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Lower and upper bound for the pull-in parameters of a micro- or nanocantilever beam immersed in liquid electrolytes

Giovanni Bianchi[a *] and Enrico Radi[b]

[a] Dipartimento di Ingegneria e Architettura, Università di Parma,
Via delle Scienze 181 / a, 43124, Parma, Italy

[b] Dipartimento di Scienze e Metodi dell'Ingegneria, Università di Modena e Reggio Emilia,
Via Amendola 2, 42122, Reggio Emilia, Italy

[a]giovanni.bianchi@unipr.it, [b]enrico.radi@unimore.it

Keywords: MEMS, NEMS, Pull-in instability, Liquid electrolytes.

Abstract. An analytical method is proposed to accurately estimate the pull-in parameters of a micro- or nanocantilever beam immersed in liquid electrolytes with a flexible support at one end. The system is actuated by electrochemical force, namely the sum of electric and osmotic forces, and is subject to Casimir or van der Waals forces according to the spacing between the two electrodes. The deflection of the beam is described by a fourth-order nonlinear boundary value problem that can be formulated by an equivalent nonlinear integral equation. At first, *a priori* upper and lower analytical estimates on the beam deflection are derived and then very accurate lower and upper bounds for the pull-in voltage and tip deflection are obtained. The analytical predictions are in excellent agreement with the numerical results provided by the shooting method. Finally, a simple closed-form relation is proposed for the pull-in voltage under the effect of bulk ion concentration.

1. Introduction

Micro- and nanoelectromechanical systems, MEMS and NEMS, have the potential to innovate many application fields: from medical and automobile to aerospace and information areas [1]. To this aim innovative NEMS devices include high precision sensors, nanoactuators and nanotweezers for miniaturized robotics and memory devices [2,3,4,5]. In addition to industrial and consumer uses, BioMEMS and BioNEMS are increasingly applied for chemical and biochemical analyses in medical diagnostics as DNA, cells, blood pressure and toxin identifications, tissue engineering, implantable pharmaceutical drug delivery, and minimal invasive surgery [6,7,8]. In these applications the components are immersed in liquid electrolytes which are typically 0.2 M ionic solutions, mainly NaCl or KCl [9]. Moreover, ionic liquid NEMS actuators find application in many other devices, such as fuel cells, batteries, supercapacitors, filters, electro-osmotic pumps, storage of hydrogen and electroactive polymer actuators [10].

These devices typically enclose a large number of nanoswitches, which are formed by a mobile electrode and a fixed one. The mobile part is modelled by a cantilever or clamped-clamped beam suspended above a conductive substrate and actuated by a voltage difference [11,12]. The application of electrostatic force then switches the mobile electrode between two stable positions. Indeed, as the voltage is increased the mobile electrode deflects towards the fixed electrode until the former suddenly collapses on the latter under a critical voltage, defined as pull-in voltage, thus closing the electric circuit. Investigation of this kind of instability, called pull-in instability, through analytical and accurate prediction of pull-in parameters is fundamental for MEMS and NEMS design in order to determine the operation voltage and power dissipation correctly [1,13,14]. Moreover, the pull-in voltage strongly limits the behaviour of the system. Indeed, to reduce the power consumption and the amount of energy stored in the system the design should aim at obtaining devices actuated by small pull-in voltage [15]. Conversely, increasing the pull-in voltage yields more stable devices. For these

reasons, the best design represents also the best compromise between power consumption and performance of the system. In particular, a decrease in the spacing between the electrodes at the nanoscale implies a drastic reduction in the actuation voltage, since the intermolecular forces, van der Waals (vdW) or Casimir, significantly influence the pull-in instability at this scale [13,16].
The nature of the intermolecular attractions depends on the separation distance between the two electrodes. If the gap is smaller than the retardation length, namely below 20 nm for metals, then the dominant attraction is the vdW force. If the gap is much larger instead, typically above 20 nm, then the Casimir force becomes dominant [17,18]. When the switch is immersed in a liquid electrolyte, the osmotic (chemical) force appears because of ion concentration difference and this new term strongly affects the pull-in voltage. A model for investigating the pull-in instability under electrochemical force, namely the sum of electric and osmotic forces, has been introduced by Boyd et al. [19,20], Noghrehabadi et al. [9,21] and Ghalambaz et al. [22]. These authors observed that the combined effect of electrochemical force and intermolecular interactions makes the problem governing the pull-in instability strongly nonlinear and, to author's knowledge, only numerical and approximate methods have been used in the Literature for solving this boundary value problem (BVP). In particular, 1D lumped models [19], modified Adomian decomposition method [9,21,22], Rayleigh-Ritz and numerical methods [23] and finite element analysis [20] have been employed for investigating the pull-in instability of NEMS devices immersed in ionic solutions. Moreover, the stability and pull-in voltage of electrostatic parallel plates in liquid solutions were experimentally investigated in [24]. The present work aims at providing an accurate analytical method, useful to estimate the pull-in parameters of electrochemically actuated nanobeam, and also to validate the various numerical approaches so far used for solving the BVP.
As reported by Radi et al. [25], the current limitations of manufacturing techniques for micro-nanoelements make inaccurate the hypothesis of perfect clamped support at one end of the real micro-nanoswitch. This involves that the stiffness of the support plays a significant role in the behaviour of the device and, in many situations, modelling a flexible boundary condition is a prerequisite for obtaining a reliable pull-in response of the system. To this scope, some electro-mechanical experiments tested the influence of flexible supports [26]. As a result, they confirm that the effective rotational stiffness of the support should be included in conditions of the problem to obtain an accurate pull-in model for devices at micro- nanoscale.
In this work, the analytical approach proposed for the assessment of accurate lower and upper bounds for the pull-in parameters of a micro- or nanocantilever electrostatically actuated by Radi et al. [25,27,28,29] is extended by taking into consideration a device immersed in liquid electrolytes with the contribution of the osmotic (chemical) force. The effects of intermolecular forces and support flexibility are also considered. The non-linear model governing the beam deflection is introduced in Section 2. An integral equation equivalent to the original BVP is derived by exploiting the Green's function of micro- nanocantilever beam in ionic solution. First, a priori bounds on the beam deflection are derived in Section 3 by exploiting positivity, monotonic behaviour and convexity of the beam deflection. These bounds are then used in Section 4 to obtain accurate lower and upper estimates of pull-in parameters, thus avoiding the difficult task of solving the strongly nonlinear BVP. Results are discussed in Section 5 where the proposed analytical technique is validated by comparing the computed estimates and the numerical results provided by the shooting method. Moreover, in Section 6, an approximated formula is provided for a fast estimation of pull-in voltage under the effect of osmotic force, thus simplifying the design of micro- nano-actuators.

2. Mathematical model

Figure 1 shows a schematic view of an elastic cantilever beam of length l, where the axial position is $0 \le z \le l$. The flexible support at the end $z = 0$ is modelled by a rotational spring of stiffness K. The deflection $v(z)$ of the beam subjected to electrochemical actuation and intermolecular surface forces can be described by a fourth-order, non-linear ODE written in terms of the nondimensional variables $u = v/d$, $x = z/l$, where d is the initial gap between the electrodes, and $\phi = z_c\, e\, \psi / (k_B\, T)$, corresponds

to the value of the non-dimensional applied voltage, as described in [9,20]. Moreover, z_c corresponds to the absolute value of the valence, $e = 1.602 \cdot 10^{-19}$ C is the electronic charge, ψ_1 and ψ_2 are the applied electric potential to the two electrodes, $k_B = 1.38054 \cdot 10^{-23}$ J K^{-1} is the Boltzmann constant and T is the absolute temperature. By following the approach presented in [9,19,20], the governing nonlinear differential equation of the nanobeam in liquid electrolytes is

$$u^{\mathrm{IV}}(x) = \frac{\beta}{\sinh^2\left(\xi_0\left[1-u(x)\right]\right)}\left[\frac{1}{2}\left(1+\left(\frac{\phi_2}{\phi_1}\right)^2\right)-\frac{\phi_2}{\phi_1}\cosh\left(\xi_0\left(1-u(x)\right)\right)\right]+\frac{\alpha_W}{[1-u(x)]^3}+\frac{\alpha_C}{[1-u(x)]^4}, \qquad (1)$$

for $x \in [0, 1]$, completed by the boundary conditions

$$u(0) = 0, \qquad u'(0) = u''(0)/k, \qquad u''(1) = u'''(1) = 0. \qquad (2)$$

The nondimensional positive parameters β, α_W and α_C, appearing in Eq. (1), are proportional to the electrochemical, van der Waals and Casimir forces, respectively. Moreover, ϕ_2/ϕ_1 is the non-dimensional voltage ratio and ξ_0 represents the non-dimensional ionic concentration.

$$\beta = w\frac{2c_b k_B T\ \phi_1^2 l^4}{dEI}, \qquad \alpha_W = \frac{A w l^4}{6\pi d^4 EI}, \qquad \alpha_C = \frac{\pi^2 h c w l^4}{240 d^5 EI}, \qquad \frac{\phi_2}{\phi_1}, \qquad \xi_0 = \kappa d\ . \qquad (3)$$

In Eq. (3) w is the cross-section width, c_b represents the bulk ion concentration, $h = 1.055 \cdot 10^{-34}$ Js is the Planck's constant divided by 2π, c = $2.998 \cdot 10^8$ m/s is the speed of light, A is the Hamaker constant (typical value $A = 0.4 \div 4 \cdot 10\text{-}20$ J), E is the Young's modulus of the beam material, I is the moment of inertia of the beam cross-section, $\kappa^2 = 2\ c_b\ (z_c\ e)^2\ /\ \varepsilon\varepsilon_0\ k_B\ T$ and $1\ /\ \kappa$ is the Debye length, finally $\varepsilon_0 = 8.854 \cdot 10^{-12}$ C^{2}N^{-1}m^{-2} is the permittivity of vacuum and ε is the relative permittivity of the dielectric medium between the two electrodes. Moreover, the constant

$$k = \frac{K\,l}{EI}, \qquad (4)$$

denotes the nondimensional ratio between the rotational spring stiffness and beam bending stiffness. Note that the built-in end condition at $x = 0$ is recovered as k approaches infinity.

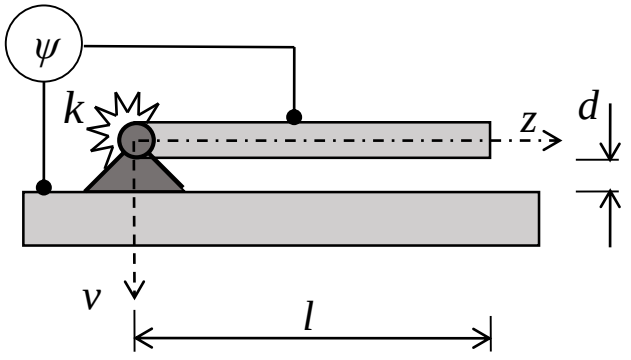


Fig. 1 A micro/nanocantilever beam on a flexible support under electrochemical loading.

Following the analytical approach introduced by Radi et al. [25,27,28,29] the BVP is equivalent to a nonlinear integral equation obtained exploiting the Green's function $G(t)$ of the differential problem (1) and (2). In particular, imposing the boundary conditions of cantilever beam and the continuity conditions, the coefficients of the general solution to this problem can be univocally determined.

$$u(x)=\int_0^x\left[\frac{t^2(3x-t)}{6}+\frac{xt}{k}\right]f_{EC}(u)dt+\int_x^1\left[\frac{x^2(3t-x)}{6}+\frac{xt}{k}\right]f_N(u)dt. \tag{5}$$

where the non-dimensional electrochemical and intermolecular surface forces are

$$f_{EC}(u)=\frac{\beta}{\sinh^2(\xi_0(1-u))}\left[\frac{1}{2}\left(1+\left(\frac{\phi_2}{\phi_1}\right)^2\right)-\frac{\phi_2}{\phi_1}\cosh(\xi_0(1-u))\right] \tag{6}$$

$$f_N(u)=\frac{\alpha_N}{[1-u]^n}\quad \text{with } N=W \text{ for } n=3 \text{ or } N=C \text{ for } n=4 \tag{7}$$

where W and C mean van der Waals and Casimir forces, respectively. Note that the function f_N is positive for $0 \le u \le 1$, namely the effect of intermolecular forces is attractive between the two electrodes. Otherwise, function f_{EC} is not defined in sign, namely the electrochemical force can produce an attractive or repulsive action between the electrodes according to the applied voltage and ionic concentration of the liquid solution. Clearly, the pull-in instability can appear only if the total force which act on the system is positive, namely attractive

$$f(u)=f_{EC}(u)+f_N(u)\ge 0, \text{for} 0\le u\le 1 \tag{8}$$

Inequality (8) allows to obtain useful a priori estimates on the solution of the BVP (1) and (2), following the procedure described in Section 3 of [25].

3. A priori estimates

Upper and lower bounds for the solution $u(x)$ to the problem (1) and (2)

$$u(x) \le u(1)\, b(x), \qquad \text{for} \quad x \in [0, 1], \tag{9}$$

$$u(x) \ge u(1)\, a_1(x) + \beta^*\, a_2(x), \qquad \text{for} \quad x \in [0, 1], \tag{10}$$

are the same provided in [25] where the extended expression for $b(x)$, $a_1(x)$ and $a_2(x)$ are reported for the elastically constrained cantilever beam. Otherwise, β^* is different from value in [25] because the load functions of the two model are different. The value of β^* for a device immersed in a liquid electrolyte is

$$\beta^* = \beta\left(\frac{1+\left(\frac{\phi_2}{\phi_1}\right)^2-2\frac{\phi_2}{\phi_1}\cosh(\xi_0)}{2\sinh^2(\xi_0)}\right)+\alpha_W+\alpha_C. \tag{11}$$

4. Bounds on the pull-in parameters

From Eq. (5) we obtain $r = u(1)$ that denotes the normalized deflection of the cantilever tip. Then, by using r and the estimates (9) and (10) on the solution of the BVP (1)-(2), lower and upper bounds can be derived for the pull-in parameters β_{PI} and r_{PI}, as described in [25].

$$r \le \frac{1}{6}\left[\beta\, L(r) + F(r)\right], \tag{12}$$

$$r \ge \frac{1}{6}\left[\beta\, K(r, \beta) + H(r, \beta)\right], \tag{13}$$

5. Results

At first, we investigate the effects of the total force affecting the micro- nanocantilever when it is immersed in a liquid electrolyte. As remarked in Sec. 2, the electrochemical force is undefined in sign, or in a more practical way it can be attractive or repulsive depending of the non-dimensional ionic concentration ξ_0 and non-dimensional voltage ratio ϕ_2/ϕ_1. When the electrochemical force is attractive, namely $f_{EC} > 0$, the total force $f = f_{EC} + f_N$ is always attractive, so the pull-in instability can appear for the cantilever actuator bended toward the substrate. Otherwise, the total force of the system can close or separate the electrodes under an attractive or a repulsive contribution. In particular, the pull-in instability is possible only for a positive, namely attractive, total force, $| f_N | > | f_{EC} |$, so for a positive deflection of the mobile electrode toward the substrate. On the contrary, the pull-in instability cannot appear for a negative, namely repulsive total force, $| f_N | < | f_{EC} |$, which bends the cantilever beam away from the substrate. Then, useful qualitative graphs, which represent regions where the total force is positive under fixed sets of nondimensional parameters, can be obtained exploiting inequality (8). The region, where the total force is attractive, is shown in Figs. 2a-2b for different values of ξ_0 and ϕ_2/ϕ_1. The coloured area provides positive attractive total force. We can observe that the region increases when the surface forces increase. In particular, for high value of surface forces, namely $\alpha_N > 0.5$, in Fig. 2a there's no more combinations of the parameters ξ_0 and ϕ_2/ϕ_1 that produce a repulsive total force. However, in Fig. 2b where higher electrochemical force β is considered, a white uncoloured region is present, so a repulsive force can appear. To reduce the white region a greater surface force, $\alpha_N > 0.5$, is necessary.

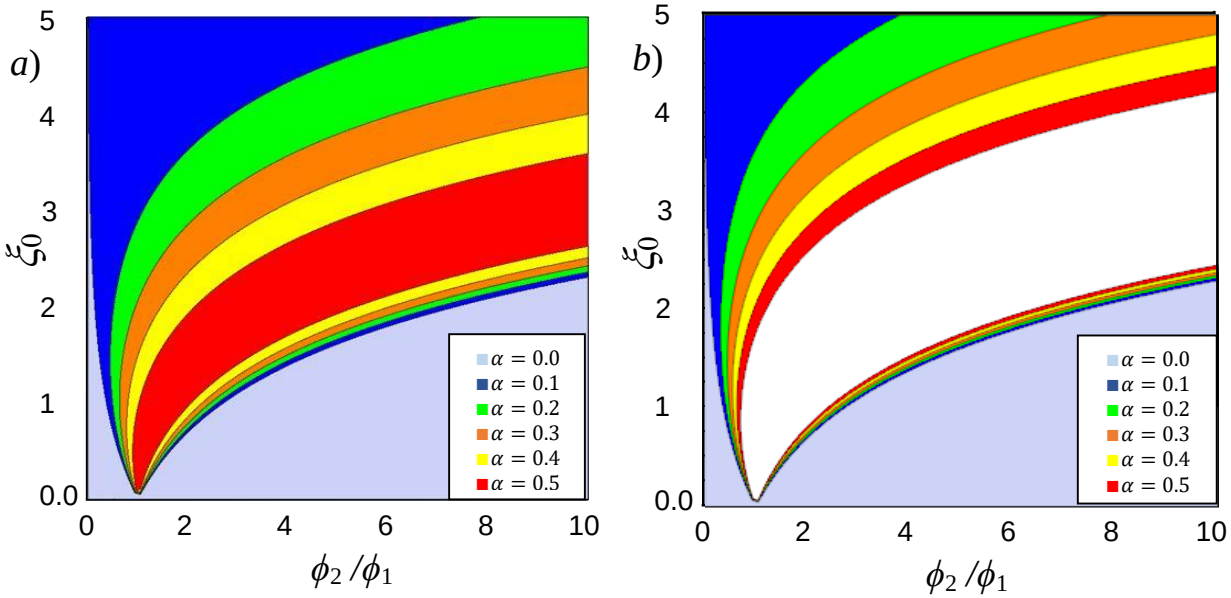


Fig. 2 Coloured region represents the relationships between ionic concentration ξ_0 and voltage ratio ϕ_2/ϕ_1 which provide positive, namely attractive, total force for different intermolecular forces α and for fixed values of $\beta = 1$, $u(1) = 0$ (*a*) and $\beta = 2$, $u(1) = 0$ (*b*).

The relationships between the non-dimensional voltage ratio ϕ_2/ϕ_1 and tip deflection of the nanocantilever $r = u(1)$, obtained from the shooting method (function NDSolve in *Mathematica*® [30]), are presented in Figs. 3a-3b for constant values of electrochemical force β, vdW force α_W and ionic concentration ξ_0, and different values of the non-dimensional support stiffness k. In Fig. 3a we can see that the cantilever tip deflection reaches zero for $\phi_2/\phi_1 = 0.518$ and $\phi_2/\phi_1 = 7.006$. Indeed, the total force at these voltage ratios is zero since the vdW and the electrochemical forces have the same intensity but opposite direction, and thus no deflection of the beam is observed at these values. Moreover, for the interval $0.518 < \phi_2/\phi_1 < 7.006$ the total force is repulsive causing the separation of the mobile electrode from the substrate, namely a negative displacement. Considering different values for β and ξ_0, in Fig. 3b, we obtain similar trends but diverse critical values of voltage ratio, namely

$\phi_2/\phi_1 = 0.633$ and $\phi_2/\phi_1 = 2.453$. In particular, the total force is repulsive force for $0.633 < \phi_2 / \phi_1 < 2.453$ that are narrow limits respect to those shown in Fig. 3a for upper values of β and ξ_0. Moreover, we can observe that the maximum normalized tip deflection $u(1)$ in module depends on the support stiffness k and it can be negative under a repulsive force, see Fig. 3a, or positive under an attractive force, see Fig. 3b. Values of maximum tip deflection can be useful design parameter in various MEMS and NEMS applications.

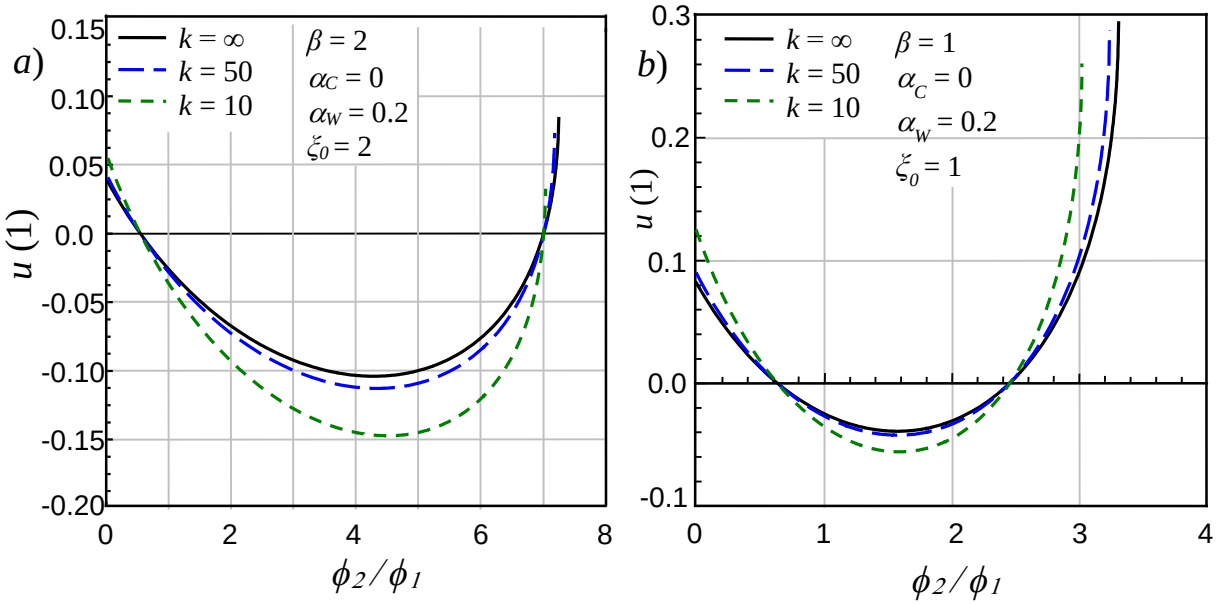


Fig. 3 Variations of the tip displacement $u(1)$ with the voltage ratio ϕ_2 / ϕ_1 obtained from the shooting method, for various values of the support stiffness k and for $\beta = 2$, $\alpha_W = 0.2$, $\alpha_C = 0$, $\xi_0 = 2$ (*a*) and $\beta = 1$, $\alpha_W = 0.2$, $\alpha_C = 0$, $\xi_0 = 1$ (*b*).

The relationships between the electrochemical parameter β and tip deflection of the beam r are presented in Fig. 4 for different values of ionic concentration ξ_0. The pull-in critical estimates predicted by using the proposed theoretical method are indicated by small (black) circles for lower bounds and by small black points for upper ones. It can be observed that the analytical lower and upper estimates of β and r are very close each other and perfectly bound the numerical pull-in parameters β_{PI} and r_{PI}, which correspond to the maximum of the curves reported in figures. In particular, observing Fig. 4 we note that increasing the ionic concentration ξ_0 of the liquid solution, the critical electrochemical pull-in values β significantly increase, whereas the corresponding values of tip deflection decrease. For different values of non-dimensional voltage ratio ϕ_2/ϕ_1, we observe very different values of the electrochemical force β and similar tip deflections, typically included in range $0.2 < u(1) < 0.5$. Furthermore, for low values of ϕ_2/ϕ_1 a very high electrochemical force is required to cause the pull-in instability.

In Table 1, upper and lower pull-in parameters are reported for the specific set of parameters. Increasing the magnitude of the intermolecular surface forces, namely for higher values of coefficients α_W and α_C, the pull-in electrochemical values β always significantly decrease, whereas the corresponding tip displacements show different trends depending on the ionic concentration and voltage ratio. In particular, for high ionic concentration and very small ϕ_2/ϕ_1 the pull-in deflection significantly increases with intermolecular forces, contrary to corresponding β reduction. Otherwise, for $\xi_0 = 0.5$ and $\phi_2/\phi_1 = 0.5$ the pull-in deflection slightly decreases with intermolecular forces.

In Table 2 critical values of β_u, β_l, r_u and r_l are shown for various values of the nondimensional support stiffness k and ionic concentration ξ_0. These estimates reveal that increasing values of the stiffness parameter k, raise the pull-in electrochemical force without producing significant effect on the pull-in tip deflection.

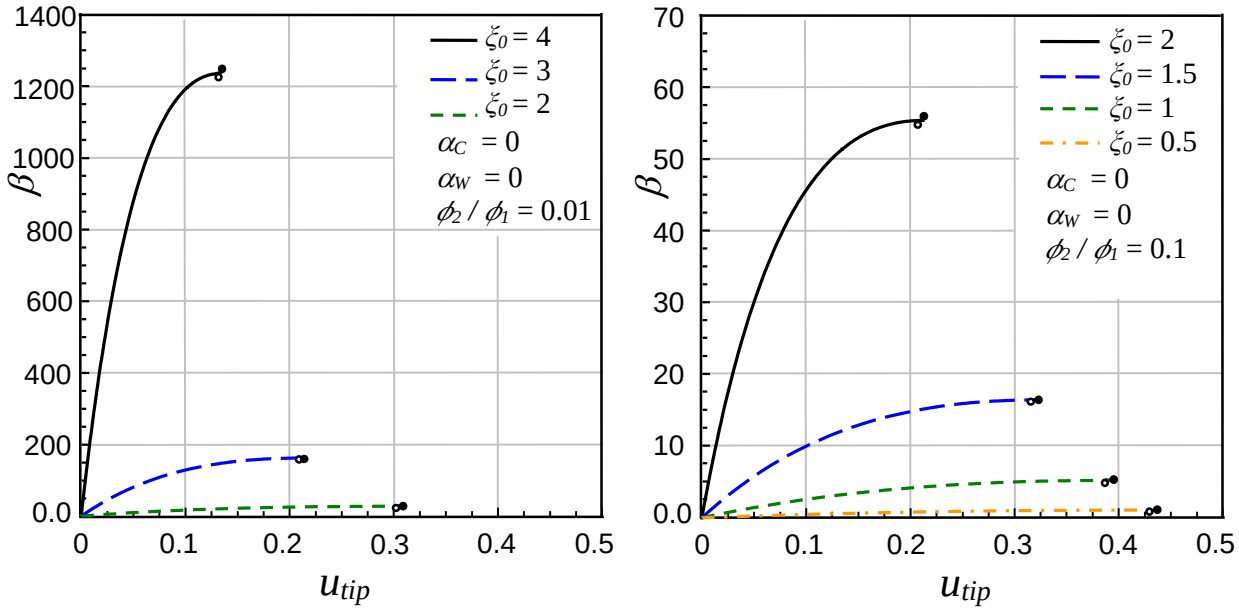


Fig. 4 Variations of the tip displacement $u(1)$ with the electrchemical parameters β obtained from the shooting method, neglecting intermolecular forces $\alpha_W = \alpha_C = 0$, for various values of the ionic concentration ξ_0 and the voltage ratio ϕ_2/ϕ_1.

$k \to \infty$		$\xi_0 = 4 \quad \phi_2/\phi_1 = 0.01$				$\xi_0 = 0.5 \quad \phi_2/\phi_1 = 0.5$			
α_W	α_C	r_l	β_l	r_u	β_u	r_l	β_l	r_u	β_u
0.0	0.0	0.132937	1223.06	0.136956	1250.1	0.349314	4.73619	0.358105	4.83197
0.0	0.2	0.156973	904.533	0.160884	929.46	0.323497	3.68087	0.331686	3.77248
0.0	0.4	0.183189	618.308	0.186965	640.929	0.304472	2.65364	0.312256	2.74188
0.0	0.6	0.212222	362.48	0.215809	382.723	0.2899323	1.64575	0.296794	1.73112
0.0	0.8	0.244988	135.411	0.248281	153.267	0.276722	0.652309	0.283933	0.735147
0.0	1.0	0.281812	-64.3341	0.285606	-48.861	0.265938	-0.329792	0.272924	-0.24924
0.0	0.0	0.132937	1223.06	0.136956	1250.1	0.349314	4.73619	0.358105	4.83197
0.2	0.0	0.158559	937.581	0.162452	962.217	0.346282	3.94379	0.354808	4.03691
0.4	0.0	0.186223	688.201	0.189972	710.256	0.343569	3.1518	0.35185	3.24234
0.6	0.0	0.216535	471.907	0.220097	491.364	0.341123	2.36015	0.349176	2.44817
0.8	0.0	0.250405	286.137	0.253697	303.064	0.338902	1.56881	0.346741	1.65437
1.0	0.0	0.28931	128.749	0.29216	143.24	0.336875	0.777725	0.344511	0.860875

Table 1 Values of lower and upper pull-in parameters estimated by using the proposed theoretical approach for a rigidly clamped nanocantilever, for various values of the parameters α_W, α_C ionic concentration ξ_0 and voltage ratio ϕ_2 / ϕ_1.

$\phi_2/\phi_1 = 0.1$		$\alpha_W = 0.5$				$\alpha_C = 0.5$			
k	ξ_0	r_l	β_l	r_u	β_u	r_l	β_l	r_u	β_u
∞	1.5	0.324531	9.50766	0.33223	9.81607	0.28623	7.52071	0.2935	7.8248
50	1.5	0.325193	8.29662	0.3336	8.6393	0.28443	6.31572	0.29209	6.63583
10	1.5	0.327684	4.83103	0.33719	5.18228	0.27769	2.87252	0.28543	3.15088
5	1.5	0.330584	2.23115	0.33919	2.48706	0.27021	0.29705	0.27629	0.45083
∞	2	0.262724	30.3253	0.26878	31.4296	0.24402	25.0186	0.24991	26.1301
50	2	0.267061	26.36	0.2737	27.5956	0.24661	20.9939	0.25287	22.183
10	2	0.284199	15.1193	0.29178	16.3991	0.25641	9.52011	0.26282	10.5756
5	2	0.305134	6.86014	0.31203	7.7845	0.26753	0.98122	0.27243	1.52032

Table 2 Values of lower and upper pull-in parameters provided by proposed theoretical approach for various values of the support stiffness k and of the ionic concentration ξ_0, fixed the voltage ratio ϕ_2 / ϕ_1 and parameters α_W, α_C.

6. Approximated analytical relations for the pull-in parameters

In this section, the lower and upper estimates computed by using the theoretical approach described above, are exploited to obtain approximated analytical relations for pull-in electrochemical force β and corresponding tip deflection r for nanocantilever beam immersed in liquid electrolytes. In particular, the variations of the parameters β and r, with the ionic concentration ξ_0, fixed $\phi_2/\phi_1 = 0.1$ and negligible intermolecular forces, are described by the following approximated relations, which hold for $0 < \xi_0 \leq 2$

$$\beta = \frac{-138.056 + 995.632\xi_0 + 3615.06\xi_0^2}{1419.31 - 563.898\xi_0 + 0.426288\xi_0^2}, \tag{14}$$

$$r = \frac{36.3559 + 31.9245\xi_0 - 17.7879\xi_0^2}{82.0033 + 66.736\xi_0 - 19.4946\xi_0^2}. \tag{15}$$

Numerical coefficients in relations (14) and (15) are set by interpolating the lower and upper bounds provided by the analytical method by using the function NonlinerModelFit available in *Mathematica*®. The analytical estimates have been computed varying the ionic concentration ξ_0 from 0.2 to 1 with a step of 0.2 and from 1 to 2 with a step of 0.1, for fixed $\phi_2/\phi_1 = 0.1$ and negligible intermolecular forces for a rigidly clamped cantilever. In Figs. 5a and 5b, the solid curves represent the approximated relations (14) and (15), whereas lower and upper bounds, β_l and β_u in Fig. 5a, r_l and r_u in Fig. 5b, are indicated by small circles and point. We observe that the approximated curves are perfectly bounded by analytical predictions, thus ensuring the accuracy of the approximated relations (14) and (15).

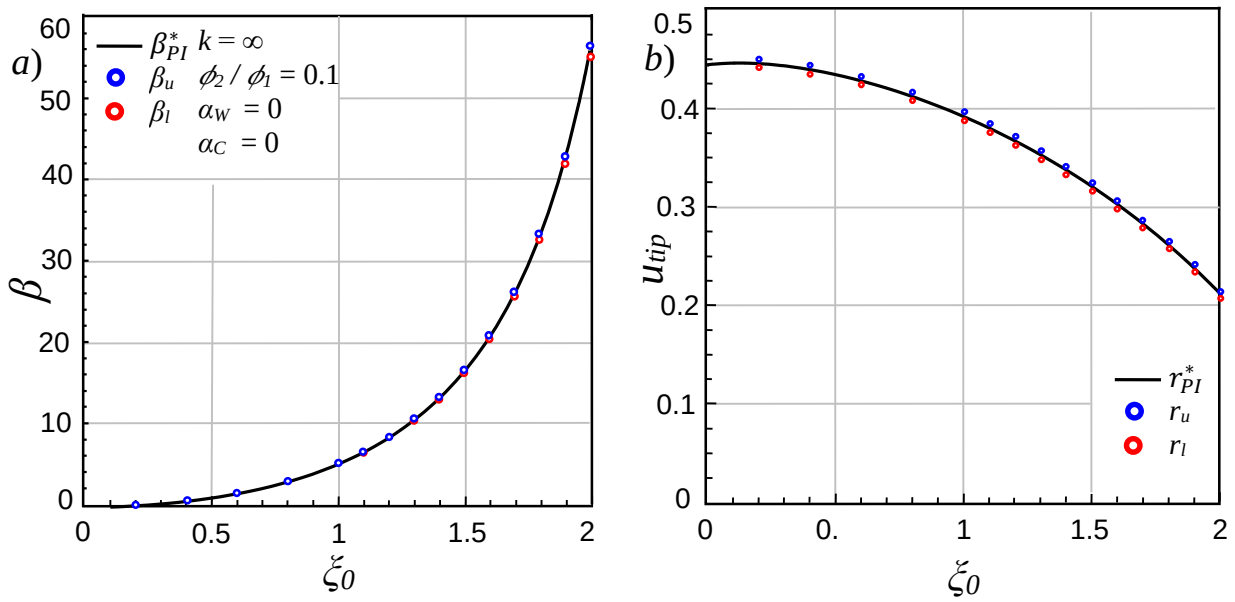


Fig. 5 Approximated variation of pull-in parameters β^*_{PI} (*a*) and r^*_{PI} (*b*) with the ionic concentration ξ_0 and nondimensional voltage ratio $\phi_2 / \phi_1 = 0.1$. The upper and lower analytical estimates are denoted by blue and red circles, respectively.

7. Conclusion

In this work, the present analytical method is extended to investigate the pull-in instability for a micro-nanocantilever actuator immersed in an electrolytic solution under the influence of the ionic concentration and the non-dimensional voltage ratio. The analytical method provides lower and upper bounds for the pull-in parameters of the initial BVP, combining the effects of vdW and Casimir forces with the influence of the electrochemical force. The accuracy is proved comparing analytical predictions with the numerical results provided by the shooting method. At first, we have observed that while the intermolecular force is always attractive, the electrochemical force can be attractive or repulsive depending on the magnitude of the electric and osmotic forces. Indeed, even if the electric force is always attractive and osmotic force always repulsive, their sum, namely the electrochemical force, is not defined at priori. Then, the pull-in instability may appear only when the total force, namely the sum of intermolecular and electrochemical forces, is attractive, bending the mobile electrode toward the substrate. On the contrary, a repulsive total force pulls apart the two electrodes and no pull-in instability is possible. Furthermore, it is noted that the combined effects of the flexible boundary conditions and intermolecular surface forces significantly reduce the pull-in voltage, whereas a higher ionic concentration increase the pull-in voltage delaying the pull-in instability. Therefore, a model complete with all these effects is required to avoid overestimation of the pull-in instability which may cause dangerous failure of the device. In this sense, some approximate relations are also provided to help the design of MEMS and NEMS actuators by simplify the prediction of pull-in parameters in order to guarantee the proper operation of these devices in various applications, avoiding unexpected faults.

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