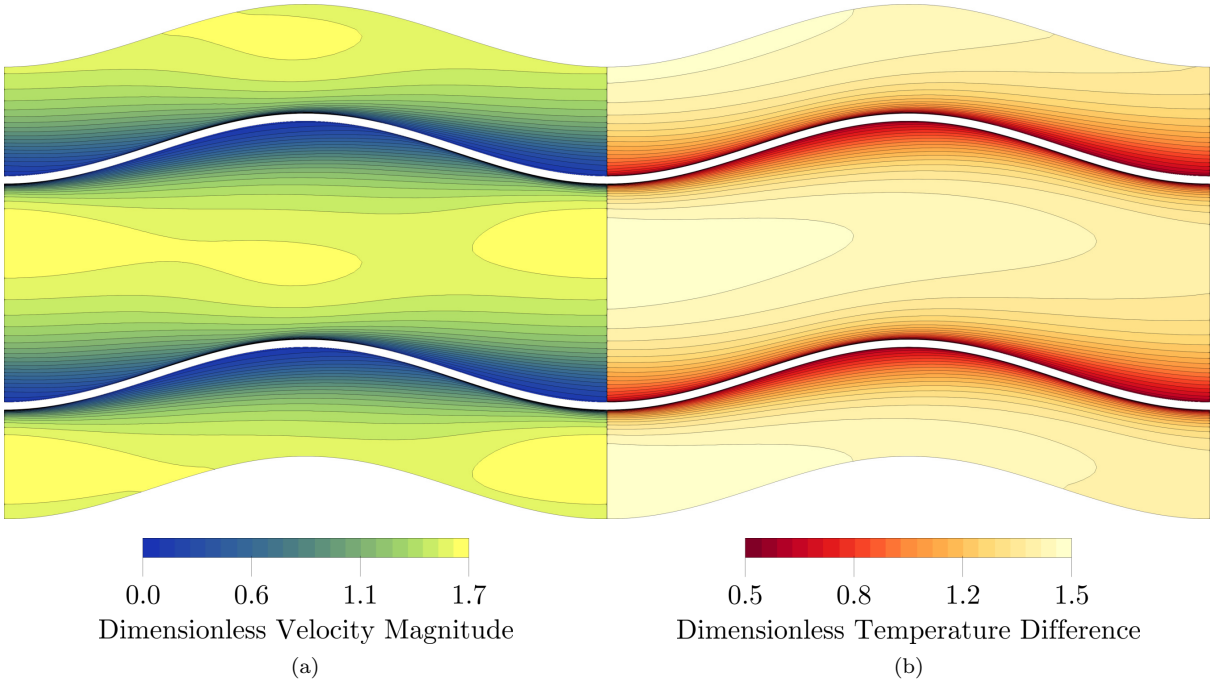


Figure 4: Contour plots of dimensionless velocity magnitude (a) and temperature difference (b), obtained on longitudinal planes located at half the height of the fins. These plots are associated with an air flow at Re = 5000 and Pr = 0.71.

5 Preliminary numerical results

Numerical flow field result are present by means of contour plots of dimensionless velocity magnitude and temperature difference, obtained on longitudinal planes positioned at half the height of the fins. The dimensionless temperature difference is given by:

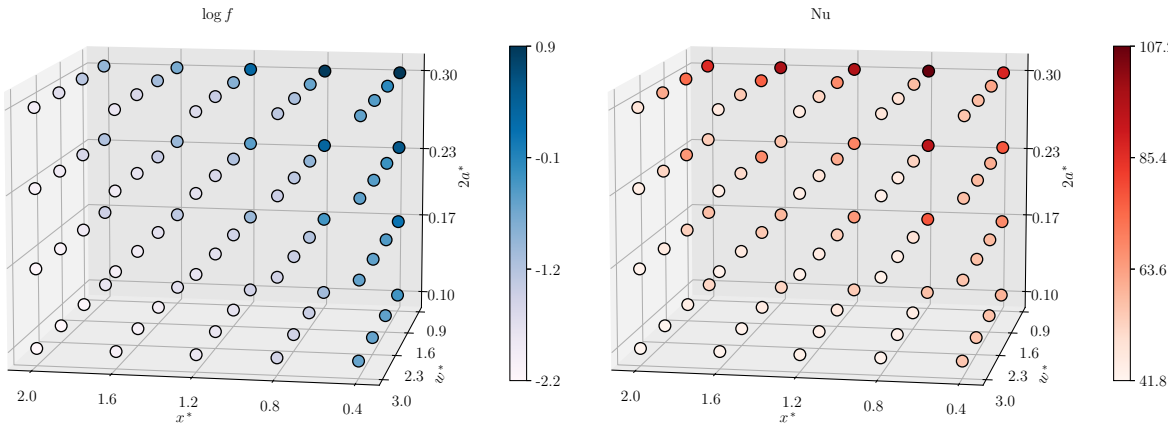
$$\vartheta = \frac{\overline{T}_{\mathrm{w}} - T}{\overline{T}_{\mathrm{w}} - \overline{T}_{\mathrm{b}}} \,, \tag{3}$$

where $\overline{T}_{\mathrm{w}}$ is the integral mean temperature over the heat transfer area A_{ref}, while $\overline{T}_{\mathrm{b}}$ is computed as the mean of the mass flow weighted averages of temperature on the inlet and outlet surfaces. Figure 4 shows an example of these contour plots related to a wavy fin defined by $x^* = 1.2$, $w^* = 1.6$, and $2a^* = 0.167$ and to an air flow at Re = 5000. In correspondence of the troughs of the fins, the flow detaches forming recirculation regions, while near the crests velocity maxima are observed. As expected, temperature hot spots are present in correspondence to the flow recirculation regions.

Integral results on head losses and heat transfer rates are expressed by the equivalent Darcy friction factor and mean Nusselt number. The former is retrieved from the Darcy-Weisbach formula, while the latter is computed on the basis of the difference between the integral mean wall temperature and arithmetic average between the inlet and outlet bulk temperature values:

$$f = \frac{2\,y\,(\bar{p}_{\mathrm{i}} - \bar{p}_{\mathrm{o}})}{w\,\rho\,\bar{u}_{\mathrm{i}}^{\,2}} \,, \qquad \mathrm{Nu} = \frac{\dot{Q}\,y}{\lambda\,A_{\mathrm{ref}}\,(\overline{T}_{\mathrm{w}} - \overline{T}_{\mathrm{b}})} \,. \tag{4}$$

Figure 5 shows four-dimensional scatter plots of f and Nu derived from numerical results at Re = 5000 and Pr = 0.71, where each point marker identifies a single geometric configuration, while the dependent variables are represented by linear colour distributions. As expected, both friction losses and heat transfer effectiveness tend to increase with the corrugation ratio of the channels, i.e. as the amplitude of the fins increases and the wavelength decreases. In addition, both f and Nu decrease with growing fin pitch x^* due to the increase of the hydraulic diameter as in straight channels [14].



(a) (b)

Figure 5: Four-dimensional scatter plots of the equivalent Darcy friction factor (a) and Nusselt number (b), derived from CFD results at Re = 5000 and Pr = 0.71. In these plots each round marker refers to one of the geometric configurations, while the dependent variable is represented by means of a colour distribution.

6 Derivation of response surfaces

Multivariate response surfaces for the equivalent Darcy friction factor and Colburn factor consist in polynomial functions of the geometric parameters:

$$\log f = b_1 (x^*)^{b_2} + b_3 (w^*)^{b_4} + b_5 (2a^*)^{b_6} + b_7 \left(\frac{2a^*}{w^*}\right) , \tag{5}$$

$$j = d_1 (x^*)^{d_2} + d_3 (w^*)^{d_4} + d_5 (2a^*)^{d_6} + \left(\frac{2a^*}{w^*}\right)^{d_7} . \tag{6}$$

The values of the b_m and d_m coefficients, which are listed in Tab. 2, are determined by fitting Eqs. (5) and (6) to numerical results, by using a numerical procedure analogous to the one presented in [13]. These response surfaces hold only for Re = 5000, however, their validity can approximately be extended to other Re-values by introducing the scaling laws derived by Stasiek *et al.* [15]:

$$f = f\big|_{\mathrm{Re}=5000} \left(\frac{\mathrm{Re}}{5000}\right)^{-1/2} , \tag{7}$$

$$\mathrm{Nu} = \mathrm{Nu}\big|_{\mathrm{Re}=5000} \left(\frac{\mathrm{Re}}{5000}\right)^{2/3} . \tag{8}$$

The validity of this approach is assessed by comparing numerical results obtained from the above-presented response surfaces with the numerical and experimental results presented in [12], for the WF4 extended surface, and G94 and G113 heat exchangers. Figure 6 shows the distributions of equivalent

Table 2: Values of b_m and d_m coefficients in Eqs. (5) and (6), respectively.

m	1	2	3	4	5	6	7
b_m	-1138.82	3.5382×10^{-4}	1295.79	8.7618×10^{-5}	-158.24	8.8538×10^{-4}	3.817
d_m	0.392	0.00247	-0.0024	0.7765	0.4115	0.00495	4.5636

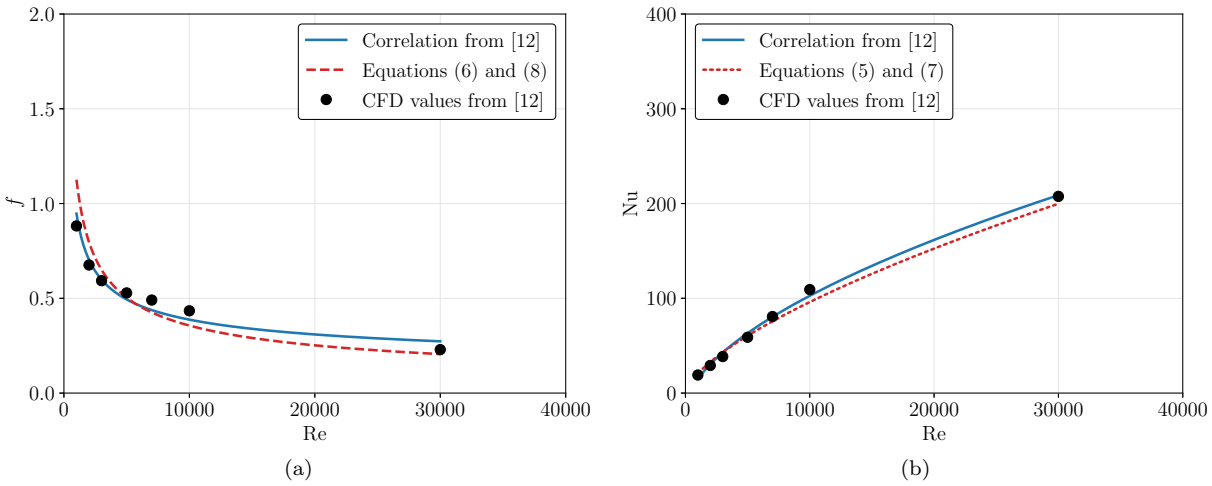


Figure 6: Comparison between the values of equivalent Darcy friction factor (a) and mean Nusselt number (b) obtained from the response surfaces derived in this work with those obtained from the dedicated CFD model developed in [12].

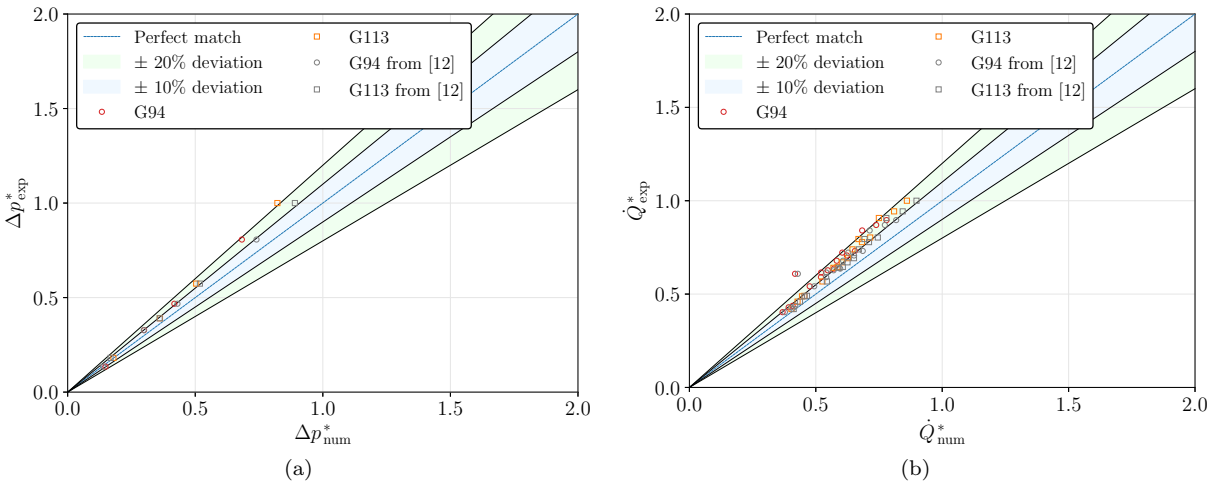
Darcy friction factor and mean Nusselt number derived from Eqs. (5) to (8) with the CFD results and associated correlations related to the WF4 fin from [12]. At Re = 5000, there is a very good agreement between the outcomes of the response surfaces and the CFD results from [12] for both f and Nu. This confirms the validity of the present parametric analysis as the response surfaces return estimates on the thermal-hydraulic performance that are effectively equivalent to those obtained from dedicated CFD models, even though the WF4 configuration was not studied directly in the present DOE. In addition, the scaling laws proposed by Stasiek *et al.* [15] seem to correctly represents the scaling of f and Nu with respect to the Reynolds number, as the correlations developed in [12] for the WF4 fin are well matched by the above-described response surfaces, particularly in case of the mean Nusselt number.

The validation of the response surfaces is expanded by comparing the numerical and experimental results on external head losses and heat transfer rates of the G94 and G113 heat exchangers from [12], with those obtained by substituting the correlations for f and Nu of WF4 derived in [12] with the generalised response surfaces developed in this work. This comparison is shown in Fig. 7, where the agreement between numerical results obtained from the present response surfaces and the experimental data is similar to that observed for the numerical results in [12], confirming the overall validity of the DOE analysis performed in this work. In particular the average relative deviations between numerical and experimental results on heat transfer and head losses are of $\pm 12.1\%$ and $\pm 10.3\%$, respectively. The mismatch between the two sets of numerical results of Fig. 7 tends to increase with growing Δp^* and $\dot{Q}^*$, i.e. as the flow rate and Reynolds number increase. These discrepancies can be traced back to the deviations between the f and Nu values provided by the response surfaces and those obtained from the dedicated correlations developed for WF4 in [12], which tend to grow with increasing Re. This shortcoming stems from the limited data set used ad a bases for the derivation of the response surfaces, which will be expanded during future research activities.

7 Concluding remarks

This work presented the first steps and preliminary results of a parametric analysis on the thermal-hydraulic performance of wavy or corrugated fins. The dependence of geometric parameters on the equivalent Darcy friction factor f and Colburn factor j was evaluated by means of a Design Of Experiment approach, which was carried out by using numerical methods. Each configuration was studied by developing a CFD model of a unitary fin module under periodic flow conditions.

Preliminary numerical results on heat transfer and head losses were employed for the derivation of basic response surfaces for f and j, where the dependency on the Reynolds number was modelled by using scaling laws available in the literature. Despite the limited data set used for the derivation of the response surfaces, these proved be quite accurate when their outcomes were compared with the numerical and experimental data obtained for single fins and complete heat exchangers presented in [12].



(a) (b)

Figure 7: Comparison between the numerically obtained values of external head losses (b) and heat transfer rates (a) with the numerical end experimental results presented in [12]. In these plots all numerical and experimental values are scaled by the maximum experimental entry.

Future research efforts will be aimed at expanding and completing the present parametric analysis by studying additional flow regimes. Further numerical results will be integrated into the data set used for the derivation of the response surface in order to improve their accuracy.

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