Dynamics of electron-electron entanglement in pulsed sinusoidal potentials

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Abstract. The surface acoustic wave (SAW) assisted charge transport represents a highly controllable mean to inject and drive electrons along quasi one-dimensional channels. By means of numerical simulations, we analyze the effect of the removal of SAW for a short time interval onto the entanglement stemming from the Coulomb scattering between two charged carriers bound by the SAW potential itself. We find that the particles get quickly entangled, thus confirming that the SAW driven transport in low-dimensional semiconductor structures permits to suppress the effects of electron-electron correlation due to the Coulomb interaction.

1. Introduction

The investigation of the entanglement formation in scattering events in semiconductor structures is not only useful for understanding the nature of the scattering process itself but it is also recognized as an important resource for the design of quantum information processing devices [1, 2, 3, 4, 5]. In the last years many theoretical efforts have been carried out [3, 4, 6, 7] and different proposals to produce entangled states between charged carriers have been presented [1, 2, 5].

The aingle-particle transport through surface acoustic waves (SAW), originally introduced in the context of metrological application for defining a new standard of electric current [8, 9, 10], constitutes also a viable means to implement quantum gate networks in semiconductor nanodevices [1, 11, 12, 13]. A number of devices exploiting SAW-electron interaction have been designed, realized and also proposed as basic building blocks for quantum computing applications, where the use of SAW has been shown to constitute a highly controllable mean to inject and drive electrons along quantum wires[12].

In this work, we analyze the time evolution of the entanglement during the scattering of two electrons interacting with an external periodic potential mimcking SAW and removed for a finite time interval. In particular we intend to examine the effect of the removal of such potential on the spreading of the wavefunction initially confined in a SAW minimum. Furthermore, we investigate how the turning off and on of the sinusoidal potential affects the building up of quantum correlations between the particles. This could provide us a clearer vision on the validity of the single-particle approximation, which leads to neglect the quantum correlations between the carriers in the study of SAW-assisted for the case of the SAW transport in low-dimensional semiconductor structures [14].

2. Physical model and results

In this section we investigate the entanglement between two electrons initially trapped into two next neighbor minima of a sinusoidal potential, when the latter is removed for a short time interval. In our approach the two particles are confined in a 1D-channel and the coupling originates from their mutual Coulomb interaction. The Hamiltonian of the system has the form

$$H(x_a, x_b) = H_{off-on}(x_a) + H_{off-on}(x_b) + \frac{e^2}{\epsilon \sqrt{(x_a - x_b)^2 + d^2}}, \tag{1}$$

where ϵ is the silicon dieletric constant and d represent the thickness of the wire, here fixed at 1 nm. $H_{off-on}(x_a)$ is the single-particle Hamiltonian given by

$$H_{off-on}(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \alpha \left[\theta(-t) + \theta(t - \Delta T)\right] \left(\sin(\frac{2\pi}{\lambda}x) + 1 \right), \tag{2}$$

with α representing the amplitude of the oscillation and λ the wavelength of the potential. Here we assume that, at the initial time $t = 0$, the external potential is abruptly switched off and then turned on again after a time intervalΔ T. As stated in the introduction, the investigation of the above model can be of interest in the contest of electron transport in quantum wires assisted by SAW. In fact, we can assume that the Hamiltonian given in Eq. (2) describes the effect of the removal of SAW potential for a timeΔ T (as obtained by a brief switching off and on of the SAW transducers or the transit of the electron in a SAW-free region) onto the dynamics of a couple of electrons.

In our numerical calculation we take the SAW sinusoidal potential with amplitude $\alpha = 50$ meV and wavelength $\lambda = 110$ nm. In fact, we found that for longer wavelengths the switching off and on of the external potential does not lead to significant effects on the entanglement since the two particles remain at a distance at which the Coulomb interaction is rather weak. We consider two electrons with the same spin (up). The quantum state describing the system at the initial time is

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\Big(|\chi\,\varphi\rangle - |\varphi\,\chi\rangle\Big)|\uparrow\uparrow\rangle. \tag{3}$$

The wavefunctions corresponding to the states $|\chi\rangle$ and $|\varphi\rangle$ are two eingestates of the SAW potential describing electrons trapped into two next minima. In Eq. (3) the ket $|\uparrow\rangle$ indicates spin up state.

To obtain the system evolution we solve the time-dependent Schröndiger equation for the two-particle wavefunction of Eq. (3). Once the real-space wavefunction is found at a given time step, we compute the two-particle density matrix $\rho = |\Psi\rangle\langle\Psi|$ and from the latter, we calculate the one-particle reduced density matrix ρ_r by tracing over the degrees of freedom of one of the two electrons. ρ_r is then used to evaluate the entanglement. In fact, it is well known that for a two-fermion system a good correlation measure is given by the von Neumann entropy of ρ_r [16, 15]:

$$\varepsilon = -\text{Tr}[\rho_r \ln \rho_r] = \sum_{i=1} |z_i|^2 \ln |z_i|^2, \tag{4}$$

where $|z_i|^2$ are the eigenvalues of the matrix ρ_r.

In Fig. 1 we report the time evolution of the entanglement for variousΔ T, ranging from 0.3 to 0.6 ps. We observe that ε increases with time and exhibits a sort of double step structure. Finally, it reaches a stationary value that strictly depends uponΔ T. In other words, the electrons get more correlated for largerΔ T. To get a better insight into the process, we need to take into account the dynamics of the real-space two-particle wavefunction. Its initial peaks, describing the localized states of the two electrons, spread as the external potential is removed.
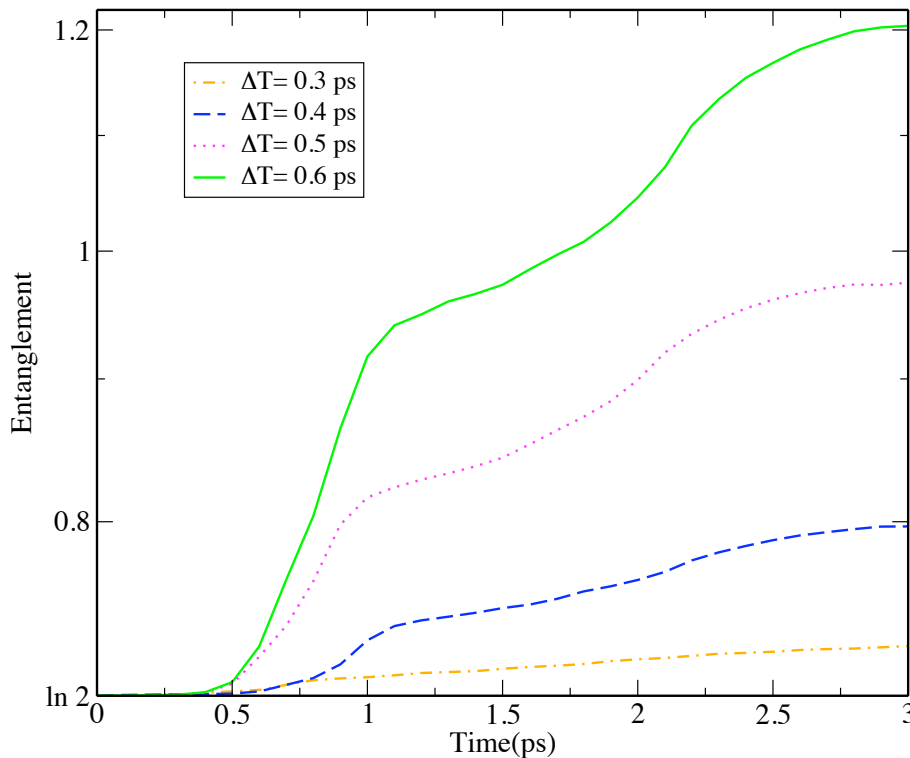
Figure 1. Entanglement as a function of time for four different values of the time interval ΔT without the sinusoidal potential. The amplitude of the oscillation is 50 meV and the wavelength 110 nm in order to mimic a SAW potential.

Therefore, for sufficiently long ΔT each peak can overlap with the other (see Fig. 2). This gives rise to a strong building up of quantum correlations, due to Coulomb interaction. Such an effect is obviously stronger when the sine-like potential is turned off for a longer time. After the potential is turned on again, i.e. for $t > \Delta T$, the wave packet is not smooth anymore, but it shows rapid oscillations. This affects the spatial overlap described above, which is responsible of the entanglement formation, thus leading to the peculiar double step structure observed in Fig. 1. In the limit of long ΔT, the two peaks of two-particle spatial wavefunction spread and quantum correlation between the electrons increases with time until maximum entanglement condition is reached.

3. Conclusions

In this paper we addressed the effect of the SAW removal onto the entanglement created by the Coulomb interaction between two electrons localized into two next neighbor minima of a standing SAW potential of wavelength comparable or smaller than the ones of Refs. [8, 9, 10]. We found that the entanglement increases with time and reaches a stationary value that is "freezed" when the potential is restored. In particular, the particles get more correlated for longer switching-off time intervals, due to the larger overlap between the spreading real-space wave functions. The appearance of a significative entanglement implies that this system cannot be described in terms of single-particle states. On the contrary, it represents a suitable candidate for the controlled generation of bipartite entanglement.

Finally, the results presented in this paper can be also considered as an additional theoretical confirmation that the use of SAW to drive electrons along quantum wires prevents the spreading of the spatial wavefunction and allows to suppress the effects of electron-electron correlation due to the Coulomb interaction. On the other hand, if the travelling periodic potential is instantaneously turned off and then on, the electrons get quickly entangled.
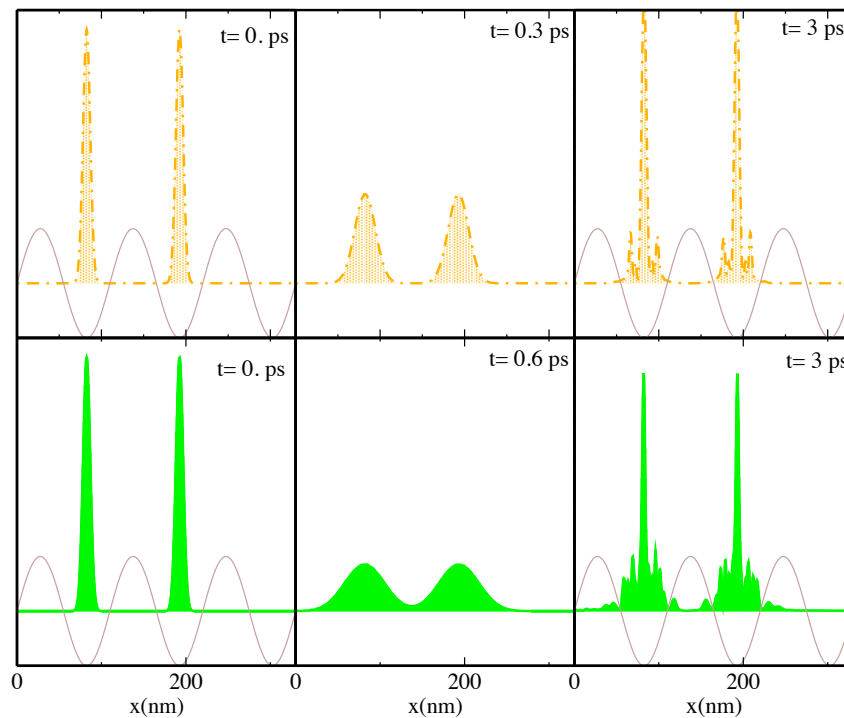
Figure 2. Single-particle probability density $\int |\langle x_a\, x_b|\chi\varphi\rangle|^2 dx_a$ evaluated at different time steps for two values of $\Delta\ T$: 0.3 ps (dash-dotted line in the upper row of graphs) and 0.6 ps (solid-shaded curve in the lower row graphs). The thin solid line sketches the SAW sinusoidal potential, when present.

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