“Going straight”: discussions and experience at primary school

Maria Favilla, Maria Luppi, Michela Maschietto

“Going straight”: discussions and experience at primary school

Maria Elena Favilla[1], Maria Luppi[2] and Michela Maschietto[1],

[1]University of Modena e Reggio Emilia, Italy; elena.favilla@unimore.it, michela.maschietto@unimore.it

[2]Scuola primaria “Madonna Pellegrina”, Modena, Italy; marialuppi8@gmail.com

Our research concerns the relationship between spatial and geometrical knowledge, assuming that it is essential to have information on the children’s stored linguistic and conceptual experience associated with the terms defining geometrical concepts. This paper presents the first phase of a design experiment in which 8-year-old pupils are involved in two discussions about going straight, before and after the experience of leaving footprints. The analysis provides some elements about pupils’ conceptualization when approaching geometrical concepts.

Keywords: Alignment, line, linguistic, space, trace.

Introduction.

In Working Group 4 at CERME10 (Jones, Maschietto, & Mithalal-Le Doze, 2017) various papers focus on the relations between spatial sense, spatial knowledge and geometrical knowledge. For instance, Houdement (2017) discusses the different perspectives in which research on the relation of a subject with the world is carried out, highlighting the use of non homogeneous terminology. Other authors, as Vandeira and Coutat (2017), point out the institutional request of working on shapes and space from a perceptive approach at kindergarten and a subsequent rupture when the attention is put on geometry, above all between primary and secondary school education. In their work, they propose tasks of shape recognition in different spaces (micro-space and meso-space, following Berthelot and Salin, 1993), taking into account semiotic and manipulative aspects.

In our work, we are interested in the analysis of pupils’ construction of geometrical concepts and in the relationship between spatial and geometrical knowledge (Berthelot & Salin, 1993) at primary school in real-world experiments recalled in the call for paper of this WG4. We propose a point of view on tackling geometry at primary school, which takes into account the ways in which people communicate and think about space and shapes, both verbally and nonverbally, in order to study how to enhance geometrical understanding. For this reason, we refer to the theory of word learning (Tomasello, 2003), in which learning can be facilitated by the “exposition” to a wide range of linguistic contexts and experiences that elicitate those contexts, in order to enrich the children’s linguistic and conceptual experience and favor the construction of increasingly richer and connected semantic representations.

This paper presents a preliminary analysis of a case study, corresponding to the first phase of a design experiment on points and lines, planned in cooperation with a teacher of Italian at grade 3 of an Italian primary school.

Research perspective.

Several studies in mathematics education concern the analysis of cognitive processes involved in mathematical thinking, paying increasingly more attention to the role of movement – considered in

a large sense, not only in gesture by hands – and conceptual constructions – as metaphors, in the learning and teaching of mathematical concepts. For instance, Nemirovsky (2003) assumes an indissoluble link between perception, motion and thinking, highlighting that perceptuo-motor activity is a constitutive element in the conceptualization of mathematics. This is reinforced by works based on a multimodal approach to mathematics knowledge and on embodied mathematics (Arzarello & Robutti, 2004). Radford (2009) highlights that mathematical meanings are constituted of actions accomplished by the subjects on material objects: thinking also occurs through a semiotic coordination of speech, body, gestures, symbols and tools. These perspectives involve an epistemological dimension related to the debate on mathematics foundations.

The next two sections present some basic components of our theoretical background: epistemological elements for mathematics, in particular for point and line, and a theory of word learning.

Mathematics.

In his reflection on mathematics foundation, Longo (2016) proposes to avoid an axiomatic vision of mathematics, arguing that mathematics is primarily the analysis of the invariants and transformations that keep them. Following Poincaré's philosophical position, Longo states that mathematics is not grounded on arbitrary conventions, but that these conventions correspond to the most convenient choices for human beings, who live in a certain world with a specific biological structure. In this sense, mathematical concepts are constructed in the interaction between us and the world around us. In particular, Longo (1997) discusses the origin of geometry, which is considered

> as a science of action and of *prediction* of movement in space: line segments, curves, circles are neither the "abstract form" (?) of a material object, nor "ideal shapes" (?), but rather the *prediction* of a path. And *a prediction is in itself an abstraction; the trajectory that is predicted or anticipated by gaze and gesture* is *abstract* (*Ib*., p. 29, italic in the original text; our transl.).

In the analysis of the conceptualisation of line and point, Longo (2016) claims that: "The "big-bang" of Western mathematics is perhaps in Euclid's Definition β: *A line is a breadthless length*" (*Ib.*, p. 1, our transl.). He also suggests that it is possible to reconstruct the gesture at the core of this invention:

> Such a notion of line can only be told and written, in the language, and it is the result of a philosophy of ideas, but its sense, its continuity is a practice of gesture, of drawing: if we want to understand it, we have to restart the sense, go back to the line drawn by the first teacher on the blackboard, to her/his gesture-trajectory in the space (*Ib*., p. 2, our transl.).

Concerning the meaning of point, Longo asserts that the point in Euclid's Elements is considered as a position on a line or as the intersection of two lines, a letter of the alphabet at the ends of a segment. In this perspective, the line is not necessarily made of points, as for Cantor, since the points are "put by the geometer or obtained from lines" (*Ib*., p. 3, our transl.). This type of analysis allows to shift the attention to the structure: the line and the point-sign are the structures at the origin of geometry. They are borderline objects, away from every tangible experience, but they exist in their relation to space referring to the experience. In his writings, Longo suggests a different

perspective on mathematics, focusing not only on the objects but also on the processes of their construction.

Giusti (1999) supports the strong assumption that mathematical objects are derived by a process of "objectification of procedure" (*Ib.*, p. 26). In this hypothesis, the line could be derived by the procedure of pulling a rope between two pegs, following the rope on the ground by a stick, and the points as the positions of the two pegs on the ground.

In the educational perspective of integrating the perceptive-motor and symbolic-reconstructive modalities, Arzarello, Danè, Lovera, Mosca, Nolli, and Ronco (2012) suggest to expose students to the cognitive and cultural roots of mathematical concepts. In the analysis of the concept of line they identifie three roots: symmetry, going straight and the shortest line between two points/positions.

Linguistics: context and contrast in the "miracle of word learning".

The geometrical concepts of point and line can be considered in a continuum with other concepts associated to the same terms in different contexts, as well as in actions and experiences of everyday life. In fact, when 8-year-old children meet these lexical labels at school during their first geometry lessons, they do not hear them for the first time, as they have already met them in various everyday expressions and situations, both in literal and figurative sense. Even without having any idea of what geometry is, they should for instance have some kind of conceptualization connected to the word *punto* (point), not only because they have probably learnt that it is used to refer to those small black marks that in English are called dots and spots, but also because they have met it when learning to write letters and to use punctuation (in Italian the dot on the vowel 'i' is called *puntino* and the word "point" is used in many punctuation marks, such as full stop or period, colon, semi-colon, exclamation and question mark, etc.), and also because of the wide use of this word in many frequent expressions, such as "meeting point", "starting point", "take stock of the situation" (*fare il punto della situazione*).

If we consider how children and human beings in general learn to associate meanings and concepts to lexical labels, we should not undervalue the fundamental role of the exposition to the different contexts in which words are used: as Tomasello (2003) points out, rarely do children learn words in pointing-and-naming games, with words proposed isolated from other words, as during a vocabulary lesson in a foreign language classroom. And this is particularly true if we consider that vocabulary learning does not include only concrete and content words, clearly associable to a picture, but a wider range of words and expressions, such as, for instance, prepositions or articles, and also abstract words and words used in a metaphorical way. In the process that Tomasello defines the "miracle of word learning" (*Ib.*, p. 44), most of the words are learnt by experience in everyday interactions with adults, and the children hear them used in different types of utterances and situations, mixed with other words and expressions and without special explanations by the adults. According to the so-called social-pragmatic theory of word learning adopted by Tomasello, this miracle is allowed by a series of factors. Among these, one important factor is the children's ability to conceptualize different aspects of their experiential worlds. In later stages of word learning, when the task becomes more complex and concerns more difficult words and expressions, two other factors become crucial: "context" and "contrast". On the one hand, children learn quite

early to exploit their knowledge of the other words present in the utterance in order to understand words that they do not know; on the other hand, they also learn to use their knowledge of other words present in the utterance that might contrast with a word they do not know.

Research questions.

Research in mathematics education investigates the relationships between physical and geometrical space, highlighting a gap in the teaching practice between an approach by physical experiences and the entrance in geometry (Kuzniak, 1995). This seems evident in some Italian textbooks, in which there is a sort of "Euclidean entrance" into the world of geometry, through the definition and representation of the fundamental geometrical entities.

Our hypothesis is that some expressions used in geometry teaching are associated with expressions and experiences in mathematical and non-mathematical contexts. In this perspective, it is essential to have information on the children's stored linguistic and conceptual experience associated with the terms that define these geometrical concepts, considering that their conceptual representations do not necessarily coincide with the adults' representations. This investigation is not generally carried out when the geometrical concepts are introduced to the pupils. Our broad research questions are: (a) what are the linguistic and conceptual experiences associated with geometrical concepts in 3-grade pupils before the Euclidean entrance discussed above? (b) how could those elements emerge? In other words, what kind of experiences can foster the pupils' exposition to the different graphical, verbal and gestural representations associated with geometrical concepts?

Our working hypothesis is that the emergence of these elements can be fostered by offering educational situations in which the pupils can work in different spaces (micro, meso and macro, Berthelot & Salin, 1993), not necessarily with the teacher labelled as the mathematics teacher.

Methods.

We have planned a design experiment composed of five phases, with physical experiences and collective discussions about them. According to the theoretical framework, after every new experience carried out with the class, the pupils should be helped to reflect and discuss on what they have done, seen and learnt, so as to re-order their mental representations and link their new conceptual experiences to their linguistic and conceptual knowledge. For this reason, class discussions are considered the central focus of all the activities that we propose.

The first phase, upon which we focus in this paper, is dedicated to "going straight". It has been opened by a collective discussion on the pupils' personal perception on the meaning of going straight, then the pupils have been invited to experience their ability to go straight, by leaving their footprints on a roll of paper using tempera, and finally they have been taken back to the initial question in a collective class discussion. The footprint activity has been carried out in the hall of the school, corresponding to a meso-space for the pupils. The other phases were: activities with the Bee-bot used to program paths (line and polygonal) (Phase 2); finding the shortest path between two positions by using a rope and two pegs in the macro-space of the school garden (Phase 3); activities of alignment (Phase 4); a session with a perspectograph in the lab of mathematical

machines (mmlab.unimore.it) on the meaning of point of view and of line as modelling the visual ray (Phase 5).

The activities were carried out in two classes (42 pupils, Grade 3, ages 8-9) in the second half of the school year. They were videotaped and all the pupils' worksheets have been conserved. All the sessions were managed by the same Italian language teacher of the two classes, both because we were interested in the pupils' linguistic experiences connected to geometrical concepts and we wanted to avoid that the activities were labelled as mathematical activities.

Results and discussion: "going straight" before and after the experience.

As anticipated, this paper focuses on Phase 1 of the design experiment. In the analysis we investigate the pupils' representations associated with the expression "going straight" emerging in the two discussions before and after the experience of leaving their footprints on paper.

The discussion preceding the experience is opened by the teacher's question "If I say 'going straight', what does it raise in our minds?". In the discussion the pupils describe going straight mainly as going forward without turning, changing direction, making or meeting curves, stopping, "splitting" the pathway, or without interruptions and keeping walking ignoring possible obstacles (see Figure 1 for some examples and their translation).

S.:	andare avanti però mai girare […] avanti senza interruzioni mh senza girarsi	going forward but never turning […] forward without interruptions mh without turning around
M.:	per me andare dritti è camminare con un criterio senza fare curve e senza fermarsi mai	to me going straight is walking with a criterion without making curves and never stopping
D.:	per me andare dritti significa che non bisogna mai cambiare la direzione e non fare curve cioè mh andare sempre in avanti	to me going straight means that we must never change direction and make curves that is mh going always forward
L.:	per me andare dritto significa non fare curve	to me going straight means not making curves
J.:	per me dritti vuol dire che andiamo dritti senza fare curve neanche ne neanche spezziamo la nostra via	to me straight means that we go straight without making curves nor e even splitting our pathway
M.:	per me dritto vuol dire andare sempre dritti senza incontrare curve	to me straight means going always straight without meeting curves
P.:	per me andar dritto è come se tu camminassi e però davanti a te ci sono degli ostacoli però tu continui a camminare […] e continui ad andare dritto	to me going straight is as if you walked but in front of you there are obstacles but you keep walking […] and keep going straight

Figure 1. Examples of definitions of "going straight"

The pupil who first enters the discussion is the only one who mentions a starting and arriving point, talking about a person going from one point to another. Besides, only one of the pupils, F., mentions the use of a ruler, by comparing natural entities such as a lake or a river, which "are round or make curves", to lines drawn with a ruler, which "are not curves": "to me going straight means … for instance … let's look at a lake or a river which are round or make curves … on the contrary when you use a ruler you draw a straight line … it is not a curve".

Most of the descriptions, therefore, seem to suggest the children think about going straight focusing their attention on the movement in itself and on what they have in front of them when they go straight, more than on the (concrete or ideal) line that one draws when he/she goes straight. These kinds of description are consistent with our theoretical background: the conceptualization of going straight is not mediated yet by school geometry and it is connected to everyday experiences and to

body movement. In particular, on one side, it reflects a conceptual representation connected to an embodied experience which involves an interaction between us and the world around us, on the other side, from a more linguistic point of view, it recalls everyday expressions such as "going straight home", as a synonym of "going directly", and "going straight to the point", that is without interruptions or deviations. It is in this sense, in our opinion, that expressions such as "interruption" or "obstacle" should be interpreted in the pupils' definitions.

It is quite important for the teacher to collect this kind of information, because it provides a starting point upon which the following part of the experience should be based and built. Even if here we have no space to go into the details of the footprint activity, aimed at helping the pupils to experience their ability to go straight, we would like to mention that the activity starts in fact from the teacher's invitation to see what happens and what one does when he/she tries to go straight, and that during the activity the teacher progressively tries to lead the pupils to shift their attention from the movement in itself to the traces left with the colored footprints.

Another aspect that can be mentioned here only briefly is the children's use of gestures. Even if we have noticed that they generally tend not to use gestures, in some few cases they do, and their gestures help to understand their conceptions even in the absence of labels and other clear verbal expressions. The most interesting example concerns D., who intervenes in the discussion at two different points, in both cases helping himself with gestures, once adding also some "sound effects", probably to convey swiftness as well as straightness. So, when D. says that going straight means that you should never change direction nor make curves and keep going forward (see D.'s intervention in Figure 1), while saying "keep going forward" he moves his right arm forward, upon the school desk, with flat hand and lateral palm. Then, quite interestingly, when F., speaking immediately after him, compares round and curvy lakes and rivers to straight lines made by a ruler, D. echoes F.'s definition of round and curvy by drawing an imaginary circle on the desk with his right forefinger and accompanying this movement making a circle also with his shoulders, while he puts both hands in a flat position with the palms oriented one towards the other when F. mentions the ruler. Finally, towards the end, D. enters the discussion again to give a specification about the meaning of going straight as not making curves, putting together the concepts of "curve" and "direction": "that is eh, .. you don't have to make round curves, but you can do so [...] doing the tips as well" (D.). The two gestures that he produces as if drawing on the desk to illustrate what one cannot do (round curves) and one can do (tips) make it clear that he has in mind the distinction between a round line and the representation of a polygonal with points, in which the change in direction corresponds to the meaning of angle. Even if the teacher has no time to deepen this aspect, she asks the pupil to describe how the tips are and D. limits himself to repeat the imaginary drawing, this time in the air and not on the desk, maybe in order to be seen better by the teacher, but adding a level of abstraction.

In the class discussion following the experience, the teacher starts with the same question she had asked in the discussion preceding it and tries to induce the pupils to relate their understanding of going straight with the activity just carried out.

After the experience, going straight is defined mainly as “without curves”, with its variants “crookedly” and “without zigzagging”, or as “without interruption”.

In the discussion, the teacher fosters the interpretation of “without interruption” by relating it to the two different kinds of footprints left by the pupils, attached one to each other so as to trace a line, or far from each other as in a normal walking. This way, the experience helps the teacher to shift the attention from the straight movement in itself to the trace of its trajectory and to the footprint line, which one girl then defines as “a line made by our footprints” (*è una linea fatta dall’impronta dei nostri piedi*).

The change of direction and the representation of a polygonal are, in a certain way, more specified than in the discussion before the experience, even if only one pupil explicitly refers to the angle. The main element in the pupils’ comments concerns keeping going straight after the turning point and being able to “go straight in many different directions” (*puoi andare dritto in tante parti diverse*). In this case, the interruption is interpreted in terms of changing direction.

Moreover, the work on the meso-space provides also a more tangible representation of going straight which can be used to introduce new kinds of reflections. For instance, with reference to the presence of an obstacle in the direction of movement, invited by the teacher to recall the “special solution” proposed by one girl during the activity in order to go straight on a road presenting an interruption, the pupils explicitly speak about “going back” and about “going forward and backward” followed by the specification “on the same line”.

The experience of going straight leaving footprints provides, therefore, a spatial context in which the movement can be seen and performed in the same way at different times (during and after the experience). This graphical object could have been exploited even more by the teacher, for instance to control if the movement was straight or not by checking the alignment of pupils’ footprints. It is a task that could be proposed in a next experiment.

In this discussion, the line comes out also to describe one of the strategies used during the experience to go straight. When walking on the paper the kids did not control the trace directly by looking at it, but seemed to rely more on their bodies and sense of equilibrium, on the help of other pupils or on other. One of the pupils, invited by the teacher to recall what he had said during the experience about a line he was following, says that while he was walking he could “see the line in his head” because he was “imagining something like a road next to him” and then he was “staring at a fixed element, a line, which had the same shape as a road”.

This idea of imagining a line in your mind may imply a greater engagement of cognitive control, which two pupils seem to refer to when they mention the difficulty of the task of going straight and the need for concentration when doing it, compared to when making curves.

Finally, to make at least one reference to gestures, we would like to mention the control of alignment by view spontaneously proposed by F., who is the pupil who had already used many gestures in the initial discussion. During the experience he had described (and shown) his strategy to verify the straightness of the footprint line on paper by aligning in front of one eye his outstretched arm and the corresponding shoulder, with the other eye closed. In the discussion after

the experience the teacher invites him to recall that gesture and the same gesture is then imitated by another kid.

Concluding remarks.

The analysis of the two discussions preceding and following the experience of leaving footprints on paper in the first phase of the design experiment provides various data on the conceptual representation of "going straight" that 8-year-old children might have before starting to study geometry and on how these representations, together with an experience which involve their bodies and the meso-space, can provide the basis to reflect on the concept of going straight and begin to introduce the concept of line.

Acknowledgment

This research was funded by the University of Modena e Reggio Emilia: *FAR2016 Prospettive, cornici e punti di vista: per una didattica interdisciplinare nella scuola del primo ciclo*.

References

Arzarello, F., Danè, C., Lovera, L., Mosca, M., Nolli, N., & Ronco, A. (2012). *Dalla geometria di Euclide alla geometria dell'Universo*. Collana Convergenze. Milano: Springer.

Arzarello, F., & Robutti, O. (2004). Approaching functions through motion experiments. *Educational Studies in Mathematics*, 57(3), 305–308.

Berthelot, R., & Salin, M.-H. (1993). L'enseignement de la géométrie à l'école primaire. *Grand N*, 53, 39–56.

Giusti, E. (1999). *Ipotesi sulla natura degli oggetti matematici*. Torino: Bollati Boringhieri.

Houdement, C. (2017). Traveling in spatiality, in spatial sense. In T. Dooley and G. Gueudet (Eds.), *Proceedings of CERME 10* (pp. 621–628). Dublin: DCU Institute of Education and ERME.

Jones, K., Maschietto, M., Mithalal-Le Doze, J. (2017). Introduction to the papers of WG4: Geometry Education. In T. Dooley and G. Gueudet (Eds.), *Proceedings of CERME 10* (pp. 560–563). Dublin: DCU Institute of Education and ERME.

Kuzniak, A. (1995). L'enseignement de la géométrie en formation initiale. *Documents pour la formation des professeurs d'école en didactique des mathématiques* (pp. 81–89). Angers: COPIRELEM. Retrieved from http://www.arpeme.fr/documents/546758FA4F122788C679.pdf

Longo, G. (1997). De la cognition à la géométrie. Retrieved from https://www.di.ens.fr/users/longo/files/PhilosophyAndCognition/cogni-geometrie.pdf.

Longo, G. (2016). Le conseguenze della filosofia. In R. Lanfredini e A. Peruzzi (a cura di), *A Plea for Balance in Philosophy* (vol. 2, pp. 17–44). Pisa: Edizioni ETS. Retrieved from https://www.di.ens.fr/users/longo/files/Le-conseguenze-filosofia.pdf.

Nemirovsky, R. (2003). Three Conjectures concerning the Relationship between Body Activity and Understanding Mathematics. In: N.A. Pateman, B.J. Dougherty & J.T. Zilliox (Eds.),

Proceedings of the 27th Conference of the IGPME (vol. 1, 103–135). Honolulu: University of Hawai‘i.

Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics*, 70, 111–126.

Tomasello, M. (2003). *Constructing a language. A usage-based theory of language acquisition*. MA: Harvard University Press.

Vandeira, C., & Coutat, S. (2017). Shapes recognition in early school: How to develop the dimensional deconstruction? In T. Dooley and G. Gueudet (Eds.), *Proceedings of CERME 10* (pp. 677–684). Dublin: DCU Institute of Education and ERME.