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EFFECTIVE THERMAL PROPERTIES OF FIBRE REINFORCED MATERIALS

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Abstract. *The thermal behaviour of a composite material formed by a conductive isotropic material containing groups of two parallel and insulated cylindrical fibres of different radii is investigated in the present work. In particular, the study aims at assessing the second-order resistivity contribution tensor which accounts for the effective thermal properties of the composite. For sake of simplicity, a 2D steady thermal heat flux is assumed in the matrix material and Neumann boundary conditions are imposed at the contours of the fibres. Owing to the geometry of the problem, reference is made to bipolar cylindrical coordinates. In order to evaluate the effective thermal resistivity, the temperature fields due to heat flows along two orthogonal directions is first determined as the sum of the basic temperature fields in a body without inclusions and a corrective term added to the basic solution in order to accomplish the boundary conditions on the contour of the inclusions. Then, the components of the resistivity tensors can be assessed analytically by evaluating proper contour integrals involving the temperature field along the contour of the fibres. The analysis allows obtaining the effective thermal properties of fibre reinforced materials varying both the geometry and dosage of fibres.*

1 INTRODUCTION

The problem of solid bodies with inclusions, cracks or cavities subjected to thermal loads has been widely investigated in Literature owing to its bearing in various practical applications. It is known that the mismatch between the thermal properties of the inclusions and those of the matrix produces stress and strain intensifications which can lead to premature failure of composite systems [1]. This fact has been shown, as an example, in [2], where the thermo-electro-elastic problem of an infinite matrix with an inclusion of various shapes has been addressed. In that study, based on a suitable numerical procedure, the interaction between a randomly oriented crack and a square hole embedded in an elastic matrix has been considered also and the obtained results have been compared with the solution provided by a FE code, founding a reasonable agreement. A thermoelastic analysis is mandatory in order to assess the mechanical performances of modern gas turbine blades which incorporate an array of circular holes that serve as cooling passages [3]. A recent study about the thermal stress in an elastic medium with two circular holes can be found in [4]. In that work, the path-independence of some quantities related to the stress concentration factors has been proven.

Thermal analyses play an important role also in the framework of biomechanics. As an example, temperature distribution in a biological tissue surrounding a single or multiple blood vessels can be formulated in terms of a 2D Poisson equation by considering a uniform heat source [5]. An application to a poroelastic material with two circular inclusions under various boundary conditions has been addressed in [6].

The presence of inhomogeneities alters the effective physical properties of the system, and a lot of studies deal with this topic. In particular, a procedure accounting for a 4^{th}-rank compliance contribution tensor due to presence of inhomgeneities in an elastic solid has been reported in [7]. This tensor allows evaluating the corrective term due to inclusions that must be added to the strain field and, in turn, to assess the homogenized mechanical properties of the composite system. Later, the same approach has been extended to find the overall thermal and mechanical properties of various composites including both interacting and non-interacting inhomogeneities of various shapes and orientations. In [8], such an approach has been validated by comparing the theoretical predictions with FE calculations. In a recent work, the resistivity contribution tensor for an elastic medium containing a single toroidal pore and multiple toroidal noninteracting inclusions has been evaluated varying the geometrical parameters of the inclusion [9].

The present work deals with the thermal properties of a 2D composite system which encompasses a homogeneous isotropic matrix and a pair of perfectly insulated circular pores. Such a system can reproduce the behavior of a conductive material reinforced with synthetic fibres of different size under the action of a steady state heat flow. Due to the geometrical setting, reference is made to bipolar cylindrical coordinates. Neumann BCs are considered at the contour of the fibres, which are taken as insulated inhomogeneities. The approach consists in assessing firstly the fundamental temperature field related to a remotely applied uniform heat flux in a homogeneous body. Then, in order to fulfill the BCs, an extra-term is added to the previous field, thus finding the complete solution of the problem. Once the temperature field is known, a homogeneization procedure is performed in order to find the equivalent thermal properties of the matrix reinforced with fibres, following the procedure reported in [9]. In particular, the resistivity contribution tensor is assessed analytically by calculating proper contour integrals.

The analysis can be generalized by considering conductive inhomogeneities with different thermal properties with respect to the matrix material.

2 GOVERNING EQUATIONS

Let us consider an infinite 2D body with two distinct circular inclusion of radii r_1 and r_2. Due to the geometrical layout, reference is made to bipolar cylindrical coordinates (α, β) connected to the Cartesian coordinates (x_1, x_2) through the following relations [10]

$$x_1 = a\frac{\sinh\alpha}{\cosh\alpha - \cos\beta}, \qquad x_2 = a\frac{\sin\beta}{\cosh\alpha - \cos\beta}, \tag{1}$$

being ($\pm a$, 0) the focal points of the bipolar coordinate system. A sketch representing the bipolar coordinate system is reported in Figure 1, in which the blue circles are the level curves for coordinate α, whereas the red circles are the level curves for coordinate β. Since β coordinate is multi-valued, we assume $-\pi < \beta \leq \pi$, thus avoiding indefiniteness.

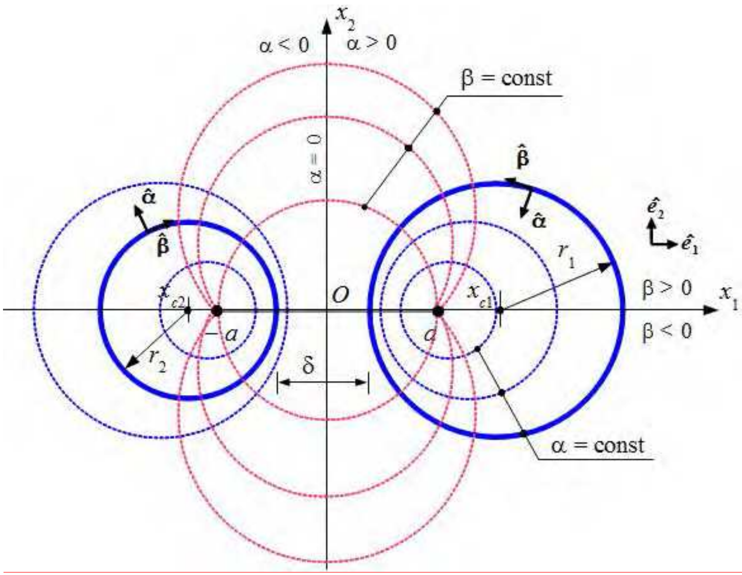

Figure 1. Sketch of the bipolar coordinate system.

In Figure 1, r_1 denotes the radius of the circular inclusion on the right ($\alpha_1 > 0$), whereas r_2 deals with the inclusion on the left ($\alpha_2 < 0$). Note that the system is completely definite through three geometric parameters, e.g. r_1, r_2 and the ligament δ between the holes, from which it follows $x_{c1} = r_1 + \delta[1-(2r_1+\delta)/(2r_1+2r_2+2\delta)]$, $\alpha_1 = \text{arccosh}\, x_{c1}/r_1$, $a = r_1 \sinh\alpha_1$, $\alpha_2 = -\text{arcsinh}\, a/r_2$ and $x_{c2} = -r_2 \cosh\alpha_2$.

2.1 Temperature distribution for heat flow in the x_1 direction

The body is subjected to a remote steady-state uniform heat flow $\boldsymbol{q}$ in the x_1-axis direction (see Figure 2*a*), i.e. $\boldsymbol{q}(x_1, x_2) = q^{\infty}(1, 0)$ or, in bipolar coordinates,

$$\boldsymbol{q}(\alpha,\beta) = q^{\infty}\left(\frac{1-\cosh\alpha\cos\beta}{\cosh\alpha-\cos\beta},\; -\frac{\sinh\alpha\sin\beta}{\cosh\alpha-\cos\beta}\right). \tag{2}$$

The temperature field T is related to the heat flux $\boldsymbol{q}$ propagating within the body through the Fourier law

$$\boldsymbol{q} = -k\ \nabla T, \tag{3}$$

being k the thermal conductivity of the body and ∇ denotes the gradient operator. Furthermore, according to the steady-state condition, the temperature field T must obey to the Laplace equation

$$\nabla^2 T = 0. \tag{4}$$

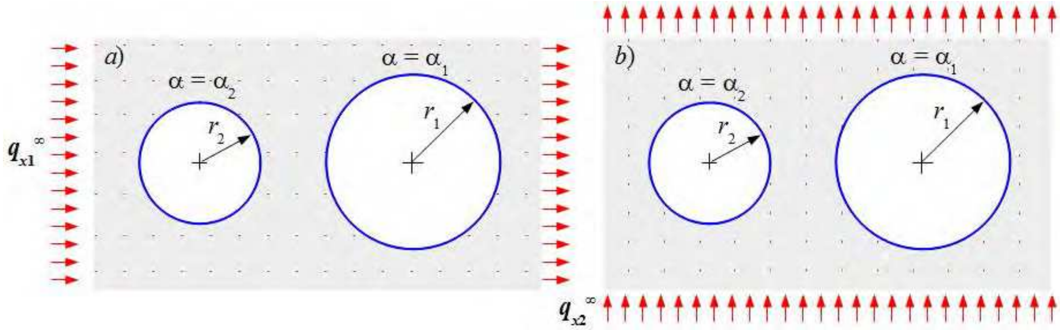

Figure 2. A 2D medium with 2 circular holes ($\alpha_1 > 0$, $\alpha_2 < 0$) subjected to a remote heat flux acting *a*) in the x_1 direction and *b*) in the x_2 direction.

Then, by using (1), the following temperature field T^∞ satisfying conditions (3)-(4) in a body without holes is retrieved

$$T^\infty(\alpha,\ \beta) = -a\frac{q^\infty}{k}\frac{\sinh\alpha}{\cosh\alpha - \cos\beta}. \tag{5}$$

The circular inclusions are perfectly insulated. This fact requires a vanishing heat flow across the boundary of the inclusions according to the following Neumann BCs:

$$q_\alpha = 0, \quad \text{for } \alpha = \alpha_1; \qquad q_\alpha = 0, \quad \text{for } \alpha = \alpha_2, \tag{6}$$

where q_α denotes the component of the heat flow along the unit vector $\boldsymbol{\alpha}$. However, the temperature field (5) does not accomplish BCs (6). Therefore, in order to satisfy BCs (6), an auxiliary steady temperature field $T^1(\alpha, \beta)$ fulfilling (4) is introduced as follows:

$$T^1(\alpha,\beta) = A + B\,\alpha + \sum_{n=1}^{\infty}\phi_n(\alpha)\cos(n\beta), \tag{7}$$

where $\phi_n(\alpha) = C_n e^{n\alpha} + D_n e^{-n\alpha}$, that produces a corrective heat flow $\boldsymbol{q}^1 = (q^1{}_\alpha,\ q^1{}_\beta)$. The component $q^1{}_\alpha$ reads, according to (3):

$$\begin{aligned} q^1{}_\alpha = {} & -k\frac{B}{a}\cosh\alpha + k\frac{B}{a}\cos\beta + \frac{k}{2a}(C_1 e^{\alpha} - D_1 e^{-\alpha}) + \frac{k}{2a}\sum_{n=1}^{\infty}[(n+1)(C_{n+1}e^{(n+1)\alpha} + \\ & -D_{n+1}e^{-(n+1)\alpha}) - 2n\cosh\alpha(C_n e^{n\alpha} - D_n e^{-n\alpha}) + (n-1)(C_{n-1}e^{(n-1)\alpha} - D_{n-1}e^{-(n-1)\alpha})]\cos n\beta. \end{aligned} \tag{8}$$

For convenience, the α-components of both the heat flows $\boldsymbol{q}^1$ and $\boldsymbol{q}^\infty$ are expanded in Fourier series. Then, by imposing $q^\infty{}_\alpha + q^1{}_\alpha = 0$ for $\alpha = \alpha_1$, α_2 (with $\alpha_1 > 0$, $\alpha_2 < 0$), from the leading-order terms one finds two independent conditions for C_1 and D_1, namely

$$q^\infty e^{-|\alpha_i|} - \frac{k}{a}B\cosh\alpha_i + \frac{k}{2a}(C_1 e^{|\alpha_i|} - D_1 e^{-|\alpha_i|}) = 0, \qquad \text{with } i = 1, 2. \tag{9}$$

Once that constants C_1 and D_1 are determined from eqns (9), the constants C_n and D_n (with $n = 2, 3, \ldots$) can be evaluated by imposing $q^\infty{}_\alpha + q^1{}_\alpha = 0$ at order $n-1$ for $\alpha = \alpha_1$, α_2. Note that the constants A and B involved in the auxiliary temperature field (7) remain unknown, and also that the corrective heat flow $\boldsymbol{q}^1$ vanishes for α, $\beta \to 0$, thus preserving the values of the remote heat flow $\boldsymbol{q}^\infty$ at infinity. For sake of definiteness, it will be assumed $A = B = 0$.

Hereinafter, results are given in terms of dimensionless temperature field $T\, k/(r_1\, q^\infty)$ and normalized heat flux $\boldsymbol{q}/q^\infty$. Also the geometric dimensions will be normalized by r_1. Therefore, the layout is completely definite by two independent dimensionless parameters: $\rho = r_2/\, r_1$ and $\gamma = \delta/r_1$.

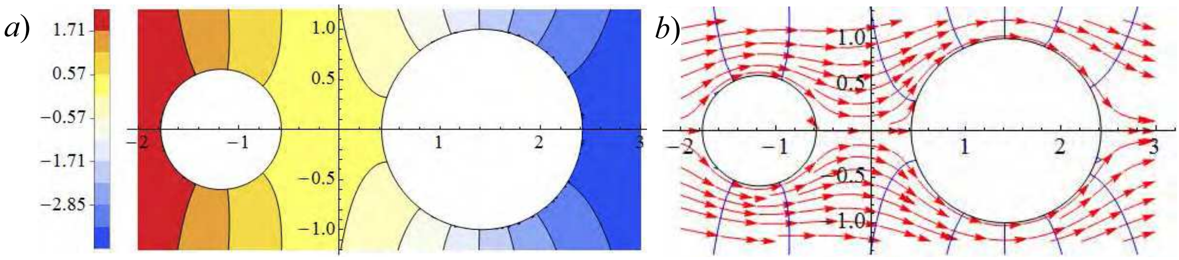

Figure 3. Representation of the dimensionless *a*) temperature distribution and *b*) heat flux of an infinite body with two insulated inclusions subjected to a remote heat flux in the x_1 direction ($\rho = 3/5$, $\gamma = 1$).

Figure 3 depicts the temperature and heat flux fields distributions around two circular inclusions for $\rho = 3/2$ and $\gamma = 1$. As demanded by BCs (6), the level curves of the temperature field $T^\infty + T^1$ (the blue lines in Figure 3*b*) are orthogonal to the contours of the inclusions. This fact is confirmed by Figure 3*b*, where the heat flow is tangent to the contour of the inhomogeneities.

2.2 Temperature distribution for heat flow in the x_2 direction

The body is subjected to a remote steady-state heat flow $\boldsymbol{q}$ in x_2–axis direction (see Figure 2*b*), i.e. $\boldsymbol{q}(x_1, x_2) = (0, q^\infty)$ or, in bipolar coordinates,

$$\boldsymbol{q}(\alpha,\beta) = q^\infty \left(-\frac{\sinh\alpha \sin\beta}{\cosh\alpha - \cos\beta}, -\frac{1-\cosh\alpha\cos\beta}{\cosh\alpha - \cos\beta} \right), \tag{10}$$

that corresponds to the following temperature field T^∞ in a body without holes:

$$T^\infty(\alpha,\ \beta) = -a\frac{q^\infty}{k}\frac{\sin\beta}{\cosh\alpha - \cos\beta}. \tag{11}$$

However, the temperature field (11) does not accomplish BCs (6). Thus, in order to satisfy these BCs, an auxiliary temperature field $T^1(\alpha, \beta)$ fulfilling (4) is introduced as follows:

$$T^1(\alpha,\beta) = B\,\beta + C\,\alpha\,\beta + \sum_{n=1}^{\infty} \phi_n(\alpha)\,\sin(n\beta)\,, \tag{12}$$

where functions $\phi_n(\alpha)$ have been defined after eqn (7).

The auxiliary temperature field (12) produces a corrective heat flow $\boldsymbol{q}^1 = (q^1{}_\alpha, q^1{}_\beta)$ whose component $q^1{}_\alpha$ is, according to (3):

$$q^1{}_\alpha = -\frac{k}{a}[2C\cosh\alpha + \frac{C}{2} + \cosh\alpha\ \phi_1'(\alpha) - \frac{1}{2}\phi_2'(\alpha)]\sin\beta + \\ + \frac{k}{2a}\sum_{n=2}^{\infty}[\frac{4C(-1)^n}{n}(\cosh\alpha - \frac{n^2}{n^2-1}) - 2\cosh\alpha\ \phi_n'(\alpha) + \phi_{n+1}'(\alpha) + \phi_{n-1}'(\alpha)]\sin n\beta, \tag{13}$$

where the apex $(\ldots)'$ denotes derivative with respect α. Also in this case, the α-component of the heat flow $\boldsymbol{q}^1$ has been expanded in Fourier series.

BCs (6) require that $q^\infty{}_\alpha + q^1{}_\alpha = 0$ for $\alpha = \alpha_1, \alpha_2$ (with $\alpha_1 > 0$, $\alpha_2 < 0$). Such conditions lead to the following equations

$$4a\,q^\infty e^{-|\alpha_i|}\sinh\alpha_i + k\,[4C\cosh\alpha_i + C + 2\cosh\alpha_i\ \phi_1'(\alpha_i) - \phi_2'(\alpha_i)] = 0,$$

$$4a\,q^\infty e^{-n|\alpha_i|}\sinh\alpha_i + k\,[\frac{4C(-1)^n}{n}(\frac{n^2}{n^2-1} - \cosh\alpha_i) + 2\cosh\alpha_i\ \phi_n'(\alpha_i) - \phi_{n+1}'(\alpha_i) - \phi_{n-1}'(\alpha_i)] = 0, \tag{14}$$

where $i = 1, 2$ and $n = 2, 3, \ldots, \infty$.

Eqn $(14)_1$ provides 2 independent conditions for constants C_1, D_1, C_2, D_2. Two further conditions can be obtained by multiplying eqn $(14)_2$ by $e^{-n|\alpha|}$ and summing from $n = 2$ up to infinite. Once constants C_1, D_1, C_2, D_2 have been determined, the remaining constants C_n, D_n can be assessed by imposing condition (14) for $\alpha = \alpha_1, \alpha_2$ at n-order. In the sequel, for sake of definiteness, it will be assumed $B = C = 0$.

Figure 4*a* shows the temperature field around the inhomogeneities, whereas Figure 4b depicts the related heat flow for ρ = 3/5 and γ = 1. Ad expected, the level curves for the temperature field are normal to the contours of the inclusions, thus confirming the accomplishment of the BCs.

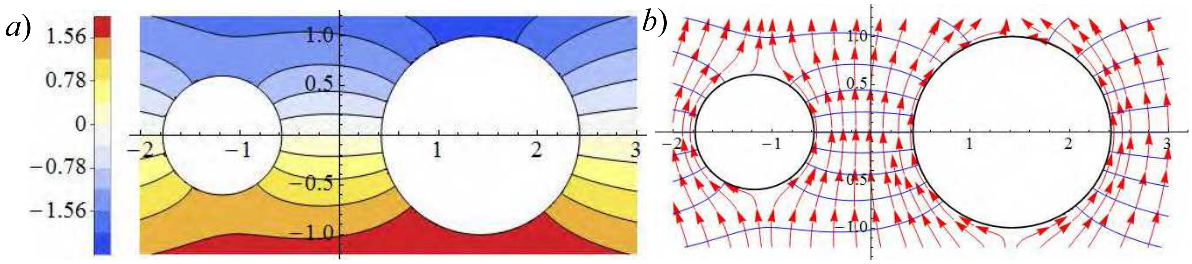

Figure 4. Representation of the dimensionless *a*) temperature distribution and *b*) heat flux of an infinite body with two insulated inclusions subjected to a remote heat flux in the x_2 direction (ρ = 3/5, γ = 1).

3 RESISTIVITY CONTRIBUTION TENSOR

Following the approach reported in [9], the corrective term about the temperature gradient Δ(∇T) due to the presence of inhomogeneities can be assessed as follows

$$\Delta(\nabla T) = \frac{A_*}{A}\boldsymbol{R}\cdot\boldsymbol{q}, \tag{15}$$

where $A_* = A_1 + A_2$ is the area of the inclusions, A is the surface of the whole system, $\boldsymbol{R}$ is the second order resistivity tensor whose components are

$$\begin{pmatrix} R_{11} \\ R_{21} \end{pmatrix} = \frac{1}{q^\infty_{x_1}A_*}\sum_{i=1}^{2}\oint_{\partial\Omega_i} T_{x_1}(\alpha_i,\beta)\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}\mathrm{d}s, \quad \begin{pmatrix} R_{12} \\ R_{22} \end{pmatrix} = \frac{1}{q^\infty_{x_2}A_*}\sum_{i=1}^{2}\oint_{\partial\Omega_i} T_{x_2}(\alpha_i,\beta)\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}\mathrm{d}s. \tag{16}$$

In eqn (20), q_{x1}^{∞}, q_{x2}^{∞} denote the magnitudes of the remotely applied heat fluxes along the x_1 and x_2 axes, respectively, and $T_x(\alpha, \beta)$ and $T_y(\alpha, \beta)$ are the corresponding total temperature fields given in Sec. 2. The contour integrals are performed around the rings $\partial\Omega_i$ ($i = 1, 2$) of the inhomogeneities, $\boldsymbol{n} = n_1\,\hat{\boldsymbol{e}}_1 + n_2\,\hat{\boldsymbol{e}}_2$ is the unitary vector normal to the contour of the inhomogeneities and outward directed, and ds is the infinitesimal arc length.

Based on expressions (5)-(7), (11)-(12), expressions (16) give:

$$
\begin{aligned}
R_{11} &= \frac{1}{k\,\pi(\mathrm{csc\,h}^2\alpha_1 + \mathrm{csc\,h}^2\alpha_2)}\{\int_{-\pi}^{\pi} \frac{(\cos\beta\cosh\alpha_1 - 1)\sinh\alpha_1}{(\cos\beta - \cosh\alpha_1)^3} - \frac{(\cos\beta\cosh\alpha_2 - 1)\sinh\alpha_2}{(\cos\beta - \cosh\alpha_2)^3}\,d\beta \\
&+ \frac{k}{a\,q^{\infty}}\sum_{n=1}^{\infty}\int_{-\pi}^{\pi}[\frac{(\cos\beta\cosh\alpha_1 - 1)\;\phi_n(\alpha_1)}{(\cos\beta - \cosh\alpha_1)^2} + \frac{(\cos\beta\cosh\alpha_1 - 1)\;\phi_n(\alpha_2)}{(\cos\beta - \cosh\alpha_2)^2}]\cos n\beta\,d\beta, \\
R_{22} &= \frac{1}{k\,\pi(\mathrm{csc\,h}^2\alpha_1 + \mathrm{csc\,h}^2\alpha_2)}\{\int_{-\pi}^{\pi}[\frac{\sinh\alpha_1}{(\cos\beta - \cosh\alpha_1)^3} - \frac{\sinh\alpha_2}{(\cos\beta - \cosh\alpha_2)^3}]\sin^2\beta\,d\beta \\
&+ \frac{k}{a\,q^{\infty}}\sum_{n=1}^{\infty}\int_{-\pi}^{\pi}[\frac{\sinh\alpha_1\;\phi_n(\alpha_1)}{(\cos\beta - \cosh\alpha_1)^2} - \frac{\sinh\alpha_2\;\phi_n(\alpha_2)}{(\cos\beta - \cosh\alpha_2)^2}]\sin n\beta\sin\beta\,d\beta.
\end{aligned} \tag{17}
$$

Note that integrals involved in expressions (17) can be evaluated in closed form through the following results:

$$
\int_{-\pi}^{\pi}\frac{\sin^2\beta}{\left(\cosh\alpha - \cos\beta\right)^3}\,d\beta = \frac{\pi}{\sinh^3|\alpha|},\quad \int_{-\pi}^{\pi}\frac{\cos n\beta}{\left(\cosh\alpha - \cos\beta\right)^2}\,d\beta = \frac{2\pi}{\sinh^2\alpha}e^{-n|\alpha|}(n + \coth|\alpha|)\,. \tag{18}
$$

Then, based on (18), after some algebraic calculations, components R_{11} and R_{22} read:

$$
\begin{aligned}
R_{11} &= \frac{1}{k}\{-1 + \frac{2k}{a\,q^{\infty}(\mathrm{csc\,h}^2\alpha_1 + \mathrm{csc\,h}^2\alpha_2)}\sum_{n=1}^{\infty} n\,[e^{-n\alpha_1}\phi_n(\alpha_1) - e^{n\alpha_2}\phi_n(\alpha_2)]\}, \\
R_{22} &= \frac{1}{k}\{-1 + \frac{2k}{a\,q^{\infty}(\mathrm{csc\,h}^2\alpha_1 + \mathrm{csc\,h}^2\alpha_2)}\sum_{n=1}^{\infty} n\,[e^{-n\alpha_1}\;\phi_n(\alpha_1) + e^{n\alpha_2}\;\phi_n(\alpha_2)]\},
\end{aligned} \tag{19}
$$

and $R_{12} = R_{21} = 0$. Note also that terms $-1/k$ in eqns (19) represent the contribution of the remote temperature fields T^{∞}. The remaining terms are due to the auxiliary solution T^1.

Figure 5 shows the components of the resistivity contribution tensor in dimensionless form, $R_{11}\,k$ and $R_{22}\,k$, for some values of γ. It is worth noting that both $R_{11}\,k$ and $R_{22}\,k$ equal -2 as $\rho \to 0$ whereas, for $\rho \to \infty$, they tend asymptotically to -1. Note also that $R_{\mathrm{ii}}(\gamma, \rho) = R_{\mathrm{ii}}(\gamma/\rho, 1/\rho)$, as expected owing to the symmetry of the system.

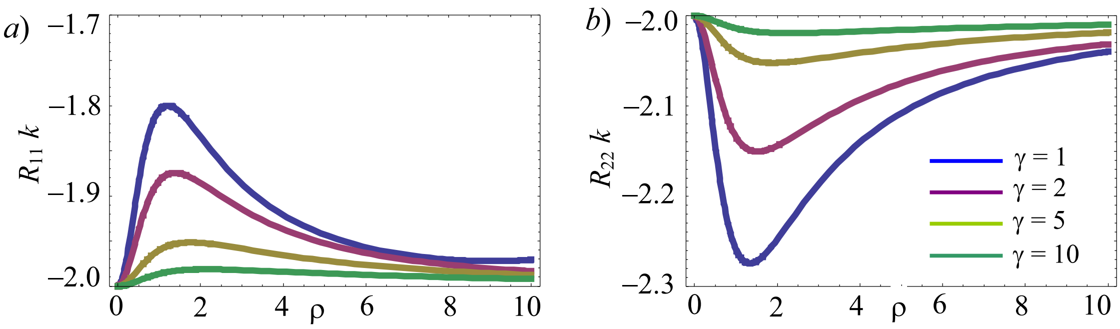

Figure 5. Dimensionless components of the resistivity contribution tensor: a) $R_{11}\,k$ and b) $R_{22}\,k$ for $\gamma = 1, 2, 5, 10$ varying ρ.

4 CONCLUSIONS

- The thermal flow and temperature fields of an elastic matrix with two circular inclusions of different size subjected to a remotely applied steady state thermal flux have been assessed analytically.
- The study allows evaluating the overall thermal properties of a composite system with insulating inhomogeneities. Indeed, through the estimation of proper contour integrals involving the temperature assessed at the ring of the inclusions, the second-order resistivity contribution tensor has been found varying the geometric parameters of the system.
- The inclusions have been taken non-conducting, thus simulating the presence of synthetic fibres or other insulating materials embedded in a matrix of a composite system. This makes the present analysis a reliable tool to investigate the effective thermal properties of a wide range of composites, like fibre reinforced concrete, fibre reinforced plastic, blocks for masonry with microvoids and cementitious composites based on synthetic reinforcements (see, e.g. [11, 12]).
- In a forthcoming work the stress and strain fields generated by a thermal flux in a plate with two overlapping inclusions will be addressed. Such a study could find noteworthy applications in evaluating stress localization induced by nucleation and coalescence of voids, a phenomenon that can drive failure and damage [13, 14] in a wide range of materials.

ACKNOWLEDGMENTS

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