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# A solitonic 3-brane in 6D bulk

## Olindo Corradini, Zurab Kakushadze

*C.N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794, USA*

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**Abstract**

We construct a solitonic 3-brane solution in the 6-dimensional Einstein–Hilbert–Gauss–Bonnet theory with a (negative) cosmological term. This solitonic brane world is $\delta$-function-like. Near the brane the metric is that for a product of the 4-dimensional flat Minkowski space with a 2-dimensional "wedge" with a deficit angle (which depends on the solitonic brane tension). Far from the brane the metric approaches that for a product of the 5-dimensional AdS space and a circle. This solitonic solution exists for a special value of the Gauss–Bonnet coupling (for which we also have a $\delta$-function-like codimension-1 solitonic solution), and the solitonic brane tension can take values in a continuous range. We discuss various properties of this solitonic brane world, including coupling between gravity and matter localized on the brane. © 2001 Published by Elsevier Science B.V.

## 1. Introduction

In the brane world scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space–time [1–16]. There is a big difference between the footings on which gauge plus matter fields and gravity come in this picture. [1] Thus, for instance, if gauge and matter fields are localized on D-branes [3], they propagate only in the directions along the D-brane world-volume. Gravity, however, is generically not confined to the branes — even if we have a graviton zero mode localized on the brane as in [14], where the volume of the extra dimension is finite, massive graviton modes are still free to propagate in the bulk. On the other hand, as was discussed in [16], in the cases with infinite volume extra dimensions, we can have almost completely localized gravity on higher codimension ($\delta$-function-like) branes with the $p^2 = 0$ modes penetrating into the bulk.

Recently in [17] it was pointed out that we can have *complete* localization of gravity on a $\delta$-function-like *solitonic* codimension-1 brane world solution. That is, there are no propagating degrees of freedom in the bulk, while on the brane we have 4-dimensional Einstein–Hilbert gravity (assuming that the solitonic brane is a 3-brane). In fact, in this solution, even though the classical solitonic background is 5-dimensional, the quantum theory perturbatively [2]

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*E-mail addresses:* olindo@insti.physics.sunysb.edu (O. Corradini), zurab@insti.physics.sunysb.edu (Z. Kakushadze).

[1] This, at least in some sense, might not be an unwelcome feature — see, e.g., [4,7,12].

[2] Non-perturbatively at the semi-classical level we can a priori have breakdown of causality via creation of "baby" branes.

is actually 4-dimensional — there are no loop corrections in the bulk as we have no propagating bulk degrees of freedom.

The setup of [17] is the 5-dimensional Einstein–Hilbert theory with a (negative) cosmological term augmented with a Gauss–Bonnet term. The solitonic brane world solution arises in this theory for a special value of the Gauss–Bonnet coupling. The fact that there are no propagating degrees of freedom in the bulk is then due to a perfect cancellation between the corresponding contributions coming from the Einstein–Hilbert and Gauss–Bonnet terms, which occurs precisely for this value of the Gauss–Bonnet coupling. Since the bulk theory does not receive loop corrections, the classical choice of parameters such as the special value of the Gauss–Bonnet coupling (or the Gauss–Bonnet combination itself) does *not* require perturbative order-by-order fine-tuning. Also, the entire setup can be supersymmetrized, and then the aforementioned solitonic solution becomes a BPS state, which preserves 1/2 of the original supersymmetries.

In this Letter we would like to address the question whether there are higher codimension solitonic brane world solutions in (the appropriate higher dimensional versions of) the setup of [17]. In fact, we do find codimension-2 *solitonic*[3] solutions, which are 3-branes if the bulk is 6-dimensional. Thus, we have a $\delta$-function-like codimension-2 solitonic solution. This solution, where the solitonic brane world-volume is flat, exists for a continuous range of values of the solitonic brane tension. However, as we explain in the following, this is not a "self-tuning" solution for two reasons. First, it turns out that to have a consistent tree-level coupling between gravity and brane matter the latter must be conformal. Second, the aforementioned special choice of the Gauss–Bonnet coupling (unlike in the codimension-1 solution of [17]) is sensitive to quantum corrections in the bulk. This is because in this codimension-2 solution we do have propagating degrees of freedom in the bulk.

The remainder of this Letter is organized as follows. In Section 2 we discuss our setup. In Section 3 we find the aforementioned solitonic codimension-2 brane world solutions and discuss their properties. In Section 4 we discuss the coupling between gravity and brane matter. Section 5 contains various remarks.

## 2. The setup

In this section we discuss the setup within which we will construct the aforementioned codimension-2 solitonic brane world solutions. The action for this model is given by (for calculational convenience we will keep the number of space–time dimensions $D$ unspecified, but we are mostly interested in the $D = 6$ case):

$$S = M_{\mathrm{P}}^{D-2} \int d^D x \sqrt{-G} \left\{ R + \lambda \left[ R^2 - 4R_{MN}^2 + R_{MNST}^2 \right] - \Lambda \right\}, \tag{1}$$

where $M_{\mathrm{P}}$ is the $D$-dimensional (reduced) Planck scale, and the Gauss–Bonnet coupling $\lambda$ has dimension (length)$^2$. Finally, the bulk vacuum energy density $\Lambda$ is a constant.

The equations of motion following form the action (1) read:

$$R_{MN} - \frac{1}{2} G_{MN} R - \frac{1}{2} \lambda G_{MN} \left( R^2 - 4R^{MN} R_{MN} + R^{MNRS} R_{MNRS} \right)$$

$$+ 2\lambda \left( R R_{MN} - 2R_{MS} R^S{}_N + R_{MRST} R_N{}^{RST} - 2R^{RS} R_{MRNS} \right) + \frac{1}{2} G_{MN} \Lambda = 0. \tag{2}$$

Note that this equation does not contain terms with third and fourth derivatives of the metric, which is a special property of the Gauss–Bonnet combination [20,21].

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[3] Codimension-2 solutions in the 6-dimensional Einstein–Hilbert gravity in the presence of source (that is, non-solitonic) branes were discussed in [18,19].

## 2.1. Codimension-1 solitonic brane-world

In [17] it was shown that, for a special combination of the Gauss–Bonnet coupling $\lambda$ and the vacuum energy density $\Lambda$, this theory possesses a codimension-1 *solitonic* brane-world solution. Since this solution will be relevant for our subsequent discussions, let us briefly review it here. Thus, let us focus on solutions to the above equations of motion with the warped [22] metric of the form

$$ds_D^2 = \exp(2A)\eta_{MN}\,dx^M\,dx^N, \tag{3}$$

where $\eta_{MN}$ is the flat $D$-dimensional Minkowski metric, and the warp factor $A$, which is a function of $z \equiv x^D$, is independent of the other $(D-1)$ coordinates $x^\mu$. With this ansatz, we have the following equations of motion for $A$ (prime denotes derivative w.r.t. $z$):

$$(D-1)(D-2)\left(A'\right)^2\left[1-(D-3)(D-4)\lambda\left(A'\right)^2\exp(-2A)\right]+\Lambda\exp(2A)=0, \tag{4}$$

$$(D-2)\left[A''-\left(A'\right)^2\right]\left[1-2(D-3)(D-4)\lambda\left(A'\right)^2\exp(-2A)\right]=0. \tag{5}$$

This system of equations has a set of solutions where the $D$-dimensional space is an AdS space for a continuous range of parameters $\Lambda$ and $\lambda$. The volume of the $z$ direction for this set of solutions is infinite.

There, however, also exists a solution where the volume of the $z$ direction is finite if we "fine-tune" the Gauss–Bonnet coupling $\lambda$ and the bulk vacuum energy density $\Lambda$ as follows: [4]

$$\Lambda = -\frac{(D-1)(D-2)}{(D-3)(D-4)}\frac{1}{4\lambda}, \tag{6}$$

where $\lambda > 0$, and $\Lambda < 0$. This solution is given by (we have chosen the integration constant such that $A(0)=0$):

$$A(z) = -\ln\left[\frac{|z|}{\Delta}+1\right], \tag{7}$$

where $\Delta$ is given by

$$\Delta^2 = 2(D-3)(D-4)\lambda. \tag{8}$$

Note that $\Delta$ can be positive or negative. In the former case the volume of the $z$ direction is finite: $v = 2\Delta/(D-1)$. On the other hand, in the latter case it is infinite.

Note that $A'$ is discontinuous at $z=0$, and $A''$ has a $\delta$-function-like behavior at $z=0$. Note, however, that (5) is still satisfied as in this solution

$$1-2(D-3)(D-4)\lambda\left(A'\right)^2\exp(-2A)=0. \tag{9}$$

Thus, this solution describes a codimension-1 *soliton*. The tension of this soliton, which is given by

$$f_{D-1} = \frac{4(D-2)}{\Delta}M_{\mathrm{P}}^{D-2}, \tag{10}$$

is positive for $\Delta > 0$, and it is negative for $\Delta < 0$. As was shown in [17], in the latter case the theory is non-unitary (which is attributed to the negativity of the brane tension). The solution with positive brane tension, on the other hand, is consistent. Here we are referring to the $z=0$ hypersurface as the brane.

It was further shown in [17] that the graviton propagator in the above solitonic solution vanishes in the bulk, while on the brane we have *completely* localized gravity. In particular, (at least perturbatively [5]) gravity on the brane is purely $(D-1)$-dimensional.

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[4] This special value of the Gauss–Bonnet coupling has appeared in a somewhat different context in [23]. In fact, it was argued in [23] that for other values of these parameters the Einstein–Hilbert–Gauss–Bonnet theory is non-unitary.

[5] As was pointed out in [17], a priori semi-classically there can be non-perturbative effects breaking causality via creation of "baby" branes, so that gravity could in this way propagate into the bulk.

## 3. Codimension-2 solitonic brane-world

In this section we would like to point out that in the above setup, precisely for the special combination of the parameters (6), there also exists a codimension-2 solitonic brane world solution. Thus, consider the following ansatz for the metric:

$$ds_D^2 = \exp(2A)\big[\eta_{\alpha\beta}\,dx^\alpha\,dx^\beta + (dr)^2 + \exp(2B)r^2\,(d\phi)^2\big], \tag{11}$$

where $\eta_{\alpha\beta}$ is the flat $(D-2)$-dimensional Minkowski metric corresponding to the first $(D-2)$ coordinates $x^\alpha$, and the other two coordinates are chosen in the polar basis $(r, \phi)$; the warp factors $A$ and $B$, which are functions of $r$, are assumed to be independent of $(x^\alpha, \phi)$ (that is, we are looking for axially symmetric solutions); the angular coordinate $\phi$ takes values between 0 and $2\pi$, while the radial coordinate $r$ takes values between 0 and $\infty$.

With the above ansatz we have the following equations of motion for $A$ and $B$ (dot denotes derivative w.r.t. $r$):

$$\left[\ddot{B} + \frac{2}{r}\dot{B} + \frac{1}{r^2}\dot{B}^2 + (D-2)(\ddot{A} - \dot{A}^2)\right]\big[1 - 2(D-3)(D-4)\lambda\dot{A}^2\exp(-2A)\big]$$
$$- 4(D-3)(D-4)\lambda\big[\ddot{A} - \dot{A}^2\big]\dot{A}\left[\dot{B} + \frac{1}{r}\right]\exp(-2A) = 0, \tag{12}$$

$$(D-1)(D-2)\dot{A}^2\big[1 - (D-3)(D-4)\lambda\dot{A}^2\exp(-2A)\big] + \Lambda\exp(2A)$$
$$+ 2(D-2)\dot{A}\left[\dot{B} + \frac{1}{r}\right]\big[1 - 2(D-3)(D-4)\lambda\dot{A}^2\exp(-2A)\big] = 0, \tag{13}$$

$$(D-1)(D-2)\dot{A}^2\big[1 - (D-3)(D-4)\lambda\dot{A}^2\exp(-2A)\big] + \Lambda\exp(2A)$$
$$+ 2(D-2)\big[\ddot{A} - \dot{A}^2\big]\big[1 - 2(D-3)(D-4)\lambda\dot{A}^2\exp(-2A)\big] = 0. \tag{14}$$

The first equation is a linear combination of the $(\alpha\beta)$ and $(rr)$ equations, the second equation is the $(rr)$ equation, and the third equation is the $(\phi\phi)$ equation. Only two of the above three equations are independent, which, as usual, is a consequence of Bianchi identities.

### 3.1. δ-function-like solitonic brane-world

Here we would like to discuss a solution of the above equations of motion corresponding to a $\delta$-function-like codimension-2 solitonic brane world. This solution is given by:

$$A(r) = -\ln\left(\frac{r}{\Delta} + 1\right), \qquad B(r) = -\beta, \tag{15}$$

where $\beta$ is a constant, which we will assume to be *positive*, while $\Delta$, which is also assumed to be positive, is related to $\lambda$ via (8). The metric is given by:

$$ds_D^2 = \left(\frac{r}{\Delta} + 1\right)^{-2}\big[\eta_{\alpha\beta}\,dx^\alpha\,dx^\beta + (dr)^2 + \exp(-2\beta)r^2\,(d\phi)^2\big]. \tag{16}$$

Note that $\Delta \to \infty$ in this solution corresponds to the flat bulk limit. Also note that the presence of the Gauss–Bonnet term (as well as the fact that the Gauss–Bonnet coupling takes a special value (6)) is crucial for the existence of the solution (15) — indeed, (13) could not be satisfied without the Gauss–Bonnet term.

Near the origin $r \to 0$ the above metric takes the following form:

$$ds_D^2 = \eta_{\alpha\beta}\,dx^\alpha\,dx^\beta + (dr)^2 + \exp(-2\beta)r^2\,(d\phi)^2. \tag{17}$$

This metric describes a product of the $(D-2)$-dimensional flat Minkowski space with a 2-dimensional "wedge" with the deficit angle

$$\theta = 2\pi\big[1 - \exp(-\beta)\big]. \tag{18}$$

This wedge is locally $\mathbf{R}^2$ except for the origin $r = 0$, where we have a $\delta$-function-like singularity. Thus, we have a $\delta$-function-like codimension-2 *solitonic* brane located at $r = 0$.

The tension of this solitonic brane can be determined as follows. Consider the following action

$$S_1 = M_{\mathrm{P}}^{D-2} \int d^D x \sqrt{-G}\, R - f_{D-2} \int_{\Sigma} d^{D-2}x \sqrt{-\widehat{G}}, \tag{19}$$

where $\Sigma$ is a $\delta$-function-like codimension-2 *source* brane, which is the hypersurface $x^i = 0$ ($x^i$, $i = 1, 2$, are the two coordinates transverse to the brane); the tension $f_{D-2}$ of this brane is assumed to be positive; finally,

$$\widehat{G}_{\alpha\beta} \equiv \delta_\alpha{}^M \delta_\beta{}^N G_{MN}\big|_\Sigma, \tag{20}$$

where $x^\alpha$ are the $(D-2)$ coordinates along the brane (the $D$-dimensional coordinates are given by $x^M = (x^\alpha, x^i)$).

The equations of motion following from the action (19) are given by:

$$R_{MN} - \frac{1}{2} G_{MN} R + \frac{1}{2} \frac{\sqrt{-\widehat{G}}}{\sqrt{-G}} \delta_M{}^\alpha \delta_N{}^\beta \widehat{G}_{\alpha\beta}\, \widetilde{f}_{D-2}\delta^{(2)}(x^i) = 0, \tag{21}$$

where $\widetilde{f}_{D-2} \equiv f_{D-2}/M_{\mathrm{P}}^{D-2}$.

Next, consider the following ansatz for the metric:

$$ds_D^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu + \exp(2\omega)\delta_{ij}\, dx^i\, dx^j, \tag{22}$$

where $\omega$ is a function of $x^i$ but is independent of $x^\alpha$. With this ansatz we have

$$R_{\alpha\beta} = 0, \qquad R_{ij} = \widetilde{R}_{ij} = \frac{1}{2}\widetilde{G}_{ij}\widetilde{R}, \tag{23}$$

$$\sqrt{\widetilde{G}}\,\widetilde{R} = \widetilde{f}_{D-2}\delta^{(2)}(x^i), \tag{24}$$

where $\widetilde{R}$ and $\widetilde{R}_{ij}$ are the 2-dimensional Ricci scalar, respectively, Ricci tensor constructed from the 2-dimensional metric

$$\widetilde{G}_{ij} \equiv \exp(2\omega)\delta_{ij}. \tag{25}$$

Since this metric is conformally flat, we have $\sqrt{\widetilde{G}}\,\widetilde{R} = -2\partial^i\partial_i\omega$ (where the indices are lowered and raised using $\delta_{ij}$ and $\delta^{ij}$, respectively), so that

$$\partial^i\partial_i\omega = -\frac{1}{2}\widetilde{f}_{D-2}\delta^{(2)}(x^i). \tag{26}$$

The solution to this equation is given by

$$\omega = -\frac{1}{8\pi}\widetilde{f}_{D-2}\ln\!\left(\frac{x^2}{a^2}\right), \tag{27}$$

where $x^2 \equiv \delta_{ij}x^i x^j$, and $a$ is an integration constant.

Let us go to the polar coordinates $(\rho, \phi)$: $x^1 = \rho\cos(\phi)$, $x^2 = \rho\sin(\phi)$ ($\rho$ takes values from 0 to $\infty$, while $\phi$ takes values from 0 to $2\pi$). In these coordinates the two-dimensional metric is given by

$$d\widetilde{s}_2^2 = \left(\frac{a^2}{\rho^2}\right)^\nu \big[(d\rho)^2 + \rho^2\,(d\phi)^2\big], \tag{28}$$

where

$$\nu \equiv \frac{1}{4\pi} \tilde{f}_{D-2}. \tag{29}$$

Let us change the coordinates to $(r, \phi)$, where

$$r \equiv \frac{1}{1-\nu} a^\nu \rho^{1-\nu}, \tag{30}$$

where we are assuming that $\nu < 1$. Then we have

$$d\tilde{s}_2^2 = (dr)^2 + \exp(-2\beta) r^2 (d\phi)^2, \tag{31}$$

where

$$\exp(-\beta) \equiv 1 - \nu. \tag{32}$$

Thus, we see that the brane tension $f_{D-2}$ is related to the deficit angle $\theta$ (given by (18)) via

$$f_{D-2} = 2M_{\mathrm{P}}^{D-2} \theta. \tag{33}$$

In particular, this expression gives the tension of the $\delta$-function-like codimension-2 solitonic brane located at $r = 0$ in the solution described by the metric (16).

Before we end this section let us note that for large $r$ ($r \gg \Delta$) the metric (16) approaches that of $AdS_{D-1} \times S^1$

$$ds_D^2 = \frac{\Delta^2}{r^2} \left[ \eta_{\alpha\beta} \, dx^\alpha \, dx^\beta + (dr)^2 \right] + r_*^2 \, (d\phi)^2, \tag{34}$$

where the radius of $S^1$ is given by $r_* \equiv \Delta \exp(-\beta)$.

## 4. Coupling to brane matter

In this section we would like to discuss how gravity couples to matter localized on the above $\delta$-function-like codimension-2 solitonic brane. Since this brane is solitonic, it breaks $D$-dimensional diffeomorphisms only spontaneously. This has certain implications to which we now turn.

Thus, let us consider small fluctuations around the solution (16):

$$G_{MN} = \exp(2A) \left[ \overline{G}_{MN} + \overline{h}_{MN} \right], \tag{35}$$

where $\overline{G}_{MN}$ is the background metric up to the warp factor $\exp(2A)$ (that is, $\overline{G}_{\alpha\beta} = \eta_{\alpha\beta}$, $\overline{G}_{rr} = 1$, $\overline{G}_{\phi\phi} = \exp(-2\beta) r^2$, $\overline{G}_{\alpha r} = \overline{G}_{\alpha\phi} = \overline{G}_{r\phi} = 0$), and for convenience reasons instead of the metric fluctuations $h_{MN} \equiv \exp(2A) \overline{h}_{MN}$ we choose to work with $\overline{h}_{MN}$.

In terms of $\overline{h}_{MN}$ the $D$-dimensional diffeomorphisms (corresponding to $x^M \to x^M - \xi^M$) read:

$$\delta \overline{h}_{\alpha\beta} = \partial_\alpha \overline{\xi}_\beta + \partial_\beta \overline{\xi}_\alpha + 2\eta_{\alpha\beta} \dot{A} \overline{\xi}_r, \tag{36}$$

$$\delta \overline{h}_{\alpha r} = \partial_\alpha \overline{\xi}_r + \dot{\overline{\xi}}_\alpha, \tag{37}$$

$$\delta \overline{h}_{\alpha\phi} = \partial_\alpha \overline{\xi}_\phi + \partial_\phi \overline{\xi}_\alpha, \tag{38}$$

$$\delta \overline{h}_{rr} = 2\dot{\overline{\xi}}_r + 2\dot{A} \overline{\xi}_r, \tag{39}$$

$$\delta \overline{h}_{r\phi} = \dot{\overline{\xi}}_\phi - \frac{2}{r} \overline{\xi}_\phi + \partial_\phi \overline{\xi}_r, \tag{40}$$

$$\delta \overline{h}_{\phi\phi} = 2\partial_\phi \overline{\xi}_\phi + 2\exp(-2\beta) r^2 \left[ \dot{A} + \frac{1}{r} \right] \overline{\xi}_r, \tag{41}$$

where $\overline{\xi}_M \equiv \exp(-2A)\xi_M$. Note that using these diffeomorphisms we can set two of the graviscalar components ($\overline{h}_{rr}$ and $\overline{h}_{r\phi}$) as well as one of the graviphotons ($\overline{h}_{\alpha r}$) to zero. We are then left with the $(D-2)$-dimensional graviton ($\overline{h}_{\alpha\beta}$), a graviphoton ($\overline{h}_{\alpha\phi}$), and a graviscalar ($\overline{h}_{\phi\phi}$).

Next, let us assume that we have matter localized on the $\delta$-function-like codimension-2 solitonic brane. Let $T_{\alpha\beta}$ be the corresponding conserved energy momentum tensor:

$$\partial^\alpha T_{\alpha\beta} = 0. \tag{42}$$

The coupling of gravity to the brane matter is described by the following term in the action:

$$\frac{1}{2}\int_\Sigma d^{D-2}x\, T^{\alpha\beta}\overline{h}_{\alpha\beta}, \tag{43}$$

where $\Sigma$ is the $r=0$ hypersurface corresponding to the brane (note that on $\Sigma h_{\alpha\beta}$ and $\overline{h}_{\alpha\beta}$ coincide as $A(r=0)=0$). Since $\dot{A}(r=0)=-1/\Delta \neq 0$, this coupling is invariant under the aforementioned diffeomorphisms if and only if

$$T \equiv T^\alpha_\alpha = 0, \tag{44}$$

that is, if and only if the brane matter is *conformal*.

So, at the tree level, to have a consistent coupling between gravity and the brane matter we must assume that the latter is conformal.[6] Note, however, that the conformal property cannot generically persist beyond the tree-level. Indeed, the volume of the extra two dimensions in the above solution is finite:

$$\tilde{v}_2 = \int_0^\infty dr \int_0^{2\pi} d\phi\, \sqrt{-\overline{G}}\exp(DA) = 2\pi\exp(-\beta)\frac{\Delta^2}{(D-1)(D-2)}. \tag{45}$$

This implies that the $(D-2)$-dimensional Planck scale on the brane is finite:

$$\widehat{M}^{D-4}_P = M^{D-2}_P \int_0^\infty dr \int_0^{2\pi} d\phi\, \sqrt{-\overline{G}}\exp\left[(D-2)A\right] = 4\pi\lambda\exp(-\beta)M^{D-2}_P. \tag{46}$$

That is, we have a quadratically normalizable $(D-2)$-dimensional graviton zero mode. The conformal invariance of the matter localized on the brane is then generically expected to be broken by loop corrections involving gravity.

Thus, in the above brane world solution we have a quantum inconsistency discussed in [25] in other setups. Note that such an inconsistency does not arise in the setup of [17]. The key reason is that in the codimension-1 solution of [17] (which we reviewed in Section 2) there are no propagating degrees of freedom in the bulk. In the above codimension-2 solution, however, we do have such degrees of freedom, in particular, the aforementioned graviscalar degree of freedom $\overline{h}_{\phi\phi}$ propagates in the bulk. A simple way to see this is to recall that, as we pointed out at the end of the previous section, at large $r$ the metric in this solution approaches that of $AdS_{D-1} \times S^1$, so we do have a propagating degree of freedom corresponding to the reduction on $S^1$. This then implies that (unlike in the codimension-1 solution) here we do have loop corrections in the bulk, and the corresponding higher curvature terms will generically delocalize gravity [25].

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[6] This is similar to what happens in the setup of [24].

## 5. Comments

We would like to end our discussion with a few remarks. First, note that the above codimension-2 solution, where the brane world-volume is flat, exists for a continuous range of values of the solitonic brane tension.[7] However, this is not a "self-tuning" solution for two reasons. First, to have a consistent tree-level coupling between gravity and brane matter the latter must be conformal. Second, the aforementioned special choice of the Gauss–Bonnet coupling (unlike in the codimension-1 solution of [17]) is sensitive to quantum corrections in the bulk.

Note that the issues (which are expected to arise at the quantum level as we discussed at the end of the previous section) with the coupling between the brane matter and gravity as well as with delocalization of gravity by higher curvature terms in the bulk need not arise in scenarios with infinite volume extra dimensions [26–30], at least in higher codimension cases. Here the four-dimensional gravity on the brane is obtained via the Einstein–Hilbert term on the brane, which is generically expected to be generated by quantum effects on the brane as long as the brane world-volume theory is not conformal [15,16]. As was pointed out in [17], this is expected to be the case in the string theory context as well. Thus, in the orbifold examples of [31–33][8] we always have non-conformal $U(1)$ factors. Also, in other examples such as conifolds [37,38] already the non-Abelian gauge subgroups are non-conformal in the ultra-violet (albeit they are conformal in the infra-red). As was argued in [17], in the aforementioned examples (which are conformal in the infra-red, but are non-conformal in the ultra-violet) in the context of AdS/CFT correspondence [39–41] on the type IIB side various higher curvature terms intrinsically due to the compactification[9] become important. Finally, recently non-conformal theories (that is, theories that are not conformal even in the infra-red) were discussed in [45] within a modification of the setup of [33]. Some of these theories can be discussed in the context of a certain brane–bulk duality [45], which might provide a framework for computing gravitational corrections (such as the Einstein–Hilbert term) on D-branes.

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[7] More precisely, it exists for $0 < f_{D-2} < f_{crit}$; at the critical brane tension $f_{crit} = 4\pi M_P^{D-2}$ the deficit angle $\theta = 2\pi$.

[8] The following is also correct for the orientifold examples of [34]. In considering such examples with, say, $\mathcal{N} = 1$ supersymmetry, however, some caution is needed due to the subtleties discussed in [35,36].

[9] These should not be confused with the higher dimensional terms already present in ten dimensions, which do not affect conformality [42–44].

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