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## Coupling 2D-Wavelet decomposition and Multivariate Image Analysis (2D WT-MIA)

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4 **1 Coupling 2D-Wavelet decomposition and Multivariate Image Analysis (2D WT-**  
5 **2 MIA)**  
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24 **11 ABSTRACT**

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27 The use of 2D Discrete Wavelet Transform in the Feature Enhancement phase of  
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29 Multivariate Image Analysis is discussed and implemented in a comparative way with  
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31 respect to previous publications. In the proposed approach, all the resulting sub-images  
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33 obtained by Discrete Wavelet Transform decomposition are unfolded pixel-wise and  
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35 mid-level datafused to a Feature Matrix which is used for the Feature Analysis phase.  
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37 Congruent sub-images can be obtained either by reconstruction of each decomposition  
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39 block to the original pixel dimensions, or by using the Stationary Wavelet Transform  
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41 decomposition scheme. The main advantage is that all possible relationships among  
42  
43 blocks, decomposition levels and channels are assessed in a single multivariate analysis  
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45 step (Feature Analysis). This is particularly useful in a monitoring context where the  
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47 aim is to build multivariate control charts based on images. Moreover, the approach can  
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49 be versatile for contexts where several images are analysed at a time as well as in the  
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51 multispectral images analysis.  
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4 25 Both a set of simple artificial images and a set of real images, representative of the on-  
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6 26 line quality monitoring context, will be used to highlight the details of the methodology  
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8 27 and show how the wavelet transform allows extracting features which are informative of  
9  
10 28 how strong the texture of the image is and in which direction it varies.  
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15  
16 30 **Keywords:** 2D Wavelet Transform, Multivariate Image Analysis, Multi resolution,  
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18 31 Quality monitoring  
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## 1. INTRODUCTION

The use of Multivariate Analysis to evaluate images dates to the mid-late 80's, with the work of Esbensen and Geladi<sup>1</sup> who introduced the use of Principal Component Analysis for the study of multi-channel images. Multivariate Image Analysis (MIA) has soon gained boost with the application in many contexts, typically those with images of such complexity that they could benefit of a multivariate analysis approach (e.g. remote sensing<sup>2-4</sup> and medical imaging<sup>5-9</sup>). In the 90's, the pioneering works of MacGregor and his group made the field of process industry accessible by the MIA approach<sup>2, 10</sup>. The possibility to acquire images for on-line process monitoring purposes and effectively analyse them represents a viable, PAT-like sensor to investigate process changes in time in an environment, the process line, where often the room for installing new "traditional" sensors is poor, not to mention the fact that often a single image can acquire simultaneously several different potential sources of variability.

At present, several uses of MIA are reported in literature for different tasks<sup>11-29</sup>, all of which are characterised by being well described by the two main sources of information an image can carry. Textural variability, which can be gathered by analysing the "two" dimensions relationship structure of pixels and "spectral" properties variability, which is based on the "third" dimension, that is the channels acquired for each pixel. The latter aspect becomes the more relevant as the number of channels increases, moving from simple binary or grey-scale images (where not much information can be given more than texture homogeneity/non homogeneity), to RGB images (where changes in colour can be related to the presence of non homogeneous texture or underlying phenomena which alter the composition), to multi-channel images and spectral images (where chemical information can effectively be acquired for each pixel). Therefore, most

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4 56 image-based challenges, which can be addressed with MIA approach, represent the  
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6 57 detection of product defects in quality control<sup>11-18</sup>, the monitoring of changes in process  
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8 58 behaviour and its feed-back control<sup>16, 19-21</sup>, the prediction of product properties on the  
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10 59 basis of the joint evaluation of texture and channel information (in particular addressed  
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12 60 to by the development of Multivariate Image Regression – MIR – methods<sup>22-23</sup>); or  
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14 61 more recently the development of imaging biomarkers in cancer diagnosis<sup>8-9</sup>.  
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18 62 As far as the core details of the MIA approach and its evolution in time are concerned,  
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20 63 the MIA approach proposed by Bharati and MacGregor is based on a framework<sup>11-12</sup>  
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22 64 which can be summarised in two main steps: a Feature Extraction (or Enhancement)  
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24 65 phase, and a Feature Reduction (or Analysis) phase. In the Enhancement phase the  
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26 66 image (pre-processed, if necessary) is treated so that texture information carried out by  
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28 67 the pixels is made clearer. In the Analysis phase, a suitable Multivariate Analysis  
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30 68 method is applied (e.g. Principal Component Analysis, Partial Least Squares  
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32 69 Regression, Partial Least Squares Discriminant Analysis) on the feature matrix obtained  
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34 70 after the first phase. The two phases are strictly connected to each other, since the first  
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36 71 step can strongly influence the outcome of the following analysis in a way which is not  
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38 72 much different from the effect of data pre-processing in many other situations.  
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40 73 However, a certain degree of freedom can be considered when choosing the feature  
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42 74 enhancement method (whilst the feature analysis phase is more application driven). The  
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44 75 fundamental aspect to be considered in this case is that it is not only important to  
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46 76 preserve the information given by the channels, for which a simple unfolding of the  
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48 77 image structure so that each pixel becomes a sample could be sufficient, but to retain the  
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50 78 correlation among neighbouring pixels (that is, the texture information) and, most of all,  
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52 79 to present it to the following analysis step in such a way that both sources of variability  
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4 80 (texture and channel-based properties) can be easily evaluated. In the approach  
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6 81 proposed by Bharati and MacGregor<sup>2</sup> the texture information is extracted by  
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8 82 augmenting column-wise the unfolded pixel vector with a series of its copies, shifted  
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10 83 row-wise so that each row of the generated matrix is formed by a pixel and all its  
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12 84 surrounding neighbours. This corresponds to stacking copies of the original image  
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14 85 shifted according to a given step. The number of neighbours, hence of columns, of the  
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16 86 feature matrix is  $(2w + 1)^2$ , governed by the window aperture parameter  $w$ , which  
17  
18 87 indicates the dimension of the window, centred on the pixel, encompassing the  
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20 88 neighbours to be considered (typically,  $w = 1$  or  $2$ ). In Prats-Montalbán et al.<sup>17</sup> this  
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22 89 augmentation is extended to each channel of the image, and will be referred to from  
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24 90 now on as colour-textural MIA (ct-MIA). Facco et al.<sup>14</sup> proposed a method to reduce the  
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26 91 computational cost when operating with a larger window size,  $w > 2$ . Other approaches  
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28 92 have been proposed and discussed, among which the most common are based on the  
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30 93 application of a transformation of the image, again for each channel, in a different  
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32 94 domain, such as the Fourier domain (*via* the 2D Fourier Transform) or the wavelet  
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34 95 domain<sup>10-12, 18, 24-29</sup>. The wavelet advantage with respect to Fourier is that it has both  
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36 96 good frequency and spatial resolution.

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42 97 There may be several advantages by moving to wavelet domain in terms of image  
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44 98 compression, background removal and denoising. Moreover, the use of wavelet  
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46 99 transform allows a better understanding of the pixels correlation structure at different  
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48 100 scales. At each level of decomposition, the coefficients carry both the information  
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50 101 pertaining to the energy which characterise a frequency range (based on the selected  
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52 102 filter) and an indication about the orientation in which varies (according to the type of  
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54 103 decomposition block, being it Approximation or one of the three Details blocks, namely  
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4 104 Horizontal, Vertical and Diagonal). In this way, the features extracted by wavelet  
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6 105 transform are a truly enhanced visualization of how strong, and in which direction, the  
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8 106 texture of the image varies. Literature differs in the way these features could be  
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10 107 expressed and handled. Some authors have pointed the attention to the use of global  
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12 108 indicators to synthesise the relevant information for a given decomposition level and  
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14 109 block, by using, e.g. the Frobenius norm (Energy), the entropy, statistical momenta or  
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16 110 the standard deviation of the coefficients<sup>18, 26-28</sup>. In this way, a single variable  
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18 111 summarises the effect, while the orientation information is maintained by means of the  
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20 112 level-block combination at which it is computed. This approach surely reduces the  
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22 113 computational cost of the following analysis, but carries with itself the potential loss of  
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24 114 interesting information, which goes together with an “averaging” procedure of a richer  
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26 115 set of data. Also, since all the information of an image is compressed in a single vector  
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28 116 of descriptors for each decomposition block and level, a somehow conspicuous set of  
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30 117 images must be considered to create a reference set, for example of Normal Operating  
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32 118 Conditions (NOC), when moving to the following Feature Analysis phase. On the  
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34 119 contrary, working at pixel level, that is considering each pixel as a sample, opens the  
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36 120 possibility to reduce the requirements when building a reference set, often allowing the  
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38 121 use of a single representative image, being it a real one, or a combination of snapshots  
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40 122 of NOC texture areas.

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46 123 Liu and MacGregor<sup>10</sup> have proposed an approach where the wavelet transform is used  
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48 124 for Feature Enhancement of images working at pixel level, i.e. the MultiResolutional  
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50 125 Multivariate Image Analysis (MR-MIA). MR-MIA is conjugated in two frameworks  
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52 126 that differ in the stage at which the wavelet transform is applied, i.e. before (MR-MIA I)  
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54 127 or after (MR-MIA II) the Feature Analysis step (in this case PCA). In particular, in MR-  
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4 128 MIA I by applying the discrete wavelet transform (2D DWT) to each channel of the  
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6 129 image, each decomposition block, at a given level, can be seen as an image itself with  
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8 130 the same number of channels, but representing a different “resolution” and texture  
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10 131 orientation. The Feature Analysis (e.g. PCA) step is then applied to each of these  
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12 132 images, once unfolded pixel-wise, so that as many latent variable models as number of  
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14 133 blocks per decomposition level (L) are obtained. This approach relies on the  
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16 134 orthogonality of the wavelet decomposition blocks, thus implying that there is no  
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18 135 interest in evaluating correlations among blocks at different scales, and the possibility to  
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20 136 evaluate texture – channel correlation is maintained. However, the complexity aspect of  
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22 137 this approach can be a hindrance when considering how many multivariate models one  
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24 138 should compute and the necessity of a high-level data fusion step where all the results  
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26 139 are combined to create a decision rule in order to e.g. decide if a new product image has  
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28 140 to be rejected when compared to the NOC modelled one. Recently, basing on similar  
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30 141 considerations, Juneau *et al.*<sup>25</sup> proposed an approach where all sub images obtained by  
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32 142 wavelet decomposition, once unfolded pixel-wise, are merged row-wise and analysed  
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34 143 by a single PCA. However, they used the continuous wavelet transform (undecimated  
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36 144 scheme, UWT) and in this way a rather large number of features is obtained, since scale  
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38 145 and shift parameters vary continuously.

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44 146 The MSMIA approach proposed by Reis<sup>18</sup> is similar to the MR-MIA I, although more  
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46 147 images are considered at the same time as references NOC, but it differs in the way  
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48 148 information from the Feature Analysis step (e.g. PCA) of each WT decomposition block  
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50 149 is fused. In this approach, an index evaluating the distance to the scores distribution  
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52 150 histogram of the reference NOC images for each decomposition block, at a given scale,  
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54 151 is calculated in order to obtain a set of variables, which are then used for building a  
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4 152 monitoring chart. In addition to this, multivariate control charts based on PCA of  
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6 153 wavelet features (e.g. standard deviation of wavelet coefficients for each decomposition  
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8 154 block), extracted for each decomposition block, at a given scale, are also considered.

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10 155 The approach is effective in compressing information and for on-line implementation,  
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12 156 however defects location requires a further step. This step, similarly to MR-MIA II,  
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14 157 consists in building a spatial shifting feature matrix (then analysed by PCA) for each of  
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16 158 the sub-images contributing to “out of control” observations in the preceding step.  
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18 159 Moreover, correlation structure among textural/colour pattern at different scales is only  
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20 160 indirectly taken into account (the information from different scales is always merged at  
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22 161 features, not pixels level).  
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27 162 Here we present an approach, which is named 2D WT-MIA, where the Feature  
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29 163 Enhancement step is similar to MR-MIA I, but as in Juneau<sup>25</sup> all of the resulting sub-  
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31 164 images obtained by the 2D-DWT decomposition are unfolded pixel-wise and mid-level  
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33 165 datafused to a Feature Matrix which is used for the Feature Analysis phase. In order to  
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35 166 have congruent sub-images all decomposition blocks are reconstructed separately to the  
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37 167 original pixel dimensions. This reconstruction step can be omitted, when the Stationary  
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39 168 Wavelet Transform (2D-SWT) is used<sup>30-31</sup>. In this way, all possible relationships among  
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41 169 blocks, decomposition levels and channels are assessed in a single multivariate analysis  
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43 170 step (Feature Analysis). This is particularly useful, in a monitoring context, when the  
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45 171 aim is building multivariate control charts based on NOC images. Thus, our proposed  
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47 172 approach can be versatile to handle contexts where several images are analysed at a time  
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49 173 as well as in the multispectral images analysis.  
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54 174 The rest of this paper is organised as follows: in Section 2: Methods, the proposed 2D  
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56 175 WT-MIA approach is described into more details and compared to colour-textural MIA  
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4 176 approach to highlight common and differing aspects; in Section 3: Materials, the images  
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6 177 datasets will be presented, consisting in a set of simple artificial binary images, used to  
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8 178 illustrate how texture is captured within the two approaches and a set of real images,  
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10 179 and in Section 4: Results and Discussion, these images will be analysed according to the  
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12 180 two-step MR-MIA framework, using Principal Component Analysis as Feature Analysis  
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15 181 technique with the target of simulating a quality control task.  
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## 18 182 2. METHODS

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21 183 The approach here described belongs to the more general framework of MultiResolution  
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23 184 Multivariate Image Analysis, thus basing on a two-phase elaboration of the image: a  
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25 185 first step of Feature Extraction (Enhancement) and a second step of Feature Reduction  
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27 186 (Analysis) (Figure 1). In particular, the 2D WT-MIA (wavelet based feature  
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29 187 enhancement) and the colour-textural MIA (spatial shifting based feature enhancement)  
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31 188 approaches will be discussed and compared in terms of results in the present work.  
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### 34 189 2.1 Spatial Shifting Feature Enhancement

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37 190 Colour-textural MIA<sup>17</sup> is summarised in Figure 2. This approach to Feature  
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39 191 Enhancement consists in capturing, for each channel  $ch$ , the pixel proximity correlation  
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41 192 by means of a spatial shifting of neighbouring pixels with respect to each pixel of the  
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43 193 original image, according to a selected window aperture parameter,  $w$ . In practice,  
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45 194 starting from a pixel element of the image  $p_{i,j}$ , a row vector is created by adding the  
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47 195 intensity value of the channel corresponding to the closest surrounding pixels: if  $w = 1$ ,  
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49 196 the composition appears as reported in Figure 2. When this is done for all the pixels of a  
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51 197 pixel-wise unfolded channel matrix, and the feature matrices for the different channels  
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53 198 are then fused, a total Feature Matrix is obtained which has as many rows as the number  
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4 199 of pixels  $I = n_1 \times n_2$ , and as many columns as the number of channels  $ch$  times the  
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6 200 number of spatial shifted pixels, which is given by  $(2w + 1)^2$ . This means that,  
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8 201 regardless of the number of channels, for a window parameter  $w = 1$  (the closest  
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10 202 neighbours) the Feature matrix is  $9 \times ch$ , and when moving to  $w = 2$  (the closest  
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12 203 neighbours and the next surrounding layer), the Feature matrix is  $25 \times ch$ . This implies  
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14 204 a fast increase of the number of variables considered in the analysis, the higher is the  
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16 205 number of channels.  
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20 206 Since we need, at each pixel location, to use all the neighbouring pixels up to a distance  
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22 207  $w$ , this implies that we lack information for all those pixels in the borders with width  $w$ .  
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24 208 Therefore, the solution commonly adopted is to diminish the size of the image from  
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26 209  $n_1 \times n_2$  to  $(n_1 - 2w) \times (n_2 - 2w)$   
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## 29 210 2.2 2D Wavelet-based Feature Enhancement

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32 211 Figure 3 shows the general scheme of the Feature Enhancement step involving 2D –  
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34 212 DWT application, through the fast Mallat algorithm<sup>32-33</sup>, on an image. For each channel  
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36 213  $ch$ , the low-pass and high-pass filters (which are the same as in the 1D case) are first  
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38 214 operated row-wise on the image and then, after downsampling of the coefficients, in  
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40 215 each of the resulting blocks column-wise. In this way four decomposition blocks are  
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42 216 obtained: Approximations (low-pass + low-pass), namely CA; Horizontal details (low +  
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44 217 high), namely CH; Vertical details (high + low), namely CV, and Diagonal details (high  
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46 218 + high), namely CD. The procedure is then iterated by applying it to the  
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48 219 Approximations, i.e. increasing the decomposition level. Downsampling is skipped  
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50 220 when the 2D - SWT scheme is used since, instead, the filters are up-sampled<sup>26</sup>. The  
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52 221 maximum possible decomposition level,  $L$ , depends on the image size. The four  
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4 222 decomposition blocks obtained from each level of decomposition (CA, CH, CV and  
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6 223 CD) when 2D DWT is used are independently reconstructed by means of the inverse 2D  
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8 224 – DWT so that their dimensions are the same of the original image, while they are  
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10 225 already of the same size when 2D SWT is used (in fact, each block of coefficients at  
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12 226 every level maintains the same size as the original image, and congruent images are  
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14 227 obtained). This leads to a set of  $4 \times L$  images for each channel  $ch$ , which can be  
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16 228 unfolded and column-wise merged to obtain a total Feature Matrix which has as many  
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18 229 rows as the number of pixels  $I = n_1 \times n_2$ , and as many columns as  $4 \times L \times ch$ . If we  
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20 230 compare this column dimension to the one obtained with the Spatial Shifting approach,  
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22 231 which is  $(2w + 1)^2 \times ch$ , it might appear that there is little benefit in terms of reduction  
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24 232 of the Feature Matrix dimensions. However, two aspects have to be underlined:  
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29 233 i) in the spatial shifting approach the image is analysed by moving a  $(2w + 1) \times (2w + 1)$   
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31 234 pixels window by step of 1 in all image directions; on the other hand, with wavelet, a  
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33 235 *filter length x filter length* pixels window is moved by step of 1 in all image directions,  
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35 236 but using a larger filter does not increase the number of features, which remain always  
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37 237 four;  
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41 238 ii) the two approaches lead to the same number of feature descriptors if  $L = \text{round}[(w +$   
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43 239  $\frac{1}{2})^2]$ . This corresponds, for e.g. a window parameter of  $w = 2$ , to a decomposition level  
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45 240  $L = 6$ , which in terms of multiresolution means to have gone very deep in the analysis of  
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47 241 coarse and smooth aspects of the image. In other words, such a decomposition level (if  
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49 242 allowed by the nature of the chosen wavelet) usually leads to the possibility of  
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51 243 evaluating correlations and high distance relationships among pixels to an extent, which  
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53 244 is superior to the use of a moving window of fixed size.  
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4 245 When applying the wavelet transform, the selection of the most appropriate wavelet  
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6 246 filter is considered a critical issue and a limiting step in the implementation of routine  
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8 247 applications (i.e. which wavelet family and which filter length, to analyse the specific  
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10 248 characteristics of the images at hand). This issue has been dealt in literature by  
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12 249 analysing the different properties of the decomposition filter in terms of texture  
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14 250 description capability in order to propose general criteria<sup>34</sup> or focusing on goodness of  
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16 251 image reconstruction<sup>33</sup>, or proposing a design of experiments approach<sup>36</sup>. We recently  
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18 252 proposed<sup>37</sup> a methodology based on N-way modelling to provide a range of possible  
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20 253 wavelet choices (in terms of families, filters, and decomposition levels), for each image  
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22 254 and problem at hand. Any of these strategies require a preliminary analysis step to be  
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24 255 conducted by experienced people in the field, although this step is only required once in  
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26 256 model building. However, some considerations, based on our experience can be drawn:  
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28 257 i) there is in general a relationship between the decomposition level and the filter length,  
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30 258 i.e. by using a larger filter a lower decomposition level is required to capture the  
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32 259 different image aspects (coarse and smooth) and ii) taking into account the wavelet  
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34 260 families characteristics, such as degree of symmetry or regularity or number of  
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36 261 vanishing moments<sup>38-39</sup> it is possible to focus on a small number of wavelet filters to  
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38 262 test, by choosing a representative one for each type of property.  
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### 264 3. MATERIALS

#### 265 3.1 Artificial Images datasets

266 These sets are used to illustrate how the colour-textural MIA and 2D WT-MIA  
267 approaches analyse texture and their capability to detect faulty pixels. These images are

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4 268 characterized by two main features: a particularly limited pixel size, so that  
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6 269 computational time is not a relevant benchmark property at this stage, and a simple, yet  
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8 270 well defined, pattern. Also, the differences between “normal”, i.e. reference image, and  
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10 271 “defective”, i.e. image (or images) for which a perturbation of the pattern was created,  
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12 272 are well controlled, in the sense that the number and position of pixels which have been  
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14 273 changed is known, and the entity of the disturb is enough to obtain simulated test  
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16 274 images which are not too similar to their reference image. In spite of the simplicity of  
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18 275 this simulated case, the information which can be acquired from the analysis with both  
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20 276 approaches is interesting to better understand how the two methods under comparison  
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22 277 work, and the conclusions which can be drawn are helpful and can be extended, as  
23  
24 278 shown in the next section where real images are presented and dealt with, to cases of  
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26 279 higher complexity.

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31 280 The set is composed by three binary images, as reported in Figure 4, of size  $32 \times 32$   
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33 281 pixels. Figure 4 “SimSetA” reports the “normal” (reference) image, on the basis of  
34  
35 282 which an alternation pattern has been generated. In this case, the squares which alternate  
36  
37 283 in both image directions to give a chequered pattern have a dimension of  $8 \times 8$  pixels:  
38  
39 284 starting from upper left corner and moving over columns dimension, a white (1's)  $8 \times 8$   
40  
41 285 pixel square is alternated to a black (0's)  $8 \times 8$  pixel square, and the same alternated  
42  
43 286 pattern is repeated over the rows dimension. Starting from this image, two changes in  
44  
45 287 pattern were produced, leading to two “defective” (test) images. Figure 4 “SimSetB”  
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47 288 shows an overlying irregular shape which extends from the diagonal to the lower left  
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49 289 part of the image: for this figure, a total of 55 pixels have been inverted in value (from 1  
50  
51 290 to 0 or from 0 to 1) over the total of  $32 \times 32 = 1024$  pixels. Figure 4 “SimSetC” shows  
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53 291 another change in the pattern, this time according to a regular shape which is applied on  
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4 292 top of each of the  $8 \times 8$  pixel squares: for each of these squares, starting from the second  
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6 293 diagonal element, a single pixel every fourth has been modified both in the rows and in  
7  
8 294 the columns, thus resulting in a change of four pixels for each of the squares. For this  
9  
10 295 figure, a total of 64 pixels have been inverted in value (from 1 to 0 or from 0 to 1) over  
11  
12 296 the total of  $32 \times 32 = 1024$  pixels.

### 16 297 3.2 Real Images datasets

17  
18 298 To further explore the performance of the method proposed in this work and compare it  
19  
20 299 to the results of the ct-MIA approach, additional datasets have been taken into account,  
21  
22 300 belonging to different applicative contexts, tiles and bread production, respectively. In  
23  
24 301 both cases, the control of the final product undergoes visual inspection, while the  
25  
26 302 datasets differ as for image dimensions and number of channels.

#### 30 303 3.2.1 Tiles

31  
32  
33 304 These datasets come from a production of tiles of marble-like materials for surface  
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35 305 coverage: all the cases share a common issue, that is presenting product samples which  
36  
37 306 do not comply to a strict definition of “normal” images, characterized, for instance, by a  
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39 307 precise colour shade or by the absence of defects such as spots and scratches. Therefore,  
40  
41 308 it is necessary to develop a method, complementary or alternative to visual inspection,  
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43 309 which is able to: a) recognize the presence of a defectiveness when a new tile is  
44  
45 310 compared to the reference one(s); b) indicate the kind of defectiveness (e.g. colour  
46  
47 311 shade and/or presence of unwanted changes in surface pattern); c) locate on the surface  
48  
49 312 the position of the defect in order to obtain an enhanced perception of the same, so that  
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51 313 its visualization and recognition by the operator is made easier. Samples from two  
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53 314 different products were considered with different degree of irregularity of the pattern in  
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4 315 the defective tiles. They both consists of RGB images of dimensions  $256 \times 256$  pixels  
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6 316 (Figure 5). Figure 5a reports dataset 1: Blanco Zeus, from now on referred to as  
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8 317 BZdataset, which is composed by three reference images (BZN01, BZN02 and BZN03),  
9  
10 318 and three images of tiles showing defects (BZD01, BZD02, and BZD03). This kind of  
11  
12 319 tile shows a mostly homogeneous shade of grey all over its surface, so that defects (as  
13  
14 320 for instance white or dark spots and scratches) do not usually present particularly high  
15  
16 321 difficulty of detection also by visual inspection. Figure 5b reports dataset 2: Blanco  
17  
18 322 Norte, from now on referred to as BNdataset, which is composed by three reference  
19  
20 323 images (BNN01, BNN02 and BNN03), and three images of tiles showing defects  
21  
22 324 (BND01, BND02, and BND03). In this case, the tile main colour is grey, but the surface  
23  
24 325 is characterized by an inhomogeneous distribution of darker spots, in a grainy structure,  
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26 326 which makes quite difficult to detect the presence of defectiveness, both when  
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28 327 represented by darker and paler areas.  
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### 33 328 3.2.2 Bread

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36 329 This data set comes from an industrial production of bun bread, where a digital scanner  
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38 330 is already used to automatically assess defects concerning mainly bun dimensions,  
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40 331 while surface defectiveness, such as dark spots, blisters, and pale areas is still evaluated  
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42 332 by visual inspection of expert personnel. These defects arise from different causes, some  
43  
44 333 of which not perfectly known, and are also often difficult to be detected by RGB online  
45  
46 334 cameras. Thus, a feasibility study has been undertaken<sup>29</sup> by acquiring offline  
47  
48 335 multispectral images, covering the UV-visible range (from 430 to 700 nm, 10 channels)  
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50 336 and the short-wavelength NIR range (from 850 to 970 nm, 8 channels), which can  
51  
52 337 improve the acquisition of information on bread quality, combining spectral (NIR may  
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54 338 represent also a “chemical signature”) and textural information. The whole data set has  
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4 339 been analysed by WT-MIA (DWT scheme) approach and described in detail in ref. 33  
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6 340 while here a subset of images has been analysed in order to discuss comparatively the  
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8 341 performance of WT-MIA (DWT and SWT) and ct-MIA.  
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10  
11 342 The raw images were cropped to remove the distortion effect of the round bun shape,  
12  
13 343 background and noise were removed *via* preliminary wavelet analysis<sup>29</sup>, finally giving  
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15 344 images of dimensions of about 387 x 420 pixels for 18 channels. Here two non-  
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17 345 defective images (N01, used as reference, and N02) and two defective ones (D04 and  
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19 346 D07) are analysed, shown in Figure 6.  
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## 26 348 **4. RESULTS AND DISCUSSION**

### 27 28 349 4.1 Artificial Images datasets

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31 350 All the three images (SimSetA, SimSetB and SimSetC) have been treated according to  
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33 351 the same Feature Enhancement step, by considering:

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36 352 - Spatial Shifting, colour-textural MIA (ct-MIA) with window size parameter  $w =$   
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38 353 1  
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40 354 - Wavelet Decomposition (WT-MIA) by using a Daubechies 1 (db1) at  
41  
42 355 decomposition level  $L = 1$ , both DWT and SWT.  
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46 356 The Feature Enhancement step gave a Feature Matrix of dimensions  $I = 900$  rows  
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48 357 (reduction from 32 x 32 to 30 x 30 is necessary to cope with borders) and 9 columns for  
49  
50 358 the ct-MIA approach and  $I = 1024$  rows x 4 columns for the WT-MIA approaches.  
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52  
53 359 SimSetA was used as the reference set, upon which for the ct-MIA approach a Principal  
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55 360 Component Analysis model was obtained after mean centring of the Feature Matrix.  
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4 361 Figure 7 reports the PCA results, from left to right in the order are shown: scores image  
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6 362 SimSetA, loadings, projected scores images SimSetB and SimSetC (in the order PC1 to  
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8 363 PC4 from top to bottom). PC1 captures, for the reference (“normal”) image both the  
9  
10 364 difference in grey intensity value (colour) and the variation in pattern (texture) when  
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12 365 passing from the pixels having zero value to pixels with value one, i.e. it shows the  
13  
14 366 change of value when moving along the borders from one square to another, where pixel  
15  
16 367 values invert, leading to a “blurring” effect of the borders. This is the expected effect  
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18 368 since ct-MIA window of one (which is actually  $3 \times 3$  pixels) moves pixel by pixel on  
19  
20 369 the image structure, which is made of 16 squares of dimensions  $8 \times 8$ . All features  
21  
22 370 contribute similarly to the PC1 loadings since the pattern change takes place in all  
23  
24 371 directions. Thus, PC1 works as an average grey scale image, which in fact extracts out  
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26 372 the spectral information (we have no other source of spectral information than a single  
27  
28 373 grey scale channel). The following PCs capture only the borders effects, i.e. only the  
29  
30 374 frames around the squares are visible in the scores images, and by inspecting the  
31  
32 375 loadings it is possible to understand the directions of the pattern variation, e.g. to PC4  
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34 376 the features accounting for diagonal shift do not contribute.

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40 377 When the Feature Matrix corresponding to SimSetB and SimSetC are projected onto  
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42 378 this model, the same chequered pattern is correctly reproduced (Figure 7), but the  
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44 379 changes in pixel correlations due to the small scale modifications of its regularity  
45  
46 380 produce a large blurred area, which roughly encompasses the whole shape of the  
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48 381 differences but extends further with respect to the faulty pixels, i.e. an area of about  $3 \times 3$   
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50 382 pixels around each defective pixel as it is detailed in the following text. This is due to  
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52 383 the fact that the perturbation, although being well defined (in particular for SimSetC) to  
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4 384 a small number of pixels, influences the neighbouring correlation structure of all the  
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6 385 pixels, which are contained by the moving window.  
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9 386 In a monitoring context the defective images with respect to the reference one/s can be  
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11 387 identified by the Hotelling- $T^2$  and squared residuals (RSS, SPE or Q) multivariate  
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13 388 control charts by using the percentage of pixels beyond control limits<sup>16</sup>. However, in  
14  
15 389 this case being the images binary, simply the pixel by pixel difference of the residuals  
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17 390 sum of squares (RSS) of the test images with respect to the reference image (NOC) can  
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19 391 be used. Both the RSS from a one or a four components PCA model are suitable to  
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21 392 depict the faulty pixels for SimSetB and SimSetC, but also an area of about 3 x 3  
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23 393 around each faulty pixel will show up differing in RSS values with respect to NOC  
24  
25 394 (Figure S2, supplementary material). This can be expected on the basis of the  
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27 395 considerations made above on the neighbouring pixels correlation structure.  
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31  
32 396 In the WT-MIA both decomposition schemes, DWT and SWT, have been applied,  
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34 397 considering the simulated pattern, i.e. inversion of the binary value of some not  
35  
36 398 consecutive pixels, db1 seems an appropriate filter. The feature matrix, holding the four  
37  
38 399 decomposition blocks CA, CH, CV and CD (reconstructed only in DWT case), already  
39  
40 400 captures the texture pattern, as highlighted in Figure 8 (DWT) and Figure 9 (SWT)  
41  
42 401 where the sub-images corresponding to each block of the DWT and SWT  
43  
44 402 decomposition of SimSetA and SimSetB are reported. This is a first difference with  
45  
46 403 respect to ct-MIA approach where the feature matrix holds just the shifted version of the  
47  
48 404 raw image in all possible neighbouring direction (as shown in Figure S1, supplementary  
49  
50 405 material) and thus PCA (or, more in general, a multivariate decomposition technique) is  
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52 406 needed to reveal the texture pattern. Further, in this case with only one reference image  
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4 407 and one channel the feature analysis step by PCA is not needed at all (the decomposition  
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6 408 blocks are orthogonal).  
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9 409 In the DWT the Approximation Block is the only one which carries the structural  
10  
11 410 information of image SimSetA (Figure 8, top). This is explainable by considering that  
12  
13 411 the db1 filter is of length two, thus operates like a window of pixel size  $2 \times 2$  which  
14  
15 412 moves at steps of one, and due to the down-sampling scheme of DWT only one  
16  
17 413 coefficient every two is retained. Thus, since the binary values change every  $8 \times 8$   
18  
19 414 pixels, there is no blurring effect at the squares edges; also the coefficients in all the  
20  
21 415 other decomposition blocks are zeros (Figure 8, top). On the contrary, the presence of  
22  
23 416 deviations in the two test images, related to “sharper” structures i.e. alternating by one  
24  
25 417 pixel, is well captured by all decomposition blocks (Figure 8, middle). In particular  
26  
27 418 Approximation (CA) shows both intensity change and texture, while Details blocks  
28  
29 419 capture horizontal (CH), vertical (CV) and diagonal (CD) neighbouring pixels  
30  
31 420 alternation of binary values. Thinking of a monitoring context, in this case the defects  
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33 421 can be depicted by the difference between the decomposition sub-images of the  
34  
35 422 reference and test image, as shown in Figure 8, bottom; to this aim, considering the  
36  
37 423 specific pattern of the defects in SimSetB, CA and CD are the most suitable blocks.  
38  
39 424 Figure 9 shows the results of the SWT decomposition. Similar considerations can be  
40  
41 425 drawn. The only difference is that now the effect of the variation in the binary values of  
42  
43 426 the pixels at the edges of the  $8 \times 8$  pixels squares are visible (similarly to ct-MIA). This  
44  
45 427 is well explainable by the fact that in SWT down-sampling of the coefficients is not  
46  
47 428 operated (it is worth mentioning that the window size and the moving step remain the  
48  
49 429 same). Analogous considerations hold for SimSetC decomposition (figure not shown for  
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51 430 sake of brevity).  
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4 431 The behaviour of a larger filter, i.e. Daubechies 2 (db2) of length 4, has been also  
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6 432 inspected by using the SWT scheme on SimSetB (Figure S3, supplementary materials).  
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8 433 The db2 operates as a  $4 \times 4$  window moving at steps of one pixel: the result is similar to  
9  
10 434 the one obtained by ct-MIA, leading to a blurring of the borders among squares and  
11  
12 435 around the area (of wideness about  $3 \times 3$ ) which is interested by the defect.  
13  
14  
15 436 Finally in Figure 10 the performance of ct-MIA and WT-MIA (SWT, db1) are compared  
16  
17 437 in terms of capability of detection and localization of the faulty pixels for SimSetB  
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19 438 (Figure 10a) and SimSetC (Figure 10b), respectively. The detection is good in both  
20  
21 439 approaches being all the faulty pixels correctly identified, the difference is in the  
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23 440 blurring area, which is strictly connected to the wideness of the analysing window, i.e.  $3$   
24  
25 441  $\times 3$  for ct-MIA and  $2 \times 2$  for db1. This is a general known advantage of WT of being  
26  
27 442 more efficient for feature enhancement because of the availability of several filter  
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29 443 shapes and length compared to spatial shifting approach.  
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## 37 445 4.2 Real Images datasets

### 38 39 446 4.2.1 Tiles

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41  
42 447 Several wavelet filters, belonging to Daubechies (filter length from 1 to 5), Symlet  
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44 448 (filter length from 1 to 5), Coiflet (filter length from 1 to 3) and biorthogonal (1.3 and  
45  
46 449 1.5) families were tested (decomposition levels from 1 to maximum), by using an  
47  
48 450 approach as described in ref. 30. For both BZdataset and BNdataset Daubechies filter  
49  
50 451 length 1 (db1 or Haar) resulted among the best performing wavelet filters and we report  
51  
52 452 results relative to this filter, at decomposition level  $L = 3$ . While for the ct-MIA  
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54 453 approach window size 1 and 2 were considered, better performance was obtained with  $w$   
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4 454 = 1 for BZdataset and  $w = 2$  for BNdataset. This led, considering the three RGB  
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6 455 channels, to an unfolded feature matrix of size  $65536 \times 27$  ( $w = 1$ ), or  $65536 \times 75$  ( $w =$   
7  
8 456 2) in the ct-MIA case, and  $65536 \times 36$  in the WT-MIA case.

9  
10  
11 457 In both datasets, a single reference image has been used to calibrate the PCA models  
12  
13 458 and build the Hotelling's  $T^2$  and Q statistics (control charts). Autoscaling pretreatment  
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15 459 gave for both datasets and approaches the best results.

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17  
18 460 The choice of model dimensionality, i.e. number of principal components, in this  
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20 461 context cannot be automated, i.e. assessed on the basis of a priori fixed criterion, since it  
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22 462 is problem dependent<sup>40</sup>. General guidelines that we adopted in this work consist in: i)  
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24 463 inspecting how spatial features of the image are accounted for in scores images and ii)  
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26 464 scree-plot to ensure the systematic variation is modelled. Further, when enough defects  
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28 465 images are available, to preserve some for model validation, few can be used to see  
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30 466 which are the components that maximize detection capacity. It is worth noticing that  
31  
32 467 minimizing the squared prediction error in cross validation, as most used in PCA  
33  
34 468 modelling, is not appropriate in this context, because it is not necessarily related to the  
35  
36 469 capability of fault detection which is the objective pursued in process monitoring.

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38  
39 470 Image BZN01 has been used as reference NOC image for BZdataset. The PCA model  
40  
41 471 dimensionalities were 4 PC's for both approaches ct-MIA (captured variance 77%) and  
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43 472 WT-MIA (captured variance 44%), which correspond to a number of components each  
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45 473 explaining more than 1% variance (ct-MIA) and to the first minimum in the scree-plot,  
46  
47 474 i.e. number of components vs. eigenvalues plot, (WT-MIA), respectively. All the  
48  
49 475 remaining images of the dataset were projected onto the models and distances were  
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51 476 calculated. Table 1 reports the results in terms of percentage of pixels scoring above the  
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4 477 critical limits, which were chosen on the basis of the reference image by obtaining the  
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6 478 99<sup>th</sup> percentile values of its distances distributions. Both models are able to accept as  
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8 479 normal behaving images such as BZN02 and BZN03, which are actually defectless, and  
9  
10 480 indicate, especially in terms of  $T^2$  distance, the presence of anomalies on all of the three  
11  
12 481 defective tiles, BZD01, BZD02 and BZD03; albeit the results are quite similar, a higher  
13  
14 482 percentage of pixels above the critical limits is detected by the WT-MIA approach. Both  
15  
16 483 approaches show similar results, although the WT-MIA identification of defects appears  
17  
18 484 better defined, especially for image BZD02 where more clusters of pixels are identified,  
19  
20 485 which are in particular connected to the presence of darker spots on the surface of the  
21  
22 486 tile, especially when using SWT (Figure S4, supplementary Material). SWT monitoring  
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24 487 results are also shown on Table 1 and are very close to DWT ones.

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29 488 Interpretation of the features enhancement step can be gathered by loadings analysis. ct-  
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31 489 MIA loadings are shown in Figure 11 (left) both as bar plot (top left) and refolded  
32  
33 490 (bottom left) in the corresponding position of neighbours window (the central pixel is  
34  
35 491 the pixel itself). As usual, PC1 is gathering an (approximately) average colour effect (all  
36  
37 492 loadings have the same sign). Moreover, it can be observed that colour intensity varies  
38  
39 493 left to right for red and blue channels, while green is more uniform; similarly does PC1  
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41 494 of WT-MIA (bar plot, top right, and decomposition sub-images, bottom right), to which  
42  
43 495 the Approximations of all levels and channels contribute (Approximations in fact act as  
44  
45 496 an averaging tool at each decomposition level, hence extracting out the same  
46  
47 497 phenomenon as ct-MIA). Also the WT-MIA Approximations sub-images (Figure 11,  
48  
49 498 bottom right) highlight the varying intensity from left to right, especially for  
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51 499 decomposition levels two and three (the green channel is uniform at level 1). This effect  
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4 500 may be due to illumination and eventually (not the aim here) it could be easily removed  
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6 501 in WT domain, e.g. by suppressing level 2 or 3 approximations as data pre-treatment<sup>29</sup>.  
7

8  
9 502 PC2 and PC3 show the main contrast in horizontal and vertical directions, respectively  
10  
11 503 both for ct-MIA and WT-MIA (for PC2, the Horizontal details of level 3 for all channels  
12  
13 504 are the most relevant, and for PC3 the vertical details, level 1 opposite to level 3). PC4  
14  
15 505 shows a mixed pattern, loadings sign and values vary in all directions for ct-MIA and  
16  
17 506 for PC4 in WT-MIA the vertical details of level 2 are the most relevant.  
18

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21 507 It can be noticed that the possibility to analyse the images at different resolution (the  
22  
23 508 different decomposition levels) enhances the colour-textural pattern recovery, with  
24  
25 509 respect to ct-MIA where only the neighbouring window size can be varied (that in WT-  
26  
27 510 MIA can roughly corresponds to the filter length/family).  
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31 511 *Table 1 to be inserted about here*  
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33 512 As reference NOC image for BNdataset, image **BNN01** has been used, the PCA model  
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35 513 dimensionalities were 2 PC's for both approaches ct-MIA (**variance captured 39%**) and  
36  
37 514 WT-MIA (**variance captured 26%**), which correspond to the first minimum in the scree-  
38  
39 515 plot. All the remaining images of the dataset were projected onto the models and  
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41 516 distances were calculated. Table 2 reports the results in terms of percentage of pixels  
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43 517 scoring above the critical limits, which were chosen on the basis of the reference image  
44  
45 518 by obtaining the 95<sup>th</sup> percentile values of its distances distributions.  
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50 519 *Table 2 to be inserted about here*  
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52 520 Neither of the models appear particularly satisfactory, since the normal behaving images  
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54 521 BNN02 and BNN03, which are defectless, appear to have Q distances higher than 5%.  
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56 522 The defective tiles, BND01 and BND03 appear above limits for both models, according  
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4 523 to  $T^2$  distance statistic. On the contrary, defective BND02 is only detected by WT-MIA,  
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6 524  $T^2$  distance, albeit close to the limit.  
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9 525 By considering the  $T^2$  distance values reshaped in the original pixel domain it is  
10  
11 526 possible to identify the groups of pixels which have distances higher than the critical  
12  
13 527 values. Figure 12a) and 12b) shows the comparison of defective images and the normal  
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15 528 images, with the corresponding distance images for ct-MIA and WT-MIA (DWT). The  
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17 529 WT-MIA identification of defects appears better defined, while ct-MIA seems to find  
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19 530 fewer clusters of pixels and more darker, well separated, spots all over the surface.  
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23 531 In a monitoring context, the results of Table 2 would indicate products BNN02 and  
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25 532 BNN03 as defective (false negatives) and shed doubt on rejecting or not product  
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27 533 BND02. On the other hand the possibility to look at above limits  $T^2$  distance images (or  
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29 534 in general to the images corresponding to the above limit statistic) may clarify if defects  
30  
31 535 are present or not. In particular, this is a case were the defects are mainly due to a non  
32  
33 536 uniform distribution of pixels with a given colour content and texture that if normally  
34  
35 537 distributed on the image, as in the case of BNN01, BNN02 and BNN03, would be  
36  
37 538 acceptable. In this situation, it may be useful to calculate and represent the local  
38  
39 539 entropy<sup>41</sup> of the scores images, were the defective area is a region of low entropy  
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41 540 encircled by high entropy values, as shown in Figure 13.  
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46 541 WT-MIA model based on SWT in this case yielded lower performance.  
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#### 50 51 52 543 4.2.2 Bread

53  
54 544 The Daubechies 2 (db2) wavelet filter was used up to decomposition level 5 and for  
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56 545 both DWT and SWT decomposition schemes. The feature data matrix results of  
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4 546 dimension  $I_{\text{pixels}} \times 360$  (4 blocks  $\times$  5 levels  $\times$  18 channels). In ct-MIA both a window  
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6 547 size of 1 (162 features) and 2 (450 features) were tested. Since results were similar we  
7  
8 548 will discuss the ones corresponding to  $w = 1$ , which gave a better defects localization.  
9  
10 549 The reference PCA model for non-defective image has been calculated by considering  
11  
12 550 as feature matrix the one obtained for N01 image (Figure 6). **The PCA model refers to**  
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14  
15 551 **mean centred data and model dimensionalities were 6 PC's for both approaches ct-MIA**  
16  
17 552 **(variance captured 66%) and WT-MIA (variance captured 52%), which correspond to**  
18  
19 553 **reaching the plateau in the scree-plot.** We tested also a model made on two NOC images  
20  
21 554 but the results were analogous. Q and  $T^2$  statistics were computed, and the critical limits  
22  
23 555 for each of the two statistics were computed on the basis of the 99th percentile. The  
24  
25 556 total percentage of pixels exceeding the critical limits is reported in Table 3. For all  
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27 557 approaches a clear detection of the two defective images can be obtained, with relevant  
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29 558 percentages of pixels above the critical limits for both distances, as well as N02 being  
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31 559 defectless.  
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35 560 However, when the Q and  $T^2$  values above the reference limits are refolded to the  
36  
37 561 original pixel  $\times$  pixel domain to locate the defective areas on the image (Figure 14),  
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39 562 differences among the approaches emerge. ct-MIA is less efficient to detect the  
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41 563 defective area for D07 and for D04, it is also worth noticing that ct-MIA provides these  
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43 564 results when applied on the pretreated images, i.e. after denoising and background  
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45 565 removal with WT; otherwise it detects as faulty only pixels on the borders of the image.  
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47 566 DWT seems more efficient than SWT to locate the faulty pixels, notwithstanding the  
48  
49 567 fact that the same wavelet filter and resolution have been used.  
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53 568 Now focusing on the WT-MIA DWT results, it is worth noticing that not only the  
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55 569 stains, which are also easy to detect visually, but also the blisters and tiny scratches can  
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4 570 be detected. Moreover, we can assess which features are responsible of the defects by  
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6 571 inspecting the  $T^2$ -contributions, which can be interpreted in terms of the spectral  
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8 572 channels. In particular, Figure 15 shows the  $T^2$ -contributions for some of the blisters. To  
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10 573 make the representation clearer distinct plots are made for each decomposition block,  
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12 574 and each decomposition level is represented as a distinct line, so that the x-axis reports  
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14 575 just the channels (wavelengths): the main contributions are from approximations  
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16 576 decomposition levels 1-3. Interestingly, besides the visible channels, some of the NIR  
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18 577 ones (11<sup>th</sup> to 18<sup>th</sup> corresponding to the range from 850 to 970 nm at 20 nm resolution)  
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20 578 contribute, which point to carbohydrate, fat and water bands. This may indicate a  
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22 579 segregation of some of the ingredients on surface spots where blisters appear.  
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## 29 581 CONCLUSIONS

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32 582 The artificial image datasets allowed highlighting the distinct way in which textural  
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34 583 information can be recovered by the ct-MIA and WT-MIA approaches: both are  
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36 584 efficient in depicting the salient pattern of the images and the area where the defects are  
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38 585 located. The main distinctive characteristics of the two methods are:

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41 586 i) the feature matrix obtained by ct-MIA just holds the shifted version of the raw image  
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43 587 thus always requires coupling to a multivariate decomposition technique to highlight  
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45 588 textural patterns while the feature matrix obtained by WT-MIA already captures it;  
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49 589 ii) in general WT-MIA is more efficient for feature enhancement because of the  
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51 590 availability of several filter shapes and length compared to the spatial shifting approach  
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53 591 where only the window size can be varied.  
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4 592 The analysis of the tiles data sets reveals a similar behaviour of the two considered  
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6 593 approaches although identification of defects appears better defined with the WT-MIA  
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8 594 approach. Also both decomposition schemes DWT and SWT show similar performance.  
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11 595 In a monitoring context it is worth noticing that when the defects are due to a non  
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13 596 uniform distribution of pixels, whose colour content and texture if normally distributed  
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15 597 on the image would be instead acceptable, further image analysis tools (e.g. local  
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17 598 entropy or any other to assess homogeneity or heterogeneity of pixels distribution), on  
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19 599 the beyond  $Q$  or  $T^2$  limits images, are required to avoid false negative to be detected.  
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23 600 In the analysis of multispectral images (bread data set) the WT-MIA approach  
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25 601 performed better and it was possible to highlight the full benefit of the proposed  
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27 602 approach from both the correct defects identification/location and interpretation in terms  
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29 603 of spectral features point of view.  
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32 604 A further remark is that the proposed WT-MIA approach is rather straightforward  
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34 605 requiring only the Feature Extraction (Enhancement) and Reduction (Analysis) steps, as  
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36 606 in ct-MIA; one or more NOC images can be analysed at the same time and assembled in  
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38 607 the same WT features matrix which is organized pixels wise, thus allowing defect  
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40 608 localization directly. Images denoising and background removal can be as well  
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42 609 accomplished at WT decomposition stage.  
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46 610 The proposed WT-MIA approach can be as well applied to hyperspectral images, the  
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48 611 bread data set being an example limited to eighteen channels. However, the  
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50 612 computational costs will be a limiting factor and further strategies could be considered  
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52 613 to render it more efficient, work is in progress in this direction.  
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718 **Table 1.** BZDataset. Percentage of pixels above Hotelling's  $T^2$  and Residuals Q  
 719 distances critical limits based on normal image BZN01 99<sup>th</sup> percentile

	<b>ct-MIA</b> <i>w</i> = 1, 4 PCs		<b>WT-MIA (DWT)</b> <i>Daubechies</i> 1, level = 3 4 PCs		<b>WT-MIA (SWT)</b> <i>Daubechies</i> 1, level = 3 4 PCs	
	<b>T<sup>2</sup> distance</b>	<b>Q distance</b>	<b>T<sup>2</sup> distance</b>	<b>Q distance</b>	<b>T<sup>2</sup> distance</b>	<b>Q distance</b>
<b>BZN01</b>	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%
<b>BZN02</b>	0.6%	0.6%	0.6%	0.7%	0.7%	0.6%
<b>BZN03</b>	0.9%	0.9%	0.8%	0.7%	0.8%	0.7%
<b>BZD01</b>	<b>3.6%</b>	<b>1.9%</b>	<b>5.1%</b>	<b>2.8%</b>	<b>4.1%</b>	<b>3.0%</b>
<b>BZD02</b>	<b>1.6%</b>	0.7%	<b>2.5%</b>	0.7%	<b>1.9%</b>	0.9%
<b>BZD03</b>	<b>1.9%</b>	0.9%	<b>2.4%</b>	0.7%	<b>2.1%</b>	0.8%

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722 **Table 2.** BNDataset. Percentage of pixels above Hotelling's  $T^2$  and Residuals Q  
 723 distances critical limits based on normal image BNN01 95<sup>th</sup> percentile

	<b>ct-MIA</b> <i>w</i> = 2, 2 PCs		<b>WT-MIA (DWT)</b> <i>Daubechies</i> 1, level = 3 2 PCs	
	<b>T<sup>2</sup> distance</b>	<b>Q distance</b>	<b>T<sup>2</sup> distance</b>	<b>Q distance</b>
<b>BNN01</b>	5.0%	5.0%	5.0%	5.0%
<b>BNN02</b>	5.0%	6.0%	4.9%	5.7%
<b>BNN03</b>	4.9%	6.6%	4.4%	6.5%
<b>BND01</b>	6.2%	3.8%	6.6%	4.1%
<b>BND02</b>	4.5%	2.5%	5.3%	2.5%
<b>BND03</b>	5.2%	4.3%	6.3%	4.7%

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728 **Table 3.** Bread Dataset. Percentage of pixels above Hotelling's  $T^2$  and Residuals Q  
 729 distances critical limits based on normal image N01 99<sup>th</sup> percentile

	<b>ct-MIA</b> $w = 1, 6$ PCs		<b>WT-MIA (DWT)</b> <i>Daubechies 2, level = 5</i> 6 PCs		<b>WT-MIA (SWT)</b> <i>Daubechies 2, level = 5</i> 6 PCs	
	<b><math>T^2</math> distance</b>	<b>Q distance</b>	<b><math>T^2</math> distance</b>	<b>Q distance</b>	<b><math>T^2</math> distance</b>	<b>Q distance</b>
<b>N01</b>	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%
<b>N02</b>	0.6%	1.0%	0.5%	0.7%	0.9%	0.2%
<b>D04</b>	2.7%	3.4%	3.3%	3.2%	11.9%	11.4%
<b>D07</b>	4.3%	3.0%	4.6%	4.3%	15.7%	14.3%

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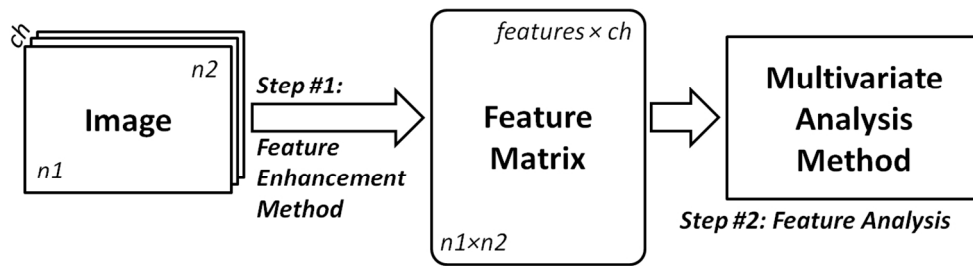


Figure 1: Feature Extraction (Enhancement) and Reduction (Analysis) steps in Multivariate Image Analysis (MIA).

381x102mm (96 x 96 DPI)

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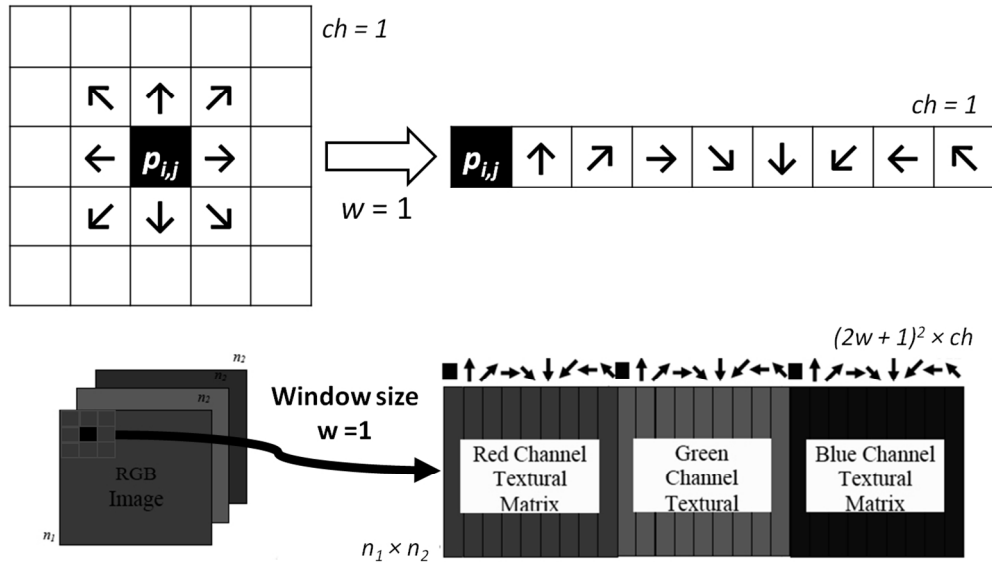


Figure 2: The Colour-textural MIA approach (ct-MIA)

389x222mm (96 x 96 DPI)

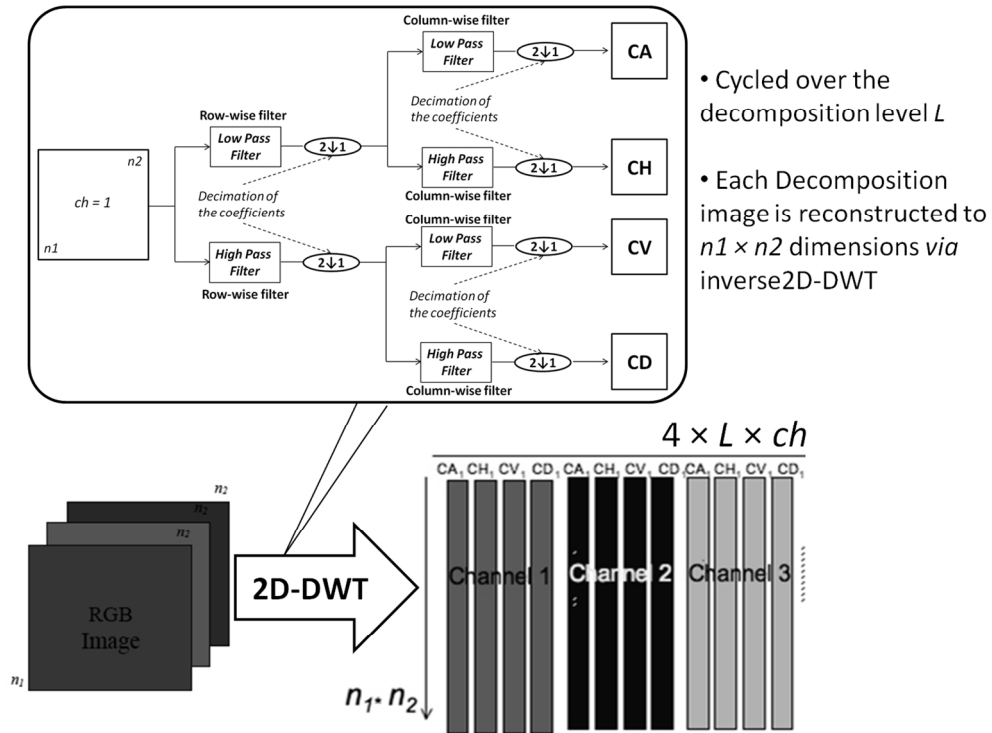


Figure 3: The Feature Extraction (Enhancement) step in the 2D WT-MIA approach, illustrated for an RGB image. The insert on the top of the figure shows the 2D DWT decomposition scheme at the first level of decomposition. CA, CH, CV and CD stand for Approximation, Horizontal details, Vertical details and Diagonal details coefficients, respectively.

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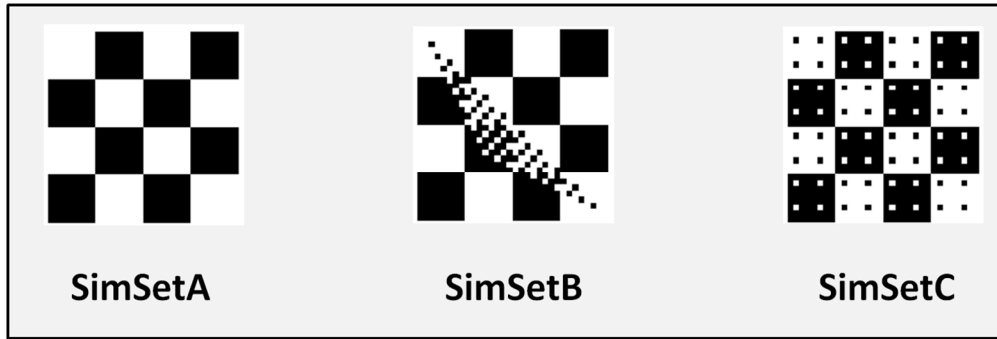
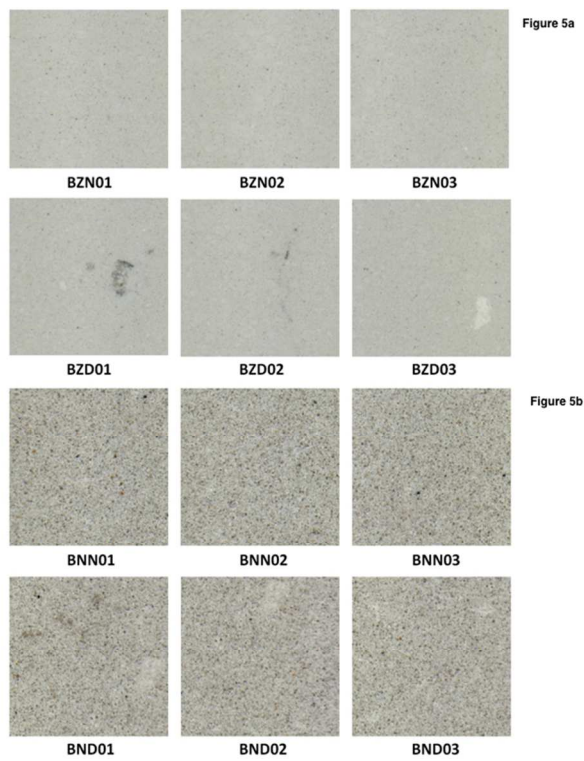


Figure 4: The SimSetA, SimSetB and SimSetC images.

376x127mm (96 x 96 DPI)

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32 Figure 5: top (5a): Blanco Zeus (BZdataset) images; bottom (5b) Blanco Norte (BNdataset) images.

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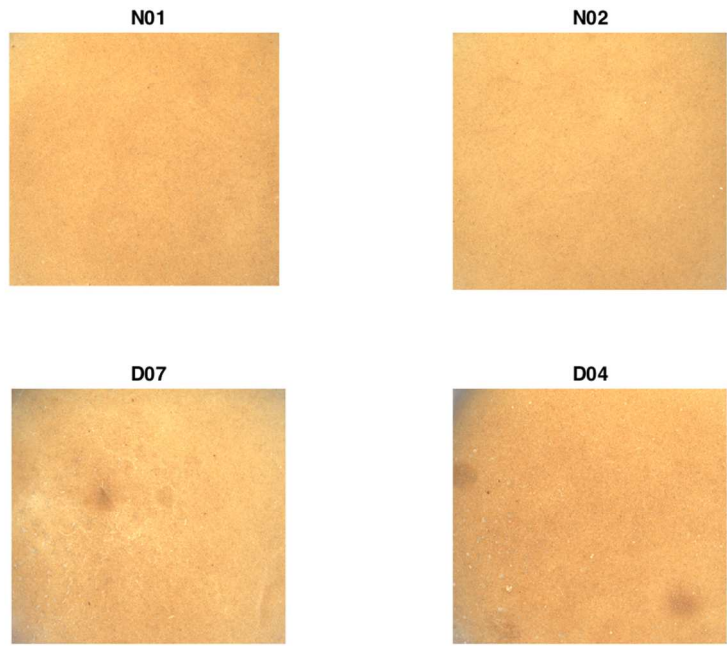


Figure 6: The Bread dataset images

395x296mm (72 x 72 DPI)

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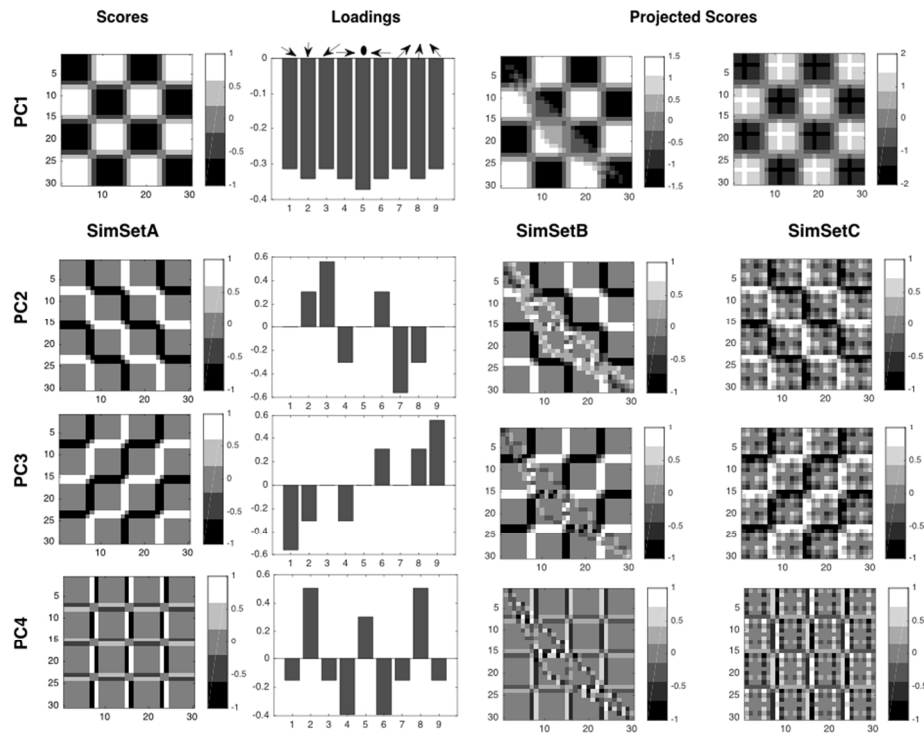


Figure 7: The features reduction (Analysis) step for ct-MIA approach. PCA model built on SimSetA from left to right in the order: scores image, loadings, projected scores images from SimSetB and SimSetC, respectively.

361x270mm (72 x 72 DPI)

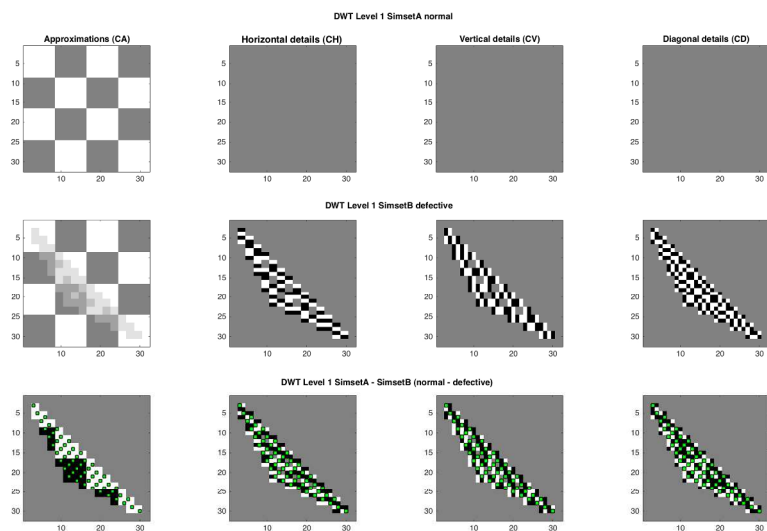


Figure 8: 2D WT-MIA approach. Decomposition blocks for level 1 by using Daubechies 1 (db1) wavelet filter and DWT scheme. In the order, from left to right, Approximations (CA), Horizontal (CH), Vertical (CV) and Diagonal (CD) details, respectively. Top) SimSetA; middle) SimSetB and bottom) difference (pixel by pixel) between SimSetA and SimSetB; small squares (green on on-line version) highlight location of the defective pixels.

903x528mm (72 x 72 DPI)

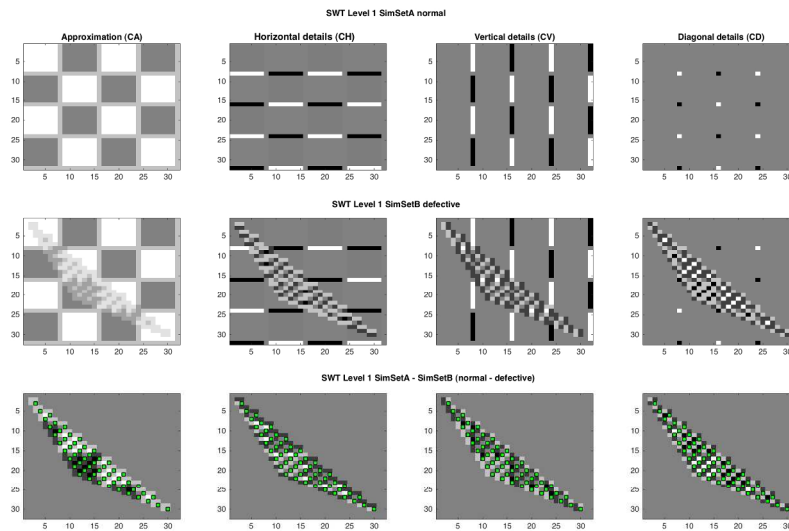


Figure 9: Figure 8: 2D WT-MIA approach. Decomposition blocks for level 1 by using Daubechies 1 (db1) wavelet filter and SWT scheme. In the order, from left to right, Approximations (CA), Horizontal (CH), Vertical (CV) and Diagonal (CD) details, respectively. Top) SimSetA; middle) SimSetB and bottom) difference (pixel by pixel) between SimSetA and SimSetB; small squares (green on on-line version) highlight location of the defective pixels.

903x528mm (72 x 72 DPI)

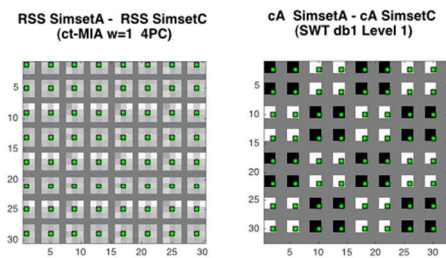
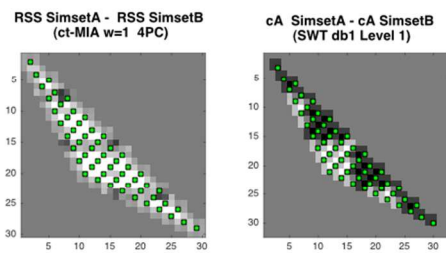


Figure 10: Differences (normal minus defective) among residuals sum of squares (RSS) images for ct-MIA approach (left) and differences among Approximations images from WT-MIA approach (right). Figure 10a (top). SimSetA (normal) minus SimSetB (defective). Figure 10b (bottom) SimSetA minus SimSetC.

361x270mm (72 x 72 DPI)

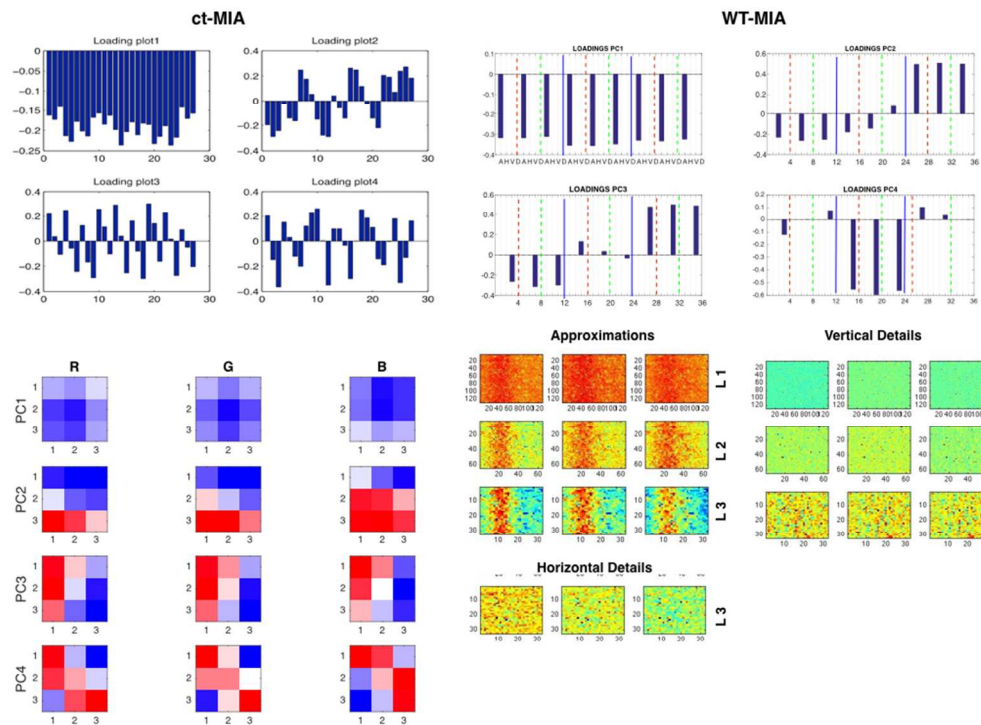
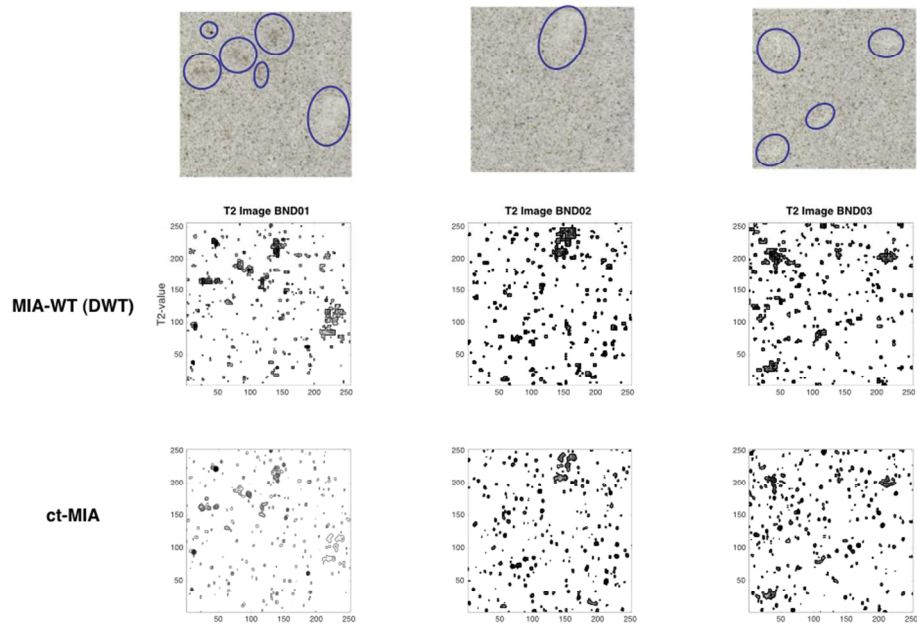


Figure 11: On the right: ct-MIA loadings for BNdataset, as bar plot (top) and as image plot (bottom); on the left WT-MIA loadings (top) and WT images for each decomposition block that it is most contributing to the loadings: Approximations from level 1 to 3 contributing to PC1; Horizontal details, level 3, contributing to PC2; Vertical details level 1 and 3 contributing to PC3 and Vertical details level 2 contributing to PC4.

361x270mm (72 x 72 DPI)



32 Figure 12: (a) Defects detection for BNdaset, from left to right the three defective images BND01, BND02  
33 and BND03, respectively. On top defective images with defect encircled. Middle T2-chart obtained by WT-  
34 MIA (daubechies 1, level 3, DWT scheme). Bottom T2-chart obtained by ct-MIA ( $w = 2$ ).

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view

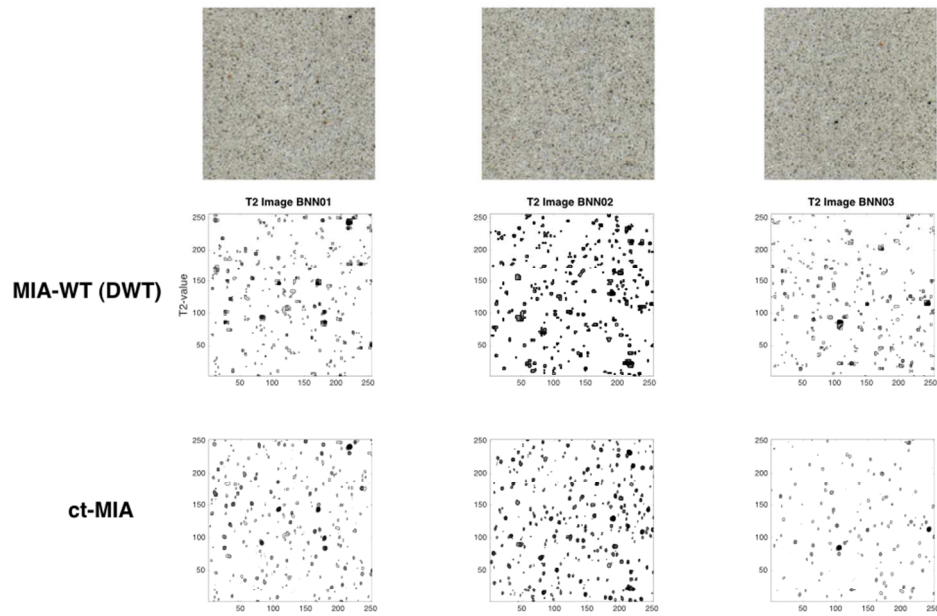


Figure 12: (b) On top normal images; Middle T2-chart obtained by WT-MIA (Daubechies 1, level 3, DWT scheme). Bottom T2-chart obtained by ct-MIA ( $w = 2$ ).!! †

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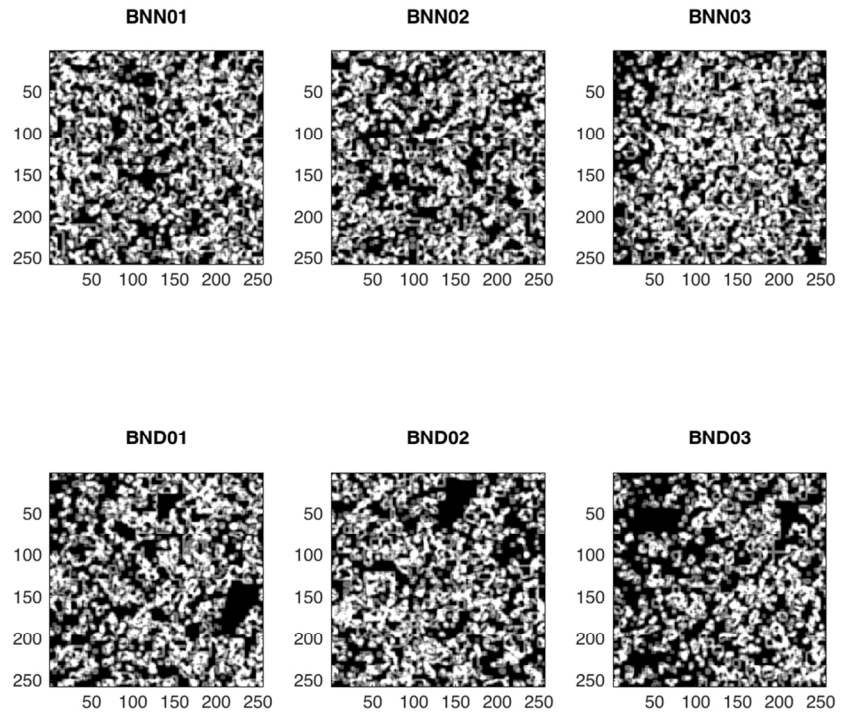


Figure 13: Results of local entropy analysis on scores images of normal and defective images for BNdataset.

398x354mm (72 x 72 DPI)

ew

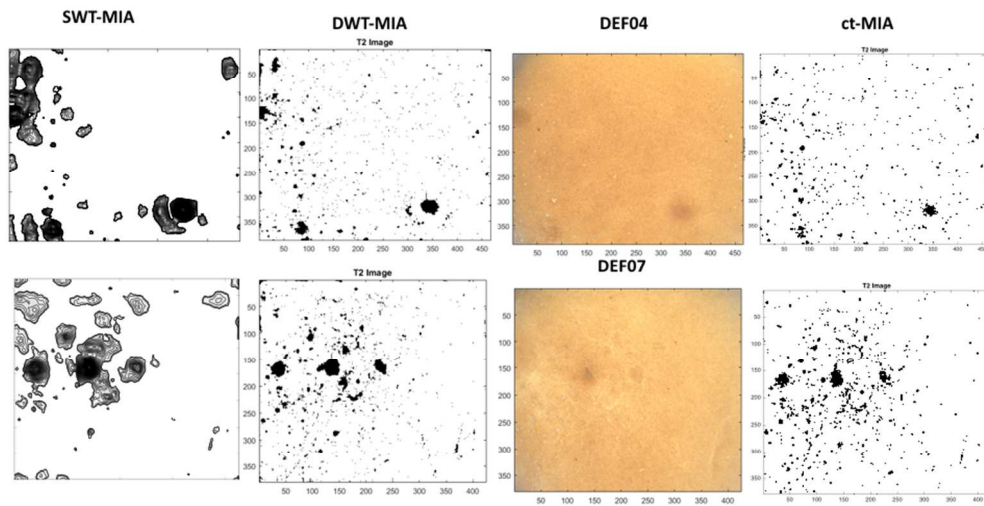


Figure 14: Bread data set: T2-chart images (from left to right SWT, DWT and ct-MIA) for defective images compared with raw images.

338x190mm (72 x 72 DPI)

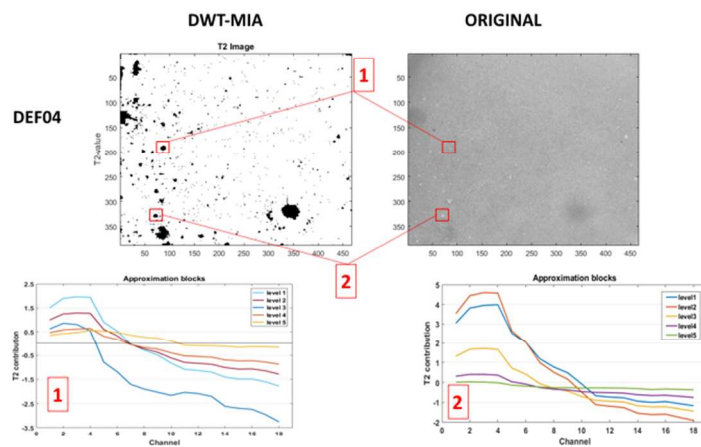


Figure 15: Bread data set, D04 image. T2-chart and contribution plot for some of the pixels highlighted as defective (also shown on the raw image), for clarity only the Approximation block, whose T2-contribution is the most relevant, is shown (each curve represent a decomposition level) on the x-axis are reported the spectral channels.

338x190mm (72 x 72 DPI)