THE USE OF CONCRETE ARTEFACTS FOR CALCULATION: A SEMIOTIC APPROACH

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My presentation deals with the use of arithmetic artefacts[i] in early years for the purpose of counting, representing whole numbers and making calculations. In most countries simple arithmetic activities have been carried on for millennia by means of concrete artefacts, which, in some sense, embody mathematics knowledge. Yet it is not enough to offer them to students to be sure that students will reconstruct the expected arithmetic meanings. In this presentation I shall introduce examples of arithmetic artefacts within the theoretical framework of semiotic mediation after a Vygotskian approach. This allows, on the one hand, to analyse the semiotic potential of each of the artefacts and, on the other hand, to design effective activities to propose in the mathematics classroom under the teacher's guidance. A discussion of the issue of cultural transposition is appended.

INTRODUCTION

The classical book of Menninger (1969) devotes many pages to concrete aids for counting, representing numbers and making calculations, from body parts to counting boards. The diffusion of these aids was boundless and concerned people from both the East and the West, from both the industrial and the developing countries. Somebody might believe that in the era of digital resources, the resort to such old fashioned tools is not useful, also because there are "virtual copies" of them (see the National Library of Virtual Manipulatives[ii]). Yet, there is evidence from neuroscience that confirm the usefulness of a multimodal approach (i. e. visual, auditive, tactile), which is better conveyed with concrete artefacts, since "concepts are deeply rooted in our sensory-motor activities" (Arzarello & Robutti, 2008, p. 719).

It is worthwhile to quote here the MIT scientist, Hiroshi Ishii, leader of the *The Tangible Media Group*[iii] that explores the tangible bits and radical atoms visions to seamlessly couple the dual world of bits and atoms by giving dynamic physical form to digital information and computation. In an interview (Moggridge, 2006), Ishii handles a soroban, the Japanese abacus, a rectangular wooden frame which is divided lengthwise into two unequal parts by a horizontal beam. The counters are doublecone-shaped wooden beads that slide on slim wooden dowels.

Ishii shows easy familiarity of a long association and talks about the meaning soroban holds for him:

> Below/in Fig 1 is an abacus, the simplest form of digital computation device. All of the information is represented in the array of the beads, in a physical way, so that people can directly touch, manipulate, and 'feel' the information. This coupling of manipulation and control is very natural in this kind of physical device, but in the digital domain, the graphical user interface introduces a great/deep divide between the pixel representation and the controllers like the mouse. Another important feature is the affordance. This is a simple mechanical structure; by grabbing this device when I was a kid, it immediately became a musical instrument, an imaginary toy train, or a backscratcher, so I could really feel and enjoy the beads. This also serves as a medium of awareness. When my mother was busy doing the accounting in our small apartment in Tokyo, I could hear the music the abacus made, which told me that I couldn't interrupt her to ask her to play with me. Knowing other people's state through some ambient sound, such as this abacus, teaches us important directions for the next generation of user interfaces. This simple historic device taught us a lot about new directions, which we call tangible user interface (Moggridge, 2006, p. 529).

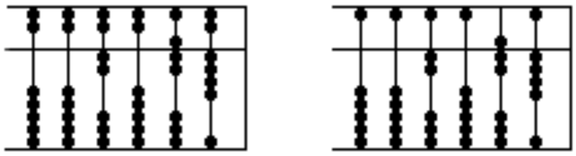

Fig. 1. Schemes of the Chinese suànpán (left) and of the Japanese soroban (right)

This anecdote does not only evoke the affective relevance of the soroban (with deep cognitive and metacognitive implications) but it also hints at the importance of such artefacts in the development of today's user interfaces. Several studies have been carried out in Japan to show how the concrete operation with the soroban, realized by means of a particular fingering (see below), allows the construction of a mental soroban which is analogous to the actual one; the mental manipulation of beads improve digit memory retention (Hatano & Osawa, 1983; Hatta, Hirose, Ikeda & Fukuhara, 1989). More recent studies (Frank & Barner, 2012) have confirmed Hatano's intuition that the mental soroban is represented non linguistically, but in a visual format. Hence they offer arguments for the school use in the digital era, as this method of mental calculation is effective in the development of the right brain and of the connection between right and left brain[iv].

Effective fingering (i. e. the role of thumb, forefinger and medium finger) is taught. Rules for addition and subtraction are presented and memorized, in order to perform the task of adding and subtracting beads mechanically, without thought or hesitation, in other words, to develop a process of thoughtlessness. Information are "felt" as Ishii expresses himself (see above).

Numbers are represented and operation are realized from left to right. It may seem a little odd at first but in this way numbers are added and subtracted in exactly the same way we read and hear them.

Western teachers are astonished by the speed in using soroban by Japanese students and even more astonished by the fact that they do not really need a concrete soroban, but simulate it in the mind moving only the fingers on the desk.

Soroban is derived from the Chinese suànpán, that is a bit more complex, as it has five beads below the beam and two beads above the beam. The rules (left to right) and fingering are similar[v]. The suànpán was among the latest 31 items added to UNESCO's list of intangible heritage on October 23, 2013. Modern versions of the suànpán have 13 columns of 7 beads.

Suànpán is introduced in Chinese primary school classrooms, with focus on the posture, the handling of the pencil and the fingering. Figure 2 is taken from a Chinese textbook for the first grade (the simpler soroban is used in this case).

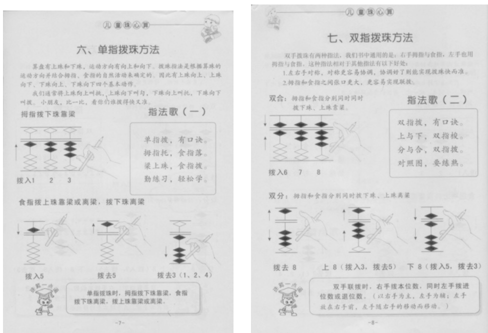

Fig. 2. Fingering in a Chinese textbook

This long introduction acknowledges the importance of the Eastern tradition in the development of some artefacts for calculation. A further example is given by the counting sticks, used in China even before the suànpán (for a semiotic analysis of them, see Mariotti, 2012). Yet these artefacts are also useful to introduce the Theory of Semiotic Mediation (TMS) after a Vygotskian approach, drawing on well known examples, before reporting on other kinds of artefacts for calculation that are related to the Western tradition.

THE THEORY OF SEMIOTIC MEDIATION

The cultural context

Mankind came to construct mathematics as a cultural object, producing artefacts which embody mathematical meanings and processes, although the emergence of meanings for users cannot be realized without specific activities (Meira, 1998). Hence we focused the function of some selected cultural artefacts, developed by mankind and the teacher's role as cultural mediator in the enculturation process.

This focus is consistent with the theoretical construct of *mathematical laboratory*, that was developed in Italy for decades under the influence of famous mathematics educators like Emma Castelnuovo and was officially acknowledged in the Mathematics curricula, developed by the Italian Mathematical Union in 2003 (see

Bartolini Bussi & Martignone, 2013, for a presentation of the mathematical laboratory within the Italian standards).

The TMS after a Vygotskian approach aims to describe and explain the process that starts with the student's use of an artefact to solve a given task and leads to the student's appropriation of a particular piece of mathematical knowledge. The TMS has been introduced by Bartolini Bussi & Mariotti (2008) drawing on several preliminary teaching experiments, carried out with groups of teacher-researchers at very different school levels and with different kinds of artefacts (from concrete ones to digital ones).

Some theoretical constructs

The TMS allows to organize a long term teaching-learning sequence by integrating the use of an artefact to solve a given task, around the key notions of *semiotic potential of an artefact* and of *didactic cycle* (Mariotti, 2012).

In TMS the notion of *artefact* is different from the notion of *instrument* (Rabardel, 1995): the artefact is an object in se, material or symbolic, designed for answering a specific need, whilst the instrument refers to mix (hybrid) entity with an artefact type component and a cognitive component, called *utilisation scheme*. This hybrid entity is the product at the same time of the subject and of the object. Different subjects might produce different utilisation schemes while using the same artefact to solve the same task. As I show below, such an approach, suggests very effective tools of analysis for outlining the semiotic potential of an artefact.

The well known rules for using a suànpán (or a soroban) mentioned above are socially shared utilisation schemes, developed over the centuries. For instance, if the number 4 is represented in the suànpán and the addition 4 + 4 has to be realized, the rule is not to add 4 beads (as in the Slavonic abacus) but to *lower 5 and take away 1*.

By semiotic potential of an artefact we mean the double semiotic link which may occur between an artefact, and the personal meanings emerging from its use to accomplish a task, and at the same time the mathematical meanings evoked by its use and recognizable as mathematics by an expert (Bartolini Bussi and Mariotti, 2008, p. 754). For instance the task *calculate 4+4* leads a novice student to use the taught rule, as she were using a technical tool. But for the expert this utilization scheme has a different status as it hints at the possibility to calculate addition by complementarity $4 + 4 = 4 + 5 - 1$. The process of semiotic mediation consists in the evolution from the initial (artefact) signs (representing the utilisation schemes realized by students to accomplish the task by means of the given artefact) to (mathematical) signs that express the relationship between artefacts and mathematics knowledge. The mathematical signs hence are related to both the use of the artefact and the mathematics knowledge to be learnt. This process may be described by the following scheme (a suànpán is used as an example of artefact, in this case)

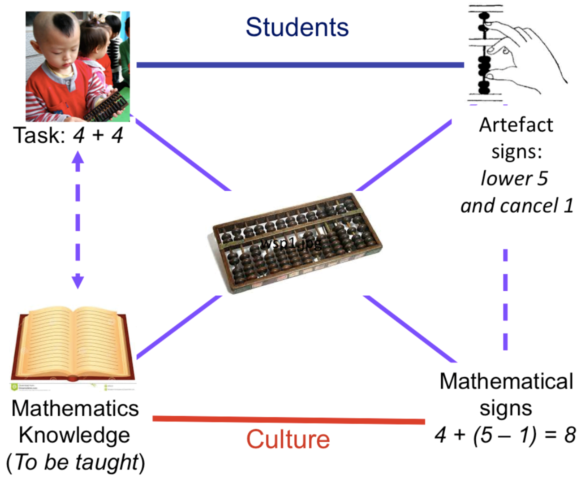

Fig. 3. The process of semiotic mediation

On the left there is the triangle of the semiotic potential of the suànpán related to the task of calculating 4 + 4. On the right there is the evolution from artefact signs to mathematical signs under the teacher's guidance. The teacher plays two different roles in this scheme: the task design role (on the left), the guidance role (on the right) in the evolution from artefact signs to mathematical signs to unfold the semiotic potential of the artefact. Students may become faster and faster to use the suànpán or the soroban and this might produce advanced skills in mental calculation (as said above). However, the construction of symbolic systems and their links with the development of mathematics as a whole, requires to construct not only the skills but also the mathematical meaning of the activity.

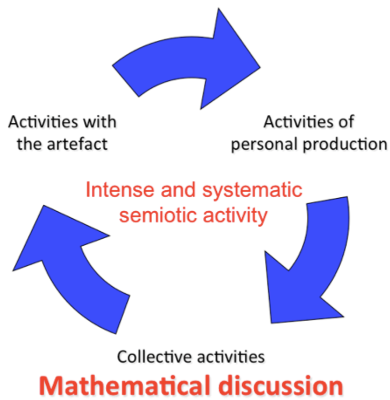

Fig. 4. The didactical cycle

The evolution from artefact signs to mathematical signs can be promoted through the iteration of didactical cycles (fig. 4), where different categories of activities take place: individual or small group activity with the artefact to solve a given task; individual production of signs of different kinds (drawing, written or oral wording, gesturing, and so on); collective production of signs, where the individual productions are shared and the semiotic potential of the artefact is unfolded. These collective phases are called *Mathematical discussions*, that are polyphonies of voices articulated on a mathematical object, which is one of the motives of the activity of teaching and learning.

This short presentation of TMS frames some examples of artefacts for calculation that have been the objects of teaching experiments in my research group.

SOME BEAD ARTEFACTS

The giant Slavonic abacus in pre-school

Since 2007 I have served as the advisor for mathematics teacher development in the more than 20 Municipal pre-schools in Modena (action-research project *Bambini che contano*, that is *Counting children*). In this project a giant Slavonic abacus has been designed by teachers to be used in all the schools. It has forty beads because this number meets the most common needs of school activity (e.g. counting children in the roll, counting the days per month in the calendar). The large size fosters large body gestures (even steps) to move the beads. A more detailed report on this project is in Bartolini Bussi (2013).

Fig. 5. The giant Slavonic abacus

The mathematics knowledge at stake in the use of Slavonic abacus for counting tasks is complex and involves: (a) partition, to separate counted beads from beads to be counted; (b) one-to-one correspondence, between beads and numerals; (c) cardinality, given by the last pronounced numeral; (d) sequence of early numerals, to be practiced in counting; (e) place value (early approach), as beads are divided in tens.

The system of tasks designed and tested in the project aims at fostering the polyphony of voices, mentioned above, when exploring the artefact: (1) the *warming up* task, representing the voice of the *narrator*: *what is this?* (2) the *artefact* task, representing the voice of the *constructor*: *how is it made?* (3) the *instrument* task, representing the voice of the *user*: *how can you use it to solve this problem* (e.g. to count the present students)? (4) the *theory* (mathematics) task, representing the voice of the *mathematician*: *why does it work to solve this task?* (5) the *problem solving* task, representing the voice of the *problem solver*: *what could you do if* …. (e.g. to count all the students of the school and not only of your class)?

This set of tasks is used in all the schools (Bartolini Bussi, 2013) and is shared with other school levels (with small changes according to students' ages, up to secondary school and university teacher education) when the exploration of an artefact is in the foreground

Superabacus

An original artefact inspired by the Japanese soroban and the Chinese suànpán, but recalling the spike abacus, that is more popular in Europe, has been designed by an Italian teacher (Bianchin, 2015).

Fig. 6. Superabacus: 138

As in the soroban, there is a rectangular wooden frame with three dowels on which counters (doublecone-shaped wooden beads) slide. The Superabacus (fig. 6) allows to represent numbers from 0 to 999. The fifth bead is dark to offer a quick perception of 9 = 4+1+4 without counting. With the help of this dark bead, it is easy for students to "know" the number of beads to be pulled or pushed without counting. In this way the shift from *perceptual subitizing* to *conceptual subitizing* (Clements, 1989) is fostered. In the initial position all the beads are at the top. To display a number the student must pull the beads towards herself. Fingering is inspired by Far East abaci: thumb is used to push the beads whilst forefinger is used to pull the beads. In this case too we have observed that students often use fingering without the concrete artefact to manipulate, as if a mental abacus has been constructed. Further studies are needed as the superabacus has been produced very recently and no careful and systematic observation has been made.

SOME NON-BEAD ARTEFACTS

The bead artefacts mentioned above share some features: first the idea of discrete representation of whole numbers (as beads) together with the emphasis on composition/decomposition and part/whole approaches; second, the user's control of the operation. In the following I give some details about two artefacts for calculation, used in Western schools, with different features.

The number line

The number line is a very popular teaching aid: whole numbers are introduced as labels on unit marks by means of a measuring process and additions and subtractions are realized, as operators, with jumps forwards and backwards. For instance, 3 + 4 is realized considering first the number 3 (first addend) and then jumping forwards 4 steps (the second addend). Hence the two addenda have different status and the idea of composition/decomposition of numbers is hidden. Traces of this early approach can be found in the teaching practices of most Western countries (Bartolini Bussi, in press), but not in the most popular Chinese textbooks (Sun, in press).

The Pascaline "zero + 1"

The pascaline is an arithmetic machine composed of gears analogous to the famous artefact, called Pascaline, invented by the French mathematician Blaise Pascal in 1642. It was produced in the period when most scholars were interested in producing more and more complex clocks and to conjecture mechanical models of the world (Rossi, 1984). It is a crucial artefact in the history of European mathematics because it represents the first example of addition performed independently of the human intellect. The pascaline Zero + 1 (Fig. 7) is a simple but ingenious tool produced by the Italian company Quercetti. It is a very robust plastic artefact (27 × 16 cm) with five gears. The 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are written on the teeth of the gears (A, B, C). Three small arrows point at a tooth (on the gears (A, B, C). In different positions, the tool may conventionally represent (by means of the digits to which these arrows point) any number from 0 (written 000) to 999.7 The gears (A, B, C) function as units, 10's, and 100's, whereas gears D and E are auxiliary driving gears to transmit the motion. Each gear may be rotated by pushing a tooth with one's finger. The rotation is not continuous but step by step (one click is one-tenth of a complete rotation). After a complete rotation of gear A (starting from 000), the bar welded to gear D makes gear B turn one step (producing 010). After a complete rotation of gear B, the bar welded to gear E makes gear C turn one step (producing 100). And so on.

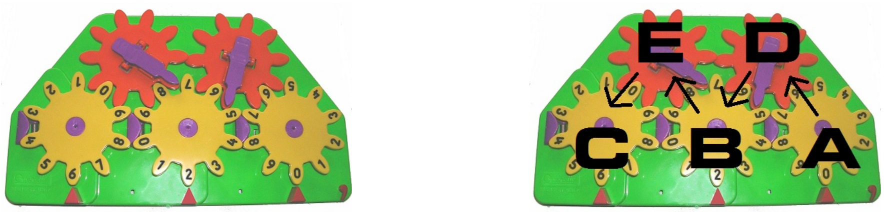

Fig. 7. The Pascaline

We have used the Pascaline in many different teaching experiments from grade 1 to grade 7 (Bartolini Bussi & Boni, 2008) and also in programs for teacher education and developments (Bartolini Bussi, 2011). An experiment in France (Maschietto & Soury-Lavergne, 2013) has been started a few years ago, with the contemporaneous use of a concrete artefact and a virtual copy of it (called e-pascaline).

The mathematics knowledge at stake when using the Pascaline is the following: the generation of the number sequence by means of the operator +1; relationship between syntactical and semantic properties of natural numbers; place value conventions; addition (and subtraction) algorithms.

In this paper I report a short excerpt of a teaching experiment carried out in a fourth grade classroom (teacher Franca Ferri). The students have just started to use the Pascaline, according to tasks similar to the ones reported for the Slavonic abacus, and are revising place value notation and the written algorithms for arithmetic operation.

The teacher has assigned a task to be solved individually with a Pascaline.

Task 1: *Write down the instructions for doing an addition with the Pascaline. For instance: 28 + 14.* Two protocols catch the teacher's attention.

Christian: *I have written the first number (28) and then I have added the second one, rotating clockwise the unit wheel four steps and the ten wheels only one step. The result is 42.*

Orlando: *I have written the number 28, then I have turned clockwise 14 times the wheel on the bottom right, the unit one. The number is 42.*

They highlight two different utilization schemes. The utilisation schemes are not only ways of using the artefact. They hint at different mathematical meanings. The teacher considers this occasion very good to discuss the different mathematical meanings and designs a new individual task for the whole classroom. She gives copies of both protocols to each student together with the following task.

Task 2: *Look carefully what Christian and Orlando have written to represent/show(?) on the Pascaline 28 + 14. Try and write the mathematical expressions that represents the two different solutions.*

The answers are very interesting. For instance a student uses only mathematical signs:

Christian: 28 + 14 = (20 + 10) + (4 + 8) = 30 + 12 = 42

Orlando: 28 + 14 = (20 + 8) + (1+1+1+1+1+1+1+1+1+1+1+1+1+1) = 20 + (8+1+1+1+1+1+1+1+1+1+1+1+1+1+1) = 20 + 22 = 42.

Other students use mathematical signs, together with verbal comments, or mathematical signs with graphical representations of the Pascaline, or all the three kinds of signs.

The different utilization schemes highlight different mathematical meanings: the decomposition of a number into units and tens, that depend on the chosen base in the place value convention (Christian) and the generation of a number according to iteration of the operator "+1" (Orlando), that allows to shift from one number to its successor (in Peano's axioms) and might work in a any base. These utilization schemes (together with others) appear everywhere, in experiments made in both

Europe and the US, with both young children and adults. Often the students observes that the order of adding units and tens is purely conventional as it would be possible also to add tens first and units later (as it happens with soroban and suànpán).

DIFFUSION TO SCHOOLS

In this presentation only concrete artefacts have been mentioned. There is now an increasing diffusion of digital artefacts which exploit the potential of technologies, including multi touch devices. It is important to look at them with no prejudice, although there are some features of the concrete artefacts that still deserve attention.

Most of the above artefacts have been studied in teaching experiments realized in many dozens of Italian schools. I mention two big projects that have been developed for pre-school (*Bambini che contano*, mentioned above) and for the early grades of primary school (*PerContare,* that is *In order to count*). The latter is an Italian inter-regional 3-year project (2011-2014) aimed at developing effective inclusive teaching strategies and materials to help primary school teachers (in grades 1, 2, and 3) address learning difficulties, especially of students who are potentially at risk of being diagnosed with developmental dyscalculia. Further information about this project are available at Baccaglini-Frank (in press, ICMI23) and at Baccaglini-Frank (in preparation). Both projects are freely available for all the Italian teachers through the portal of the National Standards[vi].

Moreover, we have opened a series of books for teachers at a very popular publisher. The name of the series is *From doing to knowing: Smart artefacts for constructing mathematical meanings* [vii], where books are sold together with a copy of the artefact.

CONCLUDING REMARKS

The delicate issue of *cultural transposition* comes to the foreground (Bartolini Bussi & Martignone, 2013, Mellone & Ramploud, in press). As discussed, there is a strong dependence of the artefacts on the cultural context: they are cultural artefacts which may reveal valuable information about the society that made or used them. The discussion about either the irrelevance of the number line in the Chinese tradition or the relevance of the Pascaline in the European tradition of the 17th century teaches us something. One might even doubt whether in mathematics education it is right and effective to transport cultural artefacts outside from their context.

Even in the use of the "same" artefact, it is important to pay attention to the ways of introducing it into the mathematics classroom. The rules for manipulating the suànpán or soroban are directly taught in Far East, whilst the rules for a Pascaline are first "invented" by the students and later shared with the whole classroom under the teacher's guidance in Europe. In our wording, in the West, different personal utilization schemes (together with the associated mathematical meanings) are likely to emerge, whilst in the Far East only the socially shared utilisation schemes are practiced. But this hints at a possible conflict since the process of collective construction of mathematical meaning in mathematical discussions is the core of the TMS. Does it make sense to speak about the TMS in China or in Japan? Does it make sense to exploit the findings about the processes of semiotic mediation in cultures so far from the one where the TMS has been developed?

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[i] The word artefact is generally used in a very general way and encompasses oral and written forms of language, texts, physical tools used in the history of tools from ICT, manipulatives, and so on. In this paper the term artefact is used after Rabardel (1995) and distinguished from instruments, as will be discussed below.

[ii] http://nlvm.usu.edu/en/nav/vlibrary.html visited on April, 8, 2015.

[iii] http://tangible.media.mit.edu visited on April, 8, 2015.

[iv] http://www.shuzan.jp/english/brain, visited on April, 8, 2015.

[v] http://www.uaq.mx/ingenieria/publicaciones/eureka/n12/abaco.html , visited on April, 8, 2015.

[vi] http://www.indicazioninazionali.it

[vii] http://www.erickson.it/Pagine/Risultati-collana.aspx?col=25521