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ADDENDUM to

"Average Internal Rate of Return and investment decisions: A new perspective"

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1. Return function. In Magni (2010) I present the AIRR approach to investment decisions. This approach shows a radical truth: a project is not uniquely associated with a return *rate*, but with a return *function*, which maps aggregate capitals to rates of return; it is an aggregate capital $PV(\boldsymbol{c}|r)$ that is associated with a unique return rate (the AIRR). (See Figure 1 in the paper).

2. Definition of rate of return. The AIRR approach enables any real number to be a rate of return, associated with a specific aggregate capital. In other words, any given number is a value taken on by the return function. This implies a radical epistemological consequence: as long as real *numbers* are used for defining a rate of return,

<u>any definition of rate of return appearing in the (past and) future literature is a particular case of AIRR</u>.

3. The incremental method. If project ranking is at issue, the investor is required to choose the stream of capitals so that all the projects to be ranked have the same present values (see Theorem 5). Implicitly, this theorem copes with choices between mutually exclusive projects as well, for this situation boils down to ranking two competing projects. However, in these situations an even simpler method may be

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used, which consists of applying the acceptability criterion (Theorem 2) to the incremental project obtained as the difference of the cash flow streams of the two projects. Consider project A and B: A is preferred to B if the incremental project $A-B$ is profitable; hence, applying Theorem 2, A is preferred to B if the AIRR of the incremental project $A-B$ exceeds the cost of capital.[1] For example, if $A=(-100,10,10,110)$, $B=(-90,\ 69,10,12,20)$, project A is preferred to B if and only if the incremental project $A-B=(-10,-59,0{,}98,-20)$ is profitable, that is, if and only if the AIRR exceeds the cost of capital. Assuming a cost of capital (COC) equal to 5%, and choosing, for example, $(10,0{,}0{,}0)$ as the sequence of capitals, one finds that the AIRR of the incremental project is 26.12%, which exceeds the COC; hence, project A is preferred to project B. The pairwise application of the incremental method enables the investor to rank a bundle of $n>2$ projects as well, but, admittedly, this method may be tedious if n is high: in this case Theorem 5 is more fruitfully employed. (It is also worth noting that, in general, the incremental method may not be used in association with the IRR, because the existence of a real-valued IRR of the incremental project is not guaranteed).

4. Rate of return on the initial investment. The IRR is not the rate of return on the initial investment; rather, the IRR represents an overall rate of return on the aggregate capital determined by the Hotelling class, which is univocally implied by the IRR itself (see Theorem 3). Therefore, important information is missed with the IRR. By contrast, with the AIRR approach, this piece of information is obtained, and in a very simple way: the analytical expression of the AIRR function at p. 160 is a useful shortcut, with the initial investment at the denominator:

$$\text{AIRR} = \text{COC} + \frac{\text{NPV} * \ (1+\text{COC})}{\text{initial investment}}.$$

For example, consider the cash flow stream $(-30,20,14,7)$: the investor invests 30 dollars. What is then the rate of return of those 30 dollars invested? Assuming

[1] If the present value of the capital stream selected is negative then, replace “exceeds” with “is smaller than”.

$\text{COC} = 3\%$, one finds $\text{NPV} = 9.02$, whence $\text{AIRR} = 0.03 + \frac{9.02(1.03)}{30} = 33.97\%$. This means that the investor earns 33.97% for each dollar of capital invested at time 0.

References

Magni, C.A. (2010). Average Internal Rate of Return and investment decisions: A new perspective. *The Engineering Economist*, 55(2), 150–180.