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**Strategic Effects of Investment and Private Information:  
The Incumbent's Curse**

Luigi Brighi \*, Marcello D'Amato \*\*

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\* University of Modena and Reggio Emilia  
RECent, Centre for Economic Research  
Address: Viale Berengario 51, 41121, Modena, Italy  
Email: [luigi.brighi@unimore.it](mailto:luigi.brighi@unimore.it)

\*\* University of Salerno  
Dipartimento di Scienze Economiche e Statistiche, Celpe, Csef  
Address: via Ponte Don Melillo 80144, Salerno, Italy  
Email: [damato@unisa.it](mailto:damato@unisa.it)

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**Dipartimento di Economia Marco Biagi**  
Università degli studi di Modena e Reggio Emilia  
Via Berengario 51 | 41121 Modena  
tel. 059 2056711 | fax. 059 2056937  
[info.economia@unimore.it](mailto:info.economia@unimore.it) | [www.economia.unimore.it](http://www.economia.unimore.it)

# Strategic Effects of Investment and Private Information:

## The Incumbent's Curse<sup>a</sup>

Luigi Brighi<sup>b</sup>

Marcello D'Amato<sup>c</sup>

December 2017

### Abstract

We study a two-period entry model where the incumbent, privately informed about his cost of production, makes a long run investment choice along with a pricing decision. Investment is cost-reducing and its effects are assumed to differ across incumbent's types, as a result investment plays a double role as a commitment variable and, along with price, as a signal. We ask whether and how investment decisions allow the incumbent to limit entry into the market. We find that the incumbent will never undertake strategic investment to deter profitable entry, because when incumbent's costs are private information the signaling role of investment cancels out its value of commitment.

*JEL Classification Numbers:* D58, L51.

*Keywords:* Entry deterrence, commitment, limit pricing, multiple signaling.

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<sup>b</sup>Dipartimento di Economia "Marco Biagi", RECent, Università di Modena e Reggio Emilia, viale Berengario 51, 41121 Modena, Italy, e-mail: luigi.brighi@unimore.it.

<sup>c</sup>Dipartimento di Scienze Economiche e Statistiche, Celpe, Csef, Università di Salerno, via Ponte Don Melillo 80144 Salerno, Italy, e-mail: damato@unisa.it.

# 1 Introduction

We study a two-period entry problem where the incumbent has the opportunity to make a long-term investment choice along with a pricing decision. The incumbent is privately informed about his cost of production and investment is cost-reducing. The level of investment is decided in the first period and affects current and future costs. We suppose that the cost reducing impact differs across types of incumbent, a key assumption suggested by the heterogeneity of production costs.

The originality of the model we set out to study in this paper rests on the double role played by investment. Being observable and having persistent effects, investment has a commitment value as in the classical models studied by Spence (1977) and Dixit (1980). On the other hand, having different cost reducing effects across types, investment is also a signal of incumbent's costs along with price. Our primary interest is to understand how the double role of investment, as commitment variable and as a signal, affects incumbent's behaviour. In particular, we are interested to know how and to what extent the commitment value of investment can be used by the incumbent for entry deterrence purposes, in combination with predatory pricing, in an environment with private information. Answering these questions is relevant for both competition policy analysis and the study of managerial strategies in imperfectly competitive sectors.

The structure of the entry problem is standard. First, Nature selects incumbent's costs which can be either low or high. After observing his cost, the incumbent makes a choice about first-period output (or equivalently price) and investment. Then, after observing incumbent's choice, a potential entrant decides whether to enter or not into the market. If entry takes place the entrant learns incumbent's cost and the firms compete à la Cournot. Otherwise, the incumbent remains a monopolist. This entry problem is formalized as a game with multiple signals. The notion of Perfect Bayesian Equilibrium is used as the solution concept and the Intuitive Criterion is applied to refine equilibria.

Our major result is that an incumbent who is privately informed about his costs can not take full advantage of investment, in fact he will never invest strategically to deter profitable entry. Our analysis shows that there exist pooling equilibria where, as in Milgrom and Roberts (1982), the

incumbent achieves deterrence. However, the most plausible outcome of the game is that the inefficient incumbent will invest to accommodate entry while the efficient one will cut down investment to zero and charge a limit price to signal his cost to the potential entrant, who will stay out. Under no circumstances profitable entry can be deterred, because the commitment value of investment to the inefficient incumbent is cancelled out by its signaling role to the efficient type. This conclusion follows from the formal analysis of the game where it is shown that there is a unique separating equilibrium which survives the Intuitive Criterion while all the pooling equilibria fail this test.

The basic insight of our result is that the opportunity to invest in an environment characterized by private information is a sort of *incumbent's curse* as it prevents the inefficient type from using investment as a commitment device. Although the opportunity to invest may raise the incentive to mimic by the inefficient incumbent and may allow him to behave strategically, the heterogeneity of the cost-reducing impact of investment makes revelation of costs cheaper to the efficient type and it is this latter force that drives the equilibrium outcome. The major policy implication of this result is that, in the presence of private information, limit pricing strategies are not likely to support deterrence of profitable entry when the incumbent has the opportunity to invest.<sup>1</sup>

Our paper contributes to the vast literature on business strategies originated from the early work by Spence (1977), Dixit (1980), Fudenberg and Tirole (1984) and Bulow, Geanakoplos and Klemperer (1985) among others. Our model extends previous analysis because, by introducing entrant's uncertainty about incumbent's cost and an heterogeneous impact of the cost reducing investment, it allows for a deeper understanding of the interplay between pricing and investment decisions.

Our model is also strictly related to the contribution by Milgrom and Roberts (1982) and the literature on multiple-signal games. Early contributions are the work by Milgrom and Roberts (1986), who studied pricing and advertising decisions as signals of quality, and the models of entry

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<sup>1</sup>Notice that this result does not necessarily mean that limit pricing will never limit entry when the incumbent has the opportunity to invest. As shown in Brighi, D'amato and Piccolo (2005), Brighi and D'Amato (2014) and In and Wright (2017), limit pricing may still have deterring effects on entry when investment is not publicly observable by potential entrants.

by Bagwell and Ramey (1988) and Bagwell (2007), where pricing and advertising are used by the incumbent to signal information about costs. Our game, however, differs from the above models because investment, unlike advertisement, has persistent effects. A broader interpretation of investment as advertising with a permanent effect on the installed base of customers, is consistent with the basic features of our model, which may be used to extend the analysis of Bagwell and Ramey (1988) to the case in which advertising, affecting post entry as well as pre-entry market demand, has a commitment value. Finally, we also notice that Bagwell (2007) obtains a result which is analogous to our finding as he shows that limit pricing and advertising do not deter profitable entry. However, his result rests on a completely different kind of analysis and specifically on the assumption that the entrant's response to signals is not binary, but smoothly changing with beliefs.

The rest of the paper is organized as follows. Section 2 sets out the entry problem and the signaling model is formalized in section 3. Section 4 contains the equilibrium analysis, while the last section provides a summary and some final remarks. All the proofs are in the Appendix.

## 2 An entry problem with investment and private information

We consider a standard two periods entry model where an incumbent firm faces the potential entry of a competing firm in a market for a homogeneous good. In the first period firm 1, the incumbent, who has private information about his costs of production, decides how much to produce,  $q \geq 0$ , and how much to invest in a cost reducing technology,  $e \geq 0$ . In the second period firm 2, the entrant, after observing incumbent's choice, decides whether to enter into the market. If entry occurs, firm 2 pays an entry cost, learns the incumbent's production costs (learning upon entry) and firms compete à la Cournot. Otherwise, firm 1 remains a monopolist and the potential entrant gains her outside option normalized to zero.

Marginal costs are constant and depend on investment, while fixed costs of production are set to zero for convenience. There are only two types of incumbent, type L with a low marginal cost (efficient incumbent) and type H with a high marginal cost (inefficient incumbent). We denote by  $\theta_t(e) \geq 0$  the marginal cost of type  $t$ , with  $t = L, H$ , hence  $\theta_H(e) < \theta_L(e)$ . The symbol  $\theta_t$  stands for

$\theta_t(0)$ . The prior probability that the incumbent is inefficient is denoted by  $\beta$ .

Heterogeneity of marginal costs suggests that the cost-reducing impact of investment differs across types of incumbent. In fact, as the more efficient firm is closer to the technological frontier, it is unlikely that his chances to enhance cost efficiency are the same as those of the other firms. It is reasonable to assume that the same level of investment is more effective in cutting costs if it is undertaken by the inefficient incumbent rather than by the efficient one. Indeed, being further away from the technological frontier the inefficient incumbent can more easily catch up with technological progress. To simplify the analysis, we will assume that the cost-reducing effect of investment for the efficient incumbent is negligible and precisely that  $\theta_L(e) = \theta_L$  for all  $e$ . The inefficient incumbent, instead, has the opportunity to reduce the cost gap with respect to type L by investing. The cost reducing technology of the high cost type,  $\theta_H(e)$ , is represented by a strictly decreasing differentiable function, with  $\theta_H(e) > \theta_L$  for all  $e$ . The most important implication of assuming heterogeneity in the cost-reducing effect is that investment may be used by the incumbent to signal his cost of production.

Investment is made by the incumbent in the first period and affects current as well as future marginal costs. Having long-term effects, investment enables the incumbent to influence post-entry competition, hence it has a commitment value. In other words, the incumbent can invest strategically in the first period in order to limit entry of firm 2 in the second.

Market demand in each period is described by an inverse demand function,  $p(q)$ , which is assumed to be differentiable and strictly decreasing. The per period incumbent's profits (gross of investment costs  $e$ ) are given by  $\Pi_t(e, q) \equiv [p(q) - \theta_t(e)]q$ . The function  $\Pi_t(e, q)$  is differentiable and *strictly quasi-concave* in  $q$ , so that, for each level of  $e$ , the incumbent's per period profit maximization problem has a unique solution, the monopoly quantity denoted by  $m_t(e)$ . Monopoly profits, which are strictly positive, are denoted by  $M_t(e) \equiv \Pi_t(e, m_t(e))$ . For convenience of notation we define  $m_t \equiv m_t(0)$  and  $M_t \equiv M_t(0)$ . It can be noticed that the L type monopoly quantity and profit are independent of investment, i.e.  $m_L(e) = m_L$  and  $M_L(e) = M_L$ .

Incumbent's profits in the second period depend on the entry decision by firm 2, which will be

denoted by  $y \in \{0, 1\}$ , with  $y = 1$  if entry takes place and 0 otherwise. If entry does not take place, the incumbent remains a monopolist and earns  $M_t(e)$ , while firm 2 makes zero profits. If entry occurs, instead, the two firms compete in quantities and earn duopoly profits. Having persistent effects on incumbent's costs, investment affects post-entry profits of both firms. Incumbent's duopoly profits are denoted by  $D_t(e) \geq 0$ , while entrant's duopoly profits, net of entry fee, are denoted by  $D_2(e, t)$ , because they also depend on the type of incumbent she faces. We define  $D_t \equiv D_t(0)$  and  $D_2(t) \equiv D_2(0, t)$ . Incumbent profits  $D_H(e)$  are increasing and entrant's profits  $D_2(e, H)$  are decreasing in  $e$ , whereas  $D_L(e) = D_L$  and  $D_2(e, L) = D_2(L)$ .

The incumbent's decision about  $q$  and  $e$ , is based on *total profits* over the two periods that, assuming no time discounting, are given by

$$V_t(e, q, y) = \Pi_t(e, q) - e + yD_t(e) + (1 - y)M_t(e). \quad (1)$$

The function  $V_H(e, q, y)$  is assumed to be strictly quasi-concave in  $e$  and  $q$ . If the incumbent decides to accommodate entry, he will produce the monopoly output in the first period and he will choose the optimal level of investment taking into account that duopoly profits will be made in the second period. The total profits will be respectively given by  $V_L^A = V_L(0, m_L, 1) = M_L + D_L$ , for type L, and by

$$V_H^A = \max_e V_H(e, m_H(e), 1) \quad (2)$$

for type H. The maximizing level of investment in problem (2) is denoted by  $e_A$  and the associated monopoly quantity by  $m_A \equiv m_H(e_A)$ .

Firm 2 will make her entry decision on the basis of her expected profits. After observing first period incumbent's choice, the entrant will make an inference about incumbent's cost. Her *beliefs* about the probability of the incumbent being of type H are denote by  $\hat{\beta}$  and her expected post-entry profits are given by

$$\hat{\beta}D_2(e, H) + (1 - \hat{\beta})D_2(L). \quad (3)$$

Firm 2 enters only if expected profits are strictly positive.

To let investment play a role in the entry problem we have to introduce a few assumptions. First of all, we will require a positive level of investment for entry deterrence to take place and in particular a level exceeding that undertaken by the inefficient incumbent to accommodate entry. We assume that entry is never profitable against a low cost incumbent, i.e.  $D_2(L) < 0$  and that firm 2 post-entry profits are strictly positive if she faces an inefficient incumbent, regardless of the level of investment undertaken, i.e.  $D_2(e, H) > 0$ . In addition, we suppose that firm 2 expected post-entry profits (at prior belief) are strictly positive if the incumbent invests the amount of accommodation, i.e.

$$\beta D_2(e_A, H) + (1 - \beta) D_2(L) > 0 \quad (4)$$

An immediate consequence of (4) is that deterrence of profitable entry requires a higher level of investment than  $e_A$ . Hence, to ensure that investment can actually deter entry we assume that there exists a level of investment  $e_0$ , the *investment of deterrence*, at which firm 2 expected post-entry profits vanish, i.e.

$$\beta D_2(e_0, H) + (1 - \beta) D_2(L) = 0. \quad (5)$$

The investment of deterrence  $e_0$  is strictly related to market entry conditions. High values of  $e_0$  are associated with favourable entry conditions and vice-versa.

The main consequences of the above assumptions is that  $e_0 > e_A$ , namely in order to deter entry the incumbent must over invest as compared to the level of investment undertaken to accommodate entry. The next two conditions are introduced to ensure that the choice to deter profitable entry by undertaking a level of investment  $e_0$  is preferred to accommodation by any type of incumbent. We assume that

$$V_H(e_0, m_L, 0) > V_H^A. \quad (6)$$

and

$$V_L(e_0, m_L, 0) > V_L^A. \quad (7)$$

The problem that will be addressed is whether the incumbent will over invest to deter profitable entry or whether he will invest to accommodate entry, when he owns private information about his costs. To answer this question we will formalize the problem as a signaling game.



### 3 The signaling game

The entry problem outlined in the previous section will be modeled as a multiple signaling game. In fact, not only investment can be used as a commitment variable being an irreversible and publicly observable choice affecting post-entry competition, but it can also play a role as a signal along with price. As it is well known, price acts as a signal because a price cut is cheaper for the low cost incumbent. Similarly, being purely dissipative for type L and cost reducing for type H, investment is ‘more expensive’ for the efficient incumbent, who can use it to reveal his cost to the entrant.<sup>2</sup> Therefore, our entry problem can be formalized as a game with multiple signals and in fact, it has some similarities with the models studied by Bagwell and Ramey (1988) and Bagwell (2007).

The timing of the game is as follows. Nature moves first and chooses the type of incumbent. After observing his costs, the incumbent decides his first period output (or price) and a level of investment, i.e. he sends the two signals  $q$  and  $e$ . Next the potential entrant, who does not know the type of incumbent, observes the signals and decides whether to enter or not into the market. Finally, payoffs are received. The cost reducing technology  $\theta_H(e)$ , the marginal cost  $\theta_L$ , the prior probability  $\beta$ , market demand  $p(q)$  and duopoly profits  $D_t(e)$  and  $D_2(e, t)$  are common knowledge, whereas type  $t$  is private information to the incumbent.

Players strategies and beliefs are as follows. A pure strategy for the incumbent is a function which associates with each type a level of investment and a first period quantity and consists of two pairs,  $(e_H, q_H)$  and  $(e_L, q_L)$ . A pure strategy by firm 2 associates the entry decision to any observable choice by the incumbent and it is denoted by  $y(e, q) \in \{0, 1\}$ . A system of *beliefs* for firm 2 is a function  $\hat{\beta}$  which associates the ex post probability of the H type to any observable choice by the incumbent,  $(e, q)$ . The incumbent’s payoff is given by (1), the entrant’s payoff is given by  $D_{2t}(e)$  in the case of entry and is equal to zero otherwise. To find a solution to the signaling game the standard notion of *Perfect Bayesian Equilibrium* (PBE) is applied.<sup>3</sup>

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<sup>2</sup>Notice that investment retains its character of signal even if it is cost reducing for the L type, provided that the effect of investment is larger for type H than for type L.

<sup>3</sup>See Fudenberg and Tirole (1991).

**Definition 1.** A profile of strategies  $(e_t, q_t)$  and  $y(e, q)$ , with  $t = H, L$ , is a PBE of the signaling game with investment, if there exist beliefs  $\hat{\beta}(e, q)$  satisfying the following conditions:

(i) The incumbent's strategy is optimal, i.e.  $(e_t, q_t) = \operatorname{argmax} V_t(e, q, y(e, q))$ , for  $t = H, L$

(ii) The entrant's strategy is optimal, i.e.  $y(e, q) = 1$  if and only if

$$\hat{\beta}(e, q)D_2(e, H) + (1 - \hat{\beta}(e, q))D_2(L) > 0.$$

(iii) Beliefs are consistent with Bayes' rule.

Condition 1.(iii) places restrictions only on beliefs along the equilibrium path. Specifically, if  $(e_t, q_t)$  is an incumbent's equilibrium strategy, consistency of beliefs requires that  $\hat{\beta}(e_H, q_H) = \beta$  if  $(e_H, q_H) = (e_L, q_L)$  and  $\hat{\beta}(e_H, q_H) = 1, \hat{\beta}(e_L, q_L) = 0$  if  $(e_H, q_H) \neq (e_L, q_L)$ . As typical in signaling games, the lack of restrictions on off-equilibrium beliefs generates a multiplicity of equilibria which will be refined by applying the Intuitive Criterion originally proposed by Cho and Kreps (1987).

Recall that the Intuitive Criterion is built on the notion of equilibrium domination, which states that a deviation from a given equilibrium is equilibrium dominated for type  $t$  if the equilibrium payoff is greater than the best conceivable payoff that type  $t$  may obtain from that deviation. Formally, let  $V_t^*$  denote the payoff to type  $t$  in a given equilibrium, then a deviation  $(\tilde{e}, \tilde{q})$  from the equilibrium choice  $(e_t, q_t)$  is *equilibrium dominated for type  $t$*  if  $V_t^* > V_t(\tilde{e}, \tilde{q}, 0)$ , as total profits are greater under no entry. Beliefs supporting a given equilibrium satisfy the Intuitive Criterion if, whenever a deviation is equilibrium dominated for type  $t$  and strictly preferred by type  $t'$ , the entrant does not assign that deviation to type  $t$ . For example, if the deviation  $(\tilde{e}, \tilde{q})$  is equilibrium dominated for H and strictly preferred by L given the entrant's best reply to L, i.e. if

$$V_H(e_H, q_H, y(e_H, q_H)) > V_H(\tilde{e}, \tilde{q}, 0) \tag{8}$$

$$V_L(e_L, q_L, y(e_L, q_L)) < V_L(\tilde{e}, \tilde{q}, 0), \tag{9}$$

the off-equilibrium belief must be  $\hat{\beta}(\tilde{e}, \tilde{q}) = 0$ . With these beliefs, however, the incumbent's strategy of the given equilibrium is not optimal. Thus, if (8) and (9) hold, the equilibrium strategy does not

meet the Intuitive Criterion. Following Cho and Kreps (1987, p. 202), an intuitive equilibrium is formally defined as follows.

**Definition 2.** *A Perfect Bayesian Equilibrium,  $(e_t, q_t)$  and  $y(e, q)$ , with  $t = H, L$ , is intuitive if there exists no deviation  $(\tilde{e}, \tilde{q}) \neq (e_t, q_t)$  such that (8) and (9) hold.<sup>4</sup>*

In the next section we will study pure strategy equilibria and work out the solution to the signaling game. The following standard condition will be used to guarantee the existence of separating equilibria<sup>5</sup>

$$M_L - D_L \geq M_H - D_H. \quad (10)$$

Condition (10) simply states that the low cost incumbent benefits from entry deterrence more than the high cost incumbent. Finally, in order to avoid trivial forms of separation, we assume that the high cost incumbent has an incentive to mimic the low cost one, i.e.

$$V_H(0, m_L, 0) > V_H^A. \quad (11)$$

## 4 No deterrence of profitable entry

Let us consider separating and pooling equilibria in turn. A separating equilibrium is a PBE where different types of incumbent make different choices or, equivalently, where  $(e_H, q_H) \neq (e_L, q_L)$ . In a separating equilibrium information is fully revealed, therefore the entrant only enters when she faces a high cost incumbent. The H type accommodates entry, i.e. he produces the monopoly quantity in the first period and the duopoly quantity after entry, and decides a level of investment which maximizes total profits. Hence, the equilibrium choice for the H type is  $e_H = e_A$  and  $q_H = m_A$  and the maximum total profit from accommodation (given by (2)) is  $V_H^A = M_H(e_A) - e_A + D_H(e_A)$ .

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<sup>4</sup>Notice that we need not consider the further requirement, implied by Cho and Kreps (1987), that there is no deviation satisfying (i)  $V_L^* > V_L(\tilde{e}, \tilde{q}, 0)$  and (ii)  $V_H^* < V_H(\tilde{e}, \tilde{q}, 1)$ . Indeed, a deviation will never satisfy (ii) as, when entry takes place, total profit for H is always lower than the equilibrium payoff.

<sup>5</sup>See, for instance, Tirole (1988) or Bagwell and Ramey (1988).

The equilibrium choice by the L type,  $(e_L, q_L)$ , must satisfy the ‘incentive compatibility condition’ for the H type, i.e.

$$V_H(e_L, q_L, 0) \leq V_H^A. \quad (12)$$

In other words, the high cost incumbent should prefer accommodating entry rather than mimicking the efficient type. Furthermore, the L type total profit at equilibrium must be greater than the ‘accommodation profit’  $V_L^A$ , i.e. the total profit he may earn by undertaking no investment, producing the monopoly output in the first period and the duopoly quantity in the second. Thus, the equilibrium choice  $(e_L, q_L)$  must satisfy the ‘participation condition’

$$V_L(e_L, q_L, 0) \geq V_L^A. \quad (13)$$

It is not difficult to see that an incumbent’s strategy with  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L)$  satisfying (12) and (13) supports a separating equilibrium.<sup>6</sup> For example, consider the system of beliefs assigning to the high cost type any choice different from  $(e_L, q_L)$ , i.e.  $\hat{\beta}(e, q) = 1$  for any  $(e, q) \neq (e_L, q_L)$  and 0 otherwise, and the entrant’s strategy  $y(e, q) = 1$  for any  $(e, q) \neq (e_L, q_L)$  and 0 otherwise. This profile of strategies and the specified system of beliefs are easily seen to satisfy Definition 1.

Although the freedom in the choice of off-equilibrium beliefs gives rise to a multiplicity of equilibria, it turns out that there is a unique separating equilibrium surviving the Intuitive Criterion.<sup>7</sup>

**Proposition 1.** *In the signaling game with investment, there exists a unique separating equilibrium satisfying the Intuitive Criterion. The equilibrium is supported by the incumbent’s strategy  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L) = (0, q^*)$ , where  $q^*$  is implicitly defined by the equation*

$$V_H(0, q^*, 0) = V_H^A. \quad (14)$$

Moreover,  $q^* > m_L$ .

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<sup>6</sup>For a formal proof see Lemma 1 in the Appendix.

<sup>7</sup>By uniqueness we mean that the same incumbent strategy is shared by all the equilibria.

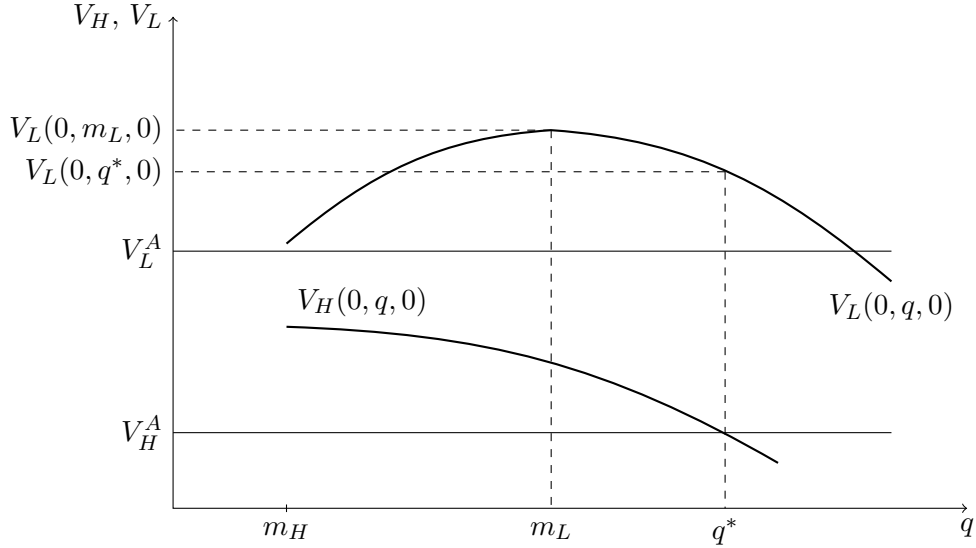


Figure 1: Intuitive separating equilibrium quantity.

In the intuitive separating equilibrium, the inefficient incumbent invests as much as needed to accommodate entry and charges his monopoly price. The efficient incumbent, instead, cuts down investment to zero and sets a limit price, i.e. a price below the monopoly level. By so doing he is able to signal his cost to the entrant who stays out. Notice that, in the separating equilibrium, limit pricing does not deter profitable entry, because firm 2 stays out exactly when entry is unprofitable. Figure 1, where total profits of both types of incumbent are depicted as functions of  $q$  at  $e = 0$ , illustrates the result of Proposition 1 by deriving the equilibrium output  $q^*$ , which is shown to exceed the level of monopoly of the low cost type.

Let us turn to the analysis of pooling equilibria. A pooling equilibrium is a PBE where all types of incumbent send the same signals:  $(e_L, q_L) = (e_H, q_H) = (e_P, q_P)$ . Since the entrant does not learn any new piece of information from the observation of the equilibrium choice, the beliefs about the incumbent type are unmodified and they are equal to prior probabilities, i.e.  $\hat{\beta}(e_P, q_P) = \beta$ . In a pooling equilibrium entry can not take place, because otherwise each type of incumbent would be better off by choosing his own monopoly output in the first period. As firm 2 expected post-entry

profits must be non positive, the incumbent is required to undertake a sufficiently high level of investment and, precisely, a level not lower than  $e_0$  as defined by (5), hence  $e_P \geq e_0$ . Furthermore, the incumbent's strategy must allow each type to obtain at least the payoff earned if entry is accommodated, which means that the following 'participation conditions' are satisfied:

$$V_H(e_P, q_P, 0) \geq V_H^A, \quad (15)$$

$$V_L(e_P, q_P, 0) \geq V_L^A. \quad (16)$$

It turns out that a pooling equilibrium is characterized by an incumbent's strategy satisfying (15) and (16), with  $e_P \geq e_0$  (see Lemma 3 in the Appendix). Under conditions (6) and (7), there is a multiplicity of pooling equilibria and, in particular, there exists a pooling equilibrium supported by the incumbent's strategy  $(e_P, q_P) = (e_0, m_L)$ , i.e. the equilibrium where both types play the minimum level of investment consistent with entry deterrence and the monopoly quantity of the low cost incumbent, while firm 2 does not enter. This is the most profitable pooling equilibrium from the point of view of the efficient incumbent, as his profit is  $V_L(0, m_L, 0) = 2M_L - e_0$ . Notice also that if the investment of deterrence is not too high, i.e. entry conditions are not much favourable, it may well happen that  $M_L - e_0 > \Pi_L(0, q^*)$ , so that total profits of the efficient incumbent are greater under the pooling equilibrium than under the intuitive separating equilibrium characterized in Proposition 1, i.e.  $V_L(0, m_L, 0) > V_L(0, q^*, 0)$ . In other words, not only the inefficient type, but also the efficient incumbent is better off when profitable entry is deterred.

In a pooling equilibrium, the two signals provide the entrant with no new piece of information about incumbent's cost, so that the level of investment exceeding  $e_0$  is able to modify the perceived profitability of entry to firm 2. Hence, matched with a suitable pricing policy, investment gains a value of commitment which allows the incumbent to deter profitable entry. In order to be supported, however, a pooling equilibrium requires the efficient type to send a completely dissipative signal in terms of investment expenditures that have no reducing impact on his costs. Is this dissipative behaviour plausible? Or, equally, is there any pooling equilibrium surviving the Intuitive Criterion? It turns out that no pooling equilibrium exists which meets the conditions set out by Definition 2.

**Proposition 2.** *In the signaling game with investment there are no pooling equilibria satisfying the Intuitive Criterion.*

The logic behind the result of Proposition 2 is quite straightforward. By cutting down investment to zero and increasing quantities above the level stipulated in the candidate pooling equilibrium, an efficient incumbent can persuade the entrant that such a low price in the deviation with no investment can not be profitable to an inefficient incumbent, whereas it allows type L to save on the dissipation of useless expenses. The use of investment as a signal by the efficient incumbent prevents the inefficient one to use investment as a commitment variable. More generally, it is the fact that investment has a different cost-reducing impact across types, and so can act as a signal, to make the pooling equilibrium impossible to survive the application of the Intuitive Criterion.<sup>8</sup>

The overall analysis of the signaling model with investment provides an unambiguous prediction of the most plausible outcome of the game, which is the unique intuitive separating equilibrium characterized in Proposition 1. Surprisingly, in the presence of private information about costs, the strategic use of investment can not be fully exploited by the incumbent. In fact, investment is undertaken only to accommodate rather than to deter profitable entry.

Finally, we notice that the main point of our analysis still survives even if the cost reducing impact of investment were strong enough to allow the inefficient incumbent to cut down to zero post-entry profits to firm 2. As can be seen, even in such a case only a separating equilibrium passes the Intuitive Criterion test and in such equilibrium the inefficient incumbent will over invest to deter *unprofitable* entry. The point remains that strategic investment can not be used to deter *profitable* entry when private information about incumbent's costs is present.

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<sup>8</sup>It is interesting to notice that the Intuitive Criterion does not rule out, in general, pooling equilibria either in the original model of limit pricing put forward by Milgrom and Roberts (1982) or in the model with advertising by Bagwell and Ramey (1988).

## 5 Summary and conclusions

We studied a model of entry in which an incumbent, who is privately informed as to his costs, has the opportunity to undertake a long run investment in a cost reducing activity. Having persistent effects on post-entry competition, investment has a commitment value. Moreover, as investment is assumed to have an heterogeneous effect across types it also acts, along with price, as a signal of incumbent's costs.

We analysed the strategic use of investment in limiting entry by means of a signaling game à la Milgrom and Roberts (1982) with multiple signals. The entry game was specified so as to leave to the inefficient incumbent two alternative options, either to invest to accommodate entry, or to over invest to deter profitable entry. Our primary interest was to find whether investment gains any further value of commitment, when the entrant is uncertain about incumbent's costs.

Although both pooling and separating equilibria are possible in our setting, we found that only a separating equilibrium can be the solution to the entry problem, because it is the sole equilibrium surviving the Intuitive Criterion. In this equilibrium, the inefficient incumbent invests to accommodate entry while the efficient one makes zero investment and charges a limit price to signal his cost to the entrant who stays out. Hence, we conclude that the strategic use of investment under conditions of private information is limited, in fact investment will only be undertaken to accommodate rather than to deter profitable entry. This result, which is due to the double role of investment as a commitment variable and as a signal, obtains because the signaling role of investment to the efficient incumbent cancels out its commitment value to the inefficient one.

The major implication of this result for the analysis of competition policy is that in markets where the costs of the incumbent are not directly observable by potential entrants, it is highly likely that strategic investment is mainly undertaken by inefficient incumbents to accommodate entry. Of course, if investment had a cost-reducing effect strong enough to bring down to zero entrant's post-entry profits, an inefficient incumbent would over invest to deter entry. In such a case, as usual, the investment behaviour of the incumbent should be evaluated by antitrust authorities on the basis of its overall welfare effects. In any case, whether investment has strong cost-reducing effects or



not, in the presence of private information about incumbent's cost, investment does not gain any commitment value that allows the incumbent to deter profitable entry.

## Appendix

**Lemma 1** *An incumbent's strategy supports a separating equilibrium if and only if  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L)$  satisfies (12) and (13).*

*Proof of Lemma A.1*

Let  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L)$  satisfy (12) and (13). We have to show that the incumbent's strategy supports a separating equilibrium. Take the beliefs which assign to the H type any choice different from  $(e_L, q_L)$ , i.e.  $\hat{\beta}(e, q) = 1$  for any  $(e, q) \neq (e_L, q_L)$  and  $\hat{\beta}(e, q) = 0$  for  $(e, q) = (e_L, q_L)$ . These beliefs obey Bayes' rule. Given these beliefs and the assumption  $D_2(e, H) > 0$ , firm 2 expected profits are strictly positive except when  $(e, q) = (e_L, q_L)$ , thus the entrant's strategy,  $y(e, q) = 0$  if  $(e, q) = (e_L, q_L)$  and 1 otherwise, satisfies Definition 1.(ii). Let us show that the incumbent's strategy is optimal given the entrant's strategy. As regard to type H, we have  $V_H(e_A, m_A, 1) \geq V_H(e, q, y(e, q))$  for all  $(e, q)$ . Indeed, for  $(e, q) \neq (e_L, q_L)$ , we have  $y(e, q) = 1$  and the inequality holds by definition of  $e_A$ ; moreover, for  $(e, q) = (e_L, q_L)$  the above inequality is satisfied because of (12). As for type L, we have  $V_L(e_L, q_L, 0) \geq V_L(e, q, y(e, q))$  for all  $(e, q)$ . Indeed, for  $(e, q) \neq (e_L, q_L)$  firm 2 enters, then  $y(e, q) = 1$  and the inequality holds because of (13) and definition of  $V_L^A$ ; if  $(e, q) = (e_L, q_L)$  firm 2 does not enter and the above inequality holds as an equality. This completes the first part of the proof.

To show the converse let us suppose that the incumbent's strategy  $(e_H, q_H)$  and  $(e_L, q_L)$  supports a separating equilibrium, so that  $\hat{\beta}(e_H, q_H) = y(e_H, q_H) = 1$  and  $\hat{\beta}(e_L, q_L) = y(e_L, q_L) = 0$ . By Definition 1.(i),  $V_H(e_H, q_H, 1) \geq V_H(e, q, y(e, q))$  and, by definition of  $e_A$  as the solution of (2), we have  $V_H(e_A, m_A, 1) \geq V_H(e_H, q_H, 1)$ . Hence, as  $(e_A, m_A)$  is optimal for type H it must be

$(e_H, q_H) = (e_A, m_A)$ . Next, we have to show that  $(e_L, q_L)$  satisfies (12) and (13). Let us suppose, to the contrary, that  $(e_L, q_L)$  violates (12); then  $(e_A, m_A)$  cannot be an optimal choice for type H, as  $(e_L, q_L)$  gives higher profits because avoids entry ( $y(e_L, q_L) = 0$ ). Now let us suppose that  $(e_L, q_L)$  violates (13); then  $(e_L, q_L)$  cannot be an optimal choice for L because  $(0, m_L)$  gives higher profits even when Firm 2 enters, therefore these contradictions complete the proof. Q.E.D.

**Lemma 2** *An incumbent's strategy,  $(e_t, q_t)$  with  $t = H, L$ , supports an intuitive separating equilibrium if and only if  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L)$  is a solution to the following maximization problem<sup>9</sup>*

$$\begin{aligned} \max_{e, q} \quad & \Pi_L(q) - e \\ \text{subject to} \quad & \Pi_H(e, q) - e + M_H(e) \leq V_H^A \end{aligned} \tag{17}$$

$$\Pi_L(q) - e \geq D_L \tag{18}$$

*Proof of Lemma 2*

Let us suppose that  $(e^*, q^*)$  is a solution to the maximization problem. We have to show that the incumbent's strategy,  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L) = (e^*, q^*)$ , supports an intuitive separating equilibrium. First, notice that  $(e^*, q^*)$  satisfies (12) and (13) since these inequalities are equivalent to (17) and (18) respectively. Therefore, by Lemma 1,  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L) = (e^*, q^*)$ , supports a separating equilibrium. To show that the equilibrium is intuitive, let us suppose that there exists a deviation  $(\tilde{e}, \tilde{q}) \neq (e^*, q^*)$ , which is equilibrium dominated for H, i.e.  $V_H(\tilde{e}, \tilde{q}, 0) < V_H(e_A, m_A, 1)$ , but not for L, i.e.  $V_L(\tilde{e}, \tilde{q}, 0) > V_L(e^*, q^*, 0)$ . Therefore,  $(\tilde{e}, \tilde{q})$  satisfies (17), but from the last inequality  $\Pi_L(\tilde{q}) - \tilde{e} > \Pi_L(q^*) - e^*$ . Therefore,  $(e^*, q^*)$  is not a solution to the maximization problem contrary to the assumption. This contradiction completes the first part of the proof.

To show the converse, let us suppose that the incumbent's strategy,  $(e_H, q_H) = (e_A, m_A)$  and  $(e_L, q_L) = (e^*, q^*)$ , supports an intuitive separating equilibrium. We have to show that  $(e^*, q^*)$  is a solution to the maximization problem. Let us proceed by contradiction and suppose that  $(e^*, q^*)$  is not

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<sup>9</sup>Since the L type profit does not depend on  $e$ , for ease of notation, we set  $\Pi_L(q) = \Pi_L(e, q)$ .

a solution, i.e. there exists a pair  $(e', q')$  satisfying (17) and (18) such that  $\Pi_L(q') - e' > \Pi_L(q^*) - e^*$ . From the last inequality it immediately follows that  $V_L(e', q', 0) > V_L(e^*, q^*, 0)$ . If the constraint (17) is not binding, so that  $V_H(e', q', 0) < V_H^A$ , the deviation  $(e', q')$  violates Definition 2 and  $(e_L, q_L) = (e^*, q^*)$  cannot support an intuitive separating equilibrium, contrary to the assumption. Therefore, (17) must be binding so that  $V_H(e', q', 0) = V_H^A$ . Then, one can find in a small neighbourhood of  $q'$  a quantity  $q''$  such that  $V_L(e', q'', 0) > V_L(e^*, q^*, 0)$  and  $V_H(e', q'', 0) < V_H^A$ , so that the deviation  $(e', q'')$  violates Definition 2 and the equilibrium cannot be intuitive contrary to the assumption. In order to show the existence of  $q''$  one can argue as follows. By continuity of  $\Pi_L$  there exists an open interval centred at  $q'$ ,  $B = \{q \mid |q - q'| < \varepsilon, \varepsilon > 0\}$ , such that  $V_L(e', q, 0) > V_L(e^*, q^*, 0)$  for all  $q \in B$ ; moreover, by strict quasi-concavity of  $\Pi_H$  there exists  $q'' \in B$  such that  $V_H(e', q'', 0) < V_H^A$ . In fact, note first that  $q'$  can not be a minimum for  $V_H(e', q, 0)$  since profits are unbounded from below. Hence  $q'$  can either be a maximum for  $V_H(e', q, 0)$  or not a maximum. If  $q'$  is a maximum, then by strict quasi-concavity it is unique and the existence of  $q''$  follows immediately by the definition of maximum. If  $q'$  is not a maximum, then by strict quasi-concavity,  $V_H(e', q, 0)$  is either a strictly increasing or a strictly decreasing function of  $q$  in an open interval centred at  $q'$ , and this is sufficient to show that  $q''$  exists. In conclusion, we have shown that  $(e^*, q^*)$  is a solution to the maximization problem and this completes the proof of the lemma. Q.E.D.

*Proof of Proposition 1.*

Let us first show that there exists a unique solution to equation (14) and that  $q^* > m_L$ . First of all we know that  $V_H(0, q, 0) = \Pi_H(0, q) + M_H$  is continuous in  $q$ . By assumption (11), we have  $V_H(0, m_L, 0) > V_H^A$ . Moreover, let  $q_c > m_L$  be the competitive market quantity, i.e. a finite quantity such that  $\Pi_L(q_c) = 0$ . Then  $\Pi_H(0, q_c) < 0$  and  $V_H(0, q_c, 0) < M_H < V_H^A$ , since  $V_H^A \geq M_H + D_H$ . Thus by continuity of  $V_H$ , equation (14) has at least a solution in the interval  $]m_L, q_c[$ . Finally, uniqueness of the solution is established by showing that  $V_H(0, q, 0)$ , or equivalently  $\Pi_H(0, q)$ , is strictly decreasing for  $q$  in the interval  $]m_L, q_c[$ . This is easily seen by noting that the profit function is strictly quasi concave and that its maximum is  $m_H < m_L$ .

Next, by Lemma 2, we have to show that  $(0, q^*)$  is a solution of the maximization problem, i.e.  $\Pi_L(q^*) > \Pi_L(q) - e$  for all  $(e, q) \neq (0, q^*)$  satisfying (17) and (18). Let us first consider the maximization problem with only the constraint (17) and separately consider the two cases  $(e, q)$  with  $q > q^*$  and  $(e, q)$  with  $q < q^*$ . If  $q > q^*$  then  $q > m_L$  and the profit function  $\Pi_L(q)$  is strictly decreasing. Therefore, we have  $\Pi_L(q^*) > \Pi_L(q)$  so that  $\Pi_L(q^*) > \Pi_L(q) - e$  for all  $(e, q)$  with  $q > q^*$ .

Let us turn to the second case, i.e.  $(e, q)$  satisfying (17) with  $q < q^*$ . By definition of  $q^*$ , i.e. by equation (14), the constraint (17) can be rewritten as follows

$$\Pi_H(e, q) - e + M_H(e) \leq \Pi_H(0, q^*) + M_H$$

or

$$\Pi_H(0, q^*) + (M_H - M_H(e)) \geq \Pi_H(e, q) - e.$$

Since monopoly profits are decreasing in costs,  $M_H - M_H(e) \leq 0$  for  $e \geq 0$ , therefore if  $(e, q)$  satisfies (17) it also satisfies

$$\Pi_H(0, q^*) \geq \Pi_H(e, q) - e$$

or

$$[p(q^*) - \theta_H]q^* \geq [p(q) - \theta_H(e)]q - e. \quad (19)$$

Moreover, since  $q < q^*$  and  $\theta_H \geq \theta_H(e)$ , it must be true that

$$(\theta_H - \theta_L)q^* \geq (\theta_H(e) - \theta_L)q \quad (20)$$

Adding term by term the inequalities (19) and (20) yields

$$(p(q^*) - \theta_L)q^* > (p(q) - \theta_L)q - e$$

or  $\Pi_L(q^*) > \Pi_L(q) - e$  for all  $(e, q)$  satisfying (17) with  $q < q^*$ .

In order to complete the proof we have to show that  $(0, q^*)$  also satisfies (18). Notice that, by definition of  $q^*$ , we have

$$\Pi_H(0, q^*) + M_H = M_H(e_A) - e_A + D_H(e_A) \quad (21)$$

By definition of  $V_H^A$  it must hold

$$M_H(e_A) - e_A + D_H(e_A) \geq M_H + D_H \quad (22)$$

Thus, (21) and (22) imply  $\Pi_H(0, q^*) \geq D_H$  and after simple manipulations

$$D_L - D_H \geq D_L - \Pi_H(0, q^*) \quad (23)$$

Next, let us consider

$$\begin{aligned} \Pi_L(q^*) - \Pi_H(0, q^*) &= (\theta_H - \theta_L)q^* \\ &> (\theta_H - \theta_L)m_L \\ &= M_L - \Pi_H(0, m_L) \\ &> M_L - M_H \end{aligned} \quad (24)$$

where the first inequality follows from  $q^* > m_L$  and the last inequality from  $M_H > \Pi_H(0, m_L)$ . By assumption (10), we have  $M_L - M_H \geq D_L - D_H$  so that from (23) and (24) it follows

$$\Pi_L(q^*) - \Pi_H(0, q^*) > D_L - \Pi_H(0, q^*)$$

and finally  $\Pi_L(q^*) > D_L$ . This completes the proof of Proposition 1. Q.E.D.

**Lemma 3** *The incumbent's strategy  $(e_t, q_t) = (e_P, q_P)$ , with  $t = H, L$ , supports a pooling equilibrium if and only if  $(e_P, q_P)$  satisfies (15), (16) and  $e_P \geq e_0$ , where  $e_0$  is the zero expected profit level of investment defined by (5).*

*Proof of Lemma 3*

Let  $(e_P, q_P)$  satisfy (15) and (16) with  $e_P \geq e_0$ . We have to show that it supports a pooling equilibrium. Let us take the beliefs  $\hat{\beta}(e, q) = \beta$  if  $(e, q) = (e_P, q_P)$  and 1 otherwise, which are consistent with Bayes rule, Definition 1.(iii). Given these beliefs, take the entrant's strategy  $y(e, q) = 0$  if  $(e, q) = (e_P, q_P)$  and 1 otherwise. The incumbent's strategy is optimal for type H. In fact, if  $(e, q) \neq (e_P, q_P)$  then  $y(e, q) = 1$  and, by definition of  $V_H^A$ , we have  $V_H^A \geq V_H(e, q, y(e, q))$  and finally,

by (15),  $V_H(e_P, q_P, 0) \geq V_H(e, q, y(e, q))$ . With a similar argument it is easily shown that  $(e_P, q_P)$  is optimal for type L, so that the incumbent's strategy satisfies Definition 1.(i). Let us show that also the entrant's strategy is optimal. If  $y(e, q) = 1$ , then  $(e, q) \neq (e_P, q_P)$  and  $\hat{\beta}(e, q) = 1$ , thus the entrant expected profits are strictly positive. Vice versa, if at  $(e, q)$  the expected profits are strictly positive then  $(e, q) \neq (e_P, q_P)$ , because  $\hat{\beta}(e_P, q_P) = \beta$  and  $e_P \geq e_0$  imply, by (5), non positive entrant's expected profits. Thus,  $(e, q) \neq (e_P, q_P)$  and by the definition of  $y$  we have  $y(e, q) = 1$  and Definition 1.(ii) is satisfied.

Let us show the converse and suppose that  $(e_H, q_H) = (e_L, q_L) = (e_P, q_P)$  supports a pooling equilibrium. Then, by Definition 1.(iii),  $\hat{\beta}(e_P, q_P) = \beta$  and  $y(e_P, q_P) = 0$ , because if  $y(e_P, q_P) = 1$  the pooling strategy would not be an optimal strategy for the incumbent. Therefore, the entrant expected profits must be non positive, which means, by (5), that  $e_P \geq e_0$ . Consider next the choice of L. Since by Definition 1.(i)  $V_L(e_P, q_P, 0) \geq V_L(0, m_L, y(0, m_L))$  and since  $V_L(0, m_L, 0)$  is the highest total profit, it must be  $y(0, m_L) = 1$  thus  $V_L(e_P, q_P, 0) \geq V_L(0, m_L, 1) = V_L^A$  and (16) holds. Consider the choice of type H. Since by Definition 1.(i)  $V_H(e_P, q_P, 0) \geq V_H(e_A, m_A, y(e_A, m_A))$  and since  $V_H(e_A, m_A, y(e_A, m_A)) \geq V_H^A$  then  $V_H(e_P, q_P, 0) \geq V_H^A$  and (15) is satisfied. This completes the proof of Lemma 3. Q.E.D.

*Proof of Proposition 2.*

Let us deal with two cases separately,  $q_P < m_L$  and  $q_P \geq m_L$ . The case  $q_P < m_L$  is straightforward. Indeed, a pooling equilibrium supported by the quantity  $q_P$  cannot be intuitive because the deviation  $\tilde{q} = m_L$  is equilibrium dominated for type H and strictly preferred by type L.

Next, turn to the case  $q_P \geq m_L$ . Let  $(e_P, q_P)$  be the pooling equilibrium strategy of the incumbent. We show that there exists a deviation  $(0, \tilde{q})$  which is equilibrium dominated for the H type and strictly preferred to the equilibrium choice by the L type. Let  $\tilde{q} > m_L$  be defined by the equality

$$\Pi_L(q_P) - \Pi_L(\tilde{q}) = e_P - \epsilon \tag{25}$$

where  $\epsilon > 0$  is arbitrarily close to zero so that  $e_P - \epsilon > 0$ . The quantity  $\tilde{q}$  is well defined and  $\tilde{q} > q_P$ .<sup>10</sup>

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<sup>10</sup>Let us consider the function of  $q$ ,  $\Pi_L(q) - [\Pi_L(q_P) - e_P + \epsilon]$  and notice that by (16) the term in square brackets

Let us consider the deviation  $(0, \tilde{q})$  and show that it is strictly preferred to the equilibrium choice by the L type. Indeed,

$$\begin{aligned}
V_L(0, \tilde{q}, 0) - V_L(e_P, q_P, 0) &= \Pi_L(\tilde{q}) + M_L - \Pi_L(q_P) + e_P - M_L \\
&= \Pi_L(\tilde{q}) - \Pi_L(q_P) + e_P \\
&= \epsilon - e_P + e_P = \epsilon > 0
\end{aligned}$$

where we used (25).

Before showing that the same deviation is equilibrium dominated for the H type let us derive the following result.

$$\begin{aligned}
\Pi_H(e_P, q_P) - \Pi_H(0, \tilde{q}) - \Pi_L(q_P) + \Pi_L(\tilde{q}) &= \\
&= -\theta_H(e_P)q_P + \theta_H\tilde{q} + \theta_Lq_P - \theta_L\tilde{q} \\
&= (\theta_H - \theta_L)(\tilde{q} - q_P) + (\theta_H - \theta_H(e_P))q_P > 0
\end{aligned} \tag{26}$$

where the second equality follows by adding and subtracting  $\theta_Hq_P$  and the last inequality from  $\tilde{q} > q_P$ . By (25) and (26) we have  $\Pi_H(0, \tilde{q}) - \Pi_H(e_P, q_P) + e_P < \epsilon$ . Subtracting to both sides  $M_H(e_P) - M_H > 0$  yields

$$\Pi_H(0, \tilde{q}) + M_H - \Pi_H(e_P, q_P) + e_P - M_H(e_P) < \epsilon - [M_H(e_P) - M_H] \tag{27}$$

where the LHS is the variation of type H total profits after the deviation  $(0, \tilde{q})$ , i.e.

$$V_H(0, \tilde{q}, 0) - V_H(e_P, q_P, 0) = \Pi_H(0, \tilde{q}) + M_H - \Pi_H(e_P, q_P) + e_P - M_H(e_P). \tag{28}$$

(27) and (28) give

$$V_H(0, \tilde{q}, 0) - V_H(e_P, q_P, 0) < \epsilon - [M_H(e_P) - M_H] < 0$$

where the last inequality follows from the fact that the term in square brackets is strictly positive and  $\epsilon$  can be taken arbitrarily close to zero. Therefore, we have shown that the deviation  $(0, \tilde{q})$  is strictly positive. Clearly, for  $q = m_L$  the function is strictly positive and at the perfectly competitive quantity,  $q_c$ , the above function is strictly negative since  $\Pi_L(q_c) = 0$ . Thus, by continuity there exists  $\tilde{q}$  satisfying (25) in the open interval  $]m_L, q_c[$ . Moreover, since for  $q > m_L$  the profit function is strictly decreasing,  $\tilde{q}$  is unique.

equilibrium dominated for the H type. Since for any pooling equilibrium one can find a deviation which is equilibrium dominated for the H type and strictly preferred by the L type, by Definition 2 there exists no pooling equilibrium satisfying the Intuitive Criterion and this completes the proof of Proposition 2. Q.E.D.

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