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Introducing Aggregate Return on Investment as a Solution to the Contradiction Between Some PME Metrics and IRR

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Abstract. The Index Comparison Method (ICM) is a well-known approach for measuring a Private Equity Investment's (PEI) performance. It is based on the construction of a benchmark portfolio that, each period, earns the index return. This generates a time series of interim net asset values that leads to a terminal NAV, from which an Internal Rate of Return is computed. However, the IRR is itself necessarily associated with its own time series of built-in NAVs, to which the IRR is applied. And, unfortunately, this series of values will be different from the aforementioned benchmark portfolio's NAVs. As a result, the ICM approach rests on two contradictory sets of values, thereby rendering it illegitimate. Furthermore, the ICM approach does not preserve additivity of the rates of return, and, in principle, might even generate multiple IRRs. This paper presents the Aggregate Return on Investment (AROI), a metric which (i) uses one consistent time series of NAVs (the benchmark portfolio's true values) (ii) preserves additivity, and (iii) does not incur the problem of multiple solutions.

Introduction

Some twenty years ago, Long & Nickels (1996) first published the Index Comparison Method (ICM), seemingly the first in a series of a class of metrics that are commonly referred to as Public Market Equivalents (PME) – see Gredil et al. (2014) for an excellent summary of many of these PME variants. The ICM answered the question: "What IRR would the cash flows associated with a private equity investment (PEI) have produced if the cash flows contributed into it (and distributed out of it) had, alternatively, been invested in a portfolio based on the performance of a benchmark index?" ICM was understandably embraced as a clever solution to the age old problem that a private equity investment's IRR often lacked a proper benchmark, particularly since most indices are essentially time-weighted rates of return. Notwithstanding the careful research of Long & Nickels, this paper shows that there is a critical flaw in their use of IRR for PME benchmarking purposes. And it has to do with the fact that every investment's IRR has an associated time series of implied beginning-of-period valuations upon which the IRR is earned, a fact unbeknownst to many practitioners but known to scholars (see Akerson 1976, Peasnell 1982, Lohmann 1988, Hazen 2003, Broverman 2008, Kellison 2009, Magni 2010). As we will show, these valuations contradict the computed periodic valuations of the ICM algorithm itself, the very algorithm which is employed in order to produce the final hypothetical ICM Net Asset Value (NAV) entry. The same criticism can be applied to all those PME metrics that compare the PEI to a benchmark portfolio and compute an IRR from it.

Fortunately, there is a broader class of money-weighted rate of return (MWRR) metrics, termed Average Internal Rate of Return (AIRR), created by Magni (2010, 2013), which includes other MWRR alternatives that are particularly well-suited to solving this PEI rate of return benchmarking problem. One such simple alternative is proposed herein. It is a variant of the AIRR class, named Aggregate Return on Investment (AROI). Unlike IRR, it computes a MWRR that is earned on the same, rather than a competing, set of interim valuations that the ICM approach uses. The AROI is defined as the ratio of the total net cash flow to the total invested capital, the latter which is the sum of the same beginning-of-period NAVs that are computed within the ICM algorithm.

In the first section, we demonstrate that the IRR devises its own built-in values and, in the second section, we show that the ICM's benchmark portfolio incurs the contradiction of resting on two sets of competing values. In the third section, we apply the AROI approach, one which easily captures the rate of return of a Private Equity Investment and unambiguously produces its excess

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return over and above that of a PME-style benchmark portfolio, the latter which is based on an index's periodic rates of return.

1. IRR-implied interim values

Let $\boldsymbol{F} = (F_0, F_1, \ldots, F_n)$ be the cashflow sequence of an investment. The investment's IRR is defined as the discount rate that makes the net present value (NPV) equal to zero:

$$F_0 + \frac{F_1}{1+x} + \frac{F_2}{(1+x)^2} + \cdots + \frac{F_n}{(1+x)^n} = 0 \tag{1}$$

While this equation only contains cash flows, these cash flows are connected to the investment's interim values[1] in a well-defined way. This is best illustrated via an example: Exhibit 1[2] assumes a simple cash flow profile for an 11-year private equity investment (PEI) - positive cash flows are end-of-period net distributions, negative cash flows are end-of-period net contributions. Applying (1), one computes an IRR of 17.91%. This figure is the rate of return that is applied to the capital that remains invested in the PEI, period by period. In other words, 17.91% is the growth rate of capital. As detailed in Exhibit 2, at the beginning of 1989, $25 is invested at 17.91% to produce $29.48. Adding the end of 1989 contribution of $25, we find that the capital invested at the beginning of 1990 becomes $54.48. The latter grows at 17.91% during 1990 to become $64.24. Adding the end of 1990 contribution of $25, we find that the capital that is invested in the PEI at the beginning of 1991 is $89.24. This process of growth continues until the liquidation of the investment. Note that, starting at the beginning of 1994, the beginning-of-year capital will be smaller, rather than larger, than the ending value of the prior year, given that end-of-year distributions, rather than end-of-year contributions, are assumed to occur. Upon liquidation at the end of 1999, the terminal capital is zero (see Exhibit 2). That is the very definition of eq. (1): applying a constant period rate equal to 17.91% to the beginning-of-period capital for n periods, and taking into account the contributions/distributions, the investor is eventually left with exactly a zero value. In general, if the IRR is x, the beginning-of-period capital C_t grows as

$$\overbrace{C_t}^{\text{capital invested at time } t} = \overbrace{C_{t-1}}^{\text{capital invested at time } t-1} + \overbrace{xC_{t-1}}^{\text{return}} - \overbrace{F_t.}^{\text{distribution/contribution}} \tag{2}$$

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[1] Due to differing contexts throughout the article, we will use the following synonymous terms interchangeably: NAV, invested capital, beginning-of-year capital, interim value, etc. All reflect what the rate of return is earned on.
[2] All exhibits appear at the end of this paper.

Boulding (1935), who was the one who devised the notion of *internal rate of return*, explicitly derived eq. (1) from eq. (2), by requiring that the terminal capital be zero: $C_n = 0$. Using (2) iteratively, one obtains

$$C_n = F_0(1+x)^n + F_1(1+x)^{n-1} + \cdots + F_n = 0 \qquad (3)$$

(see Altshuler and Magni 2012, Appendix), which is but a reframing of (1); therefore, the latter derives from (2), which shows how the IRR is applied to the beginning-of-period capital C_{t-1}. As a result, the IRR is a rate of return on an implied time series of beginning-of-period interim values that it 'internally' infers. To put it equivalently, the IRR equation, as a result of eq. (2) and the terminal condition $C_n = 0$, accomplishes an implicit estimation of the interim values, although it masks them through use of a simple discounted cash flow equation which inadvertently belies their very existence. In other words, while (2) shows that the IRR is a solution to a polynomial equation (NPV=0), if it also to be a *rate of return*, then the capitals it is earned on cannot be avoided (a rate of return is an amount of return per unit of capital invested).

2. ICM and the contradiction

The ICM proposition consists of asking "What IRR would the cash flows associated with a PEI have produced if the periodic cash flows contributed into it (and distributed out of it) had, alternatively, been invested in a benchmark portfolio with periodic returns reflecting that of a public market index?". Exhibit 3 illustrates, in step by step fashion, the mechanics of the associated ICM algorithm, under the assumption of an investment in a portfolio based on periodic rates of return from the S&P 500 total return index. We call this portfolio the (ICM) benchmark portfolio. The third column supplies the index's annual holding period rates of return during the time period indicated. The growth rate of capital in 1989 is 31.69%: $25 is invested at 31.69% to produce an ending balance of $32.92. Adding to that the end-of-year contribution of $25, the capital invested at the beginning of 1990 becomes $57.92. This amount is, in turn, invested at –3.10%, so that the ending value of the ICM benchmark portfolio decreases to $56.13. Adding to that the end of year contribution of $25, the capital invested at the beginning of 1991 becomes $81.13. This amount is then invested at 30.47% and so on and so forth iteratively. The second column of Exhibit 3 indicates the amounts of capital invested, period by period, in the benchmark portfolio, according to the ICM approach. In general, the capital invested in the ICM benchmark portfolio, which we denote as C_t^*, grows recursively as

$$\overbrace{C_t^*}^{\text{capital at time } t} = \overbrace{C_{t-1}^*}^{\text{capital at time } t-1} + \overbrace{r_t C_{t-1}^*}^{\text{return}} - \overbrace{F_t}^{\text{distribution/contribution}} \qquad (4)$$

where $r_t = (C_t^* + F_t - C_{t-1}^*)/C_{t-1}^*$ is the benchmark's holding period rate. In each period, the capital increases by the index's periodic return r_t and decreases (increases) by the distribution (contribution) F_t. Note also that, in so doing, the ICM benchmark portfolio replicates the exact cash flow profile of the PEI. The only difference lies in the terminal value, C_n^*. In our example, as shown in the last column of Exhibit 3, the benchmark portfolio's terminal value is $-\$13.49$, which means that it has produced \$13.49 less terminal value (or final net cash flow, if liquidated) than did the PEI. It is thus clear that the PEI has outperformed the benchmark portfolio by \$13.49 – in final year (1999) dollars.[3]

ICM proponents next ask the question "How much better or worse, from a rate of return standpoint, is the PEI?" And they posit that the answer can be found by computing the IRR of the ICM benchmark portfolio and then subtracting it from the 17.91% IRR of the PEI, to get a ΔIRR. As a result of that \$13.49 reduction in ending value, the benchmark portfolio's IRR, as calculated at the bottom right of Exhibit 4a, is 17.49%. So, the answer is that the PEI has outperformed the benchmark portfolio by 17.91% $-$ 17.49% = 0.42%.[4]

However, this outperformance result is predicated on the benchmark portfolio's IRR of 17.49% which is itself invalid, as the IRR presupposes (i.e., it applies to) values that are different from the ICM benchmark portfolio's values. That is, the benchmark portfolio has two competing values at any point in time. Specifically, as presented earlier, an IRR concocts its own implied interim values. Using (2), in this example, those implied beginning-of-year values associated with the benchmark portfolio are $C_{1989} = \$25, C_{1990} = 25 \cdot (1 + 17.49\%) + 25 = \$54.37, \quad C_{1991} = 54.37 \cdot (1 + 17.49\%) + 25 = \$88.88$, and so on and so forth (see the second column of Exhibit 4b for all the values). These implied valuations are at odds with the true values of the benchmark portfolio. That is, the IRR-implied values, C_t, as just computed via (2), are different from the values, C_t^*, as computed earlier via (4) above. Exhibit 5 shows these two different, and competing, sets of values side by side for our example. The conclusion is that the very same periodic NAVs that produce the final ICM cash flow of \$111.51, and hence ultimately its 17.49% IRR, are contradicted

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[3] In cases where the investment is not yet liquidated, the final cash flow may include a constructive distribution of any NAV.

[4] In geometric terms, the deviation is 0.36%.

by the implied beginning of period values that the 17.49% IRR is earned on. Hence, the notion that the ICM approach can produce a public market "equivalent" portfolio earning that specific IRR is illegitimate. To underscore the point, no investment can have two different values simultaneously. For this reason, the use of IRR within ICM-oriented algorithms should be avoided.

As a result, we have a problem: the ICM approach is based on the notion of IRR, which is a rate of return on beginning-of-period values that contradict the ICM benchmark portfolio's beginning-of-period values.

There is a second problem, a well-known, if unlikely, one: the PEI (and therefore the benchmark portfolio's) cash flows can swing considerably, which implies the possibility of multiple IRRs (or even the non-existence of IRR if the benchmark portfolio's terminal value is sufficiently negative). In such cases, the notion of the benchmark portfolio's IRR is impaired, and the approach cannot be used. So, the approach cannot be considered a robust one.

A third issue is the non-additivity of return rates, as noted occasionally in the IRR literature and quite recently by Magni (2013) and Gredil et al. (2014). In simple words, additivity of a rate of return means that if $100 are invested at 30% and generates $10 more than investing $100 in a 20% asset, then the former outperforms the latter by 10%, that is, 20%+10%=30%; IRR does not enjoy this property, except in the simple case of only one yearly period and equal capital. In the same vein, the ΔIRR (i.e., the difference between the PEI's IRR and the IRR of the investment implied in the ICM approach) does not measure the incremental return of the PEI over (or under) that of the benchmark portfolio. So, the 0.42% does not measure the true deviation of the PEI's rate of return from the benchmark portfolio's rate of return.

Note that, to the extent that they infer a hypothetical reference portfolio and compute an IRR, other PME approaches are susceptible to the same criticism as the ICM. Indeed, the IRR computed from the benchmark portfolio will internally infer values that differ from the benchmark portfolio interim values.[5]

How is one to resolve this threefold conundrum? In other words, is it possible to make use of the true NAVs associated with the ICM algorithm yet replace its rate of return algorithm with one that computes a rate of return which (i) does not contradict those values, (ii) has no problem of multiplicity and (iii) is additive? That is the subject of the following section.

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[5] Kaplan and Schoar's (2005) approach, for one, does not incur this criticism, for it does not presuppose the computation of an IRR.

3. The Aggregate Return on Investment

Fortunately, there is a broader class of money-weighted rate of return (MWRR) metrics, termed AIRR, introduced in Magni (2010, 2013), which includes other MWRR alternatives that are particularly well-suited to addressing this PEI rate of return benchmarking problem. One such simple alternative is proposed herein. The metric we propose is a modified AIRR, named Aggregate Return on Investment (AROI); it has been introduced in Magni (2011) and applied to capital asset investments in Magni (2015). The AROI approach assumes a financial portfolio replicating the project's cash flows and earning the cost of capital (i.e., the rate of return of an equivalent-risk asset), in the very same way as the benchmark portfolio in the ICM approach periodically earns the index return. In particular, the AROI is defined as the total net cash flow (whole dollar profit) divided by the total capital invested (see Magni 2015, eq. (5)). To compute the AROI of the ICM benchmark portfolio, we simply divide the index investment's whole dollar profit by its total capital:

$$\text{AROI}_{ICM} = \frac{F_0 + F_1 + \cdots + F_n + C_n^*}{C_0 + C_1^* + \cdots + C_{n-1}^*} \tag{5a}$$

Analogously, for the PEI, the AROI is

$$\text{AROI}_{PEI} = \frac{F_0 + F_1 + \cdots + F_n}{C_0 + C_1^* + \cdots + C_{n-1}^*}. \tag{5b}$$

For the example being considered, the total capital is the sum of the values in the second column of Exhibit 3; such values, which are the true values associated with the ICM algorithm, are also echoed into the second column of Exhibit 5. That sum is $1,286.92. As shown on the left hand side of Exhibit 6, AROI_{PEI} is $270.00/$1,286.92, or 20.98%. The benchmark portfolio has a numerator of $256.51, which reflects the fact that its whole dollar profit was $13.49 less than that of the PEI being adjudged. As shown on the right hand side of Exhibit 6, for this example, this yields a rate of return of $256.51/$1,286.92, or 19.93%. So, the active return, denoted as ΔAROI $(= \text{AROI}_{PEI} - \text{AROI}_{ICM})$, indicates that the PEI has outperformed the benchmark portfolio by 20.98% minus 19.93%, or 1.05%.[6]

Note that we use only true NAVs to compute the AROI result and do not have to devise fictitious interim values, the latter which is done by the IRR (see Magni, 2013, for a list of eighteen flaws of the IRR). Also, no problem of multiplicity of rate of return solutions exists for the AROI, as it is not derived from a polynomial equation. AROI is a simple ratio, expressing return per unit of

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[6] For those IRR devotees who might be tempted to ignore the aforementioned ICM/IRR illegitimacy issue and hope that the result is "close enough", we note that this outperformance is 2.5 times as large as the 0.42% result we obtained earlier, in section 2, using the invalidated IRR approach to this problem.

(total) invested capital. Furthermore, contrary to a ΔIRR calculation, the ΔAROI result preserves additivity, since the capital base is the same (see Appendix). Therefore, we may indeed subtract the benchmark portfolio's AROI from the PEI's AROI and properly compute an active return, ΔAROI, expressing the incremental return of PEI over (or under) the return of the portfolio investment that could have been made.

In closing, we note that, in terms of computational complexity, the AROI metric is simple. It is easily solved in closed form requiring math no more complex than multiplication and division and is devoid of any complex algorithms. And, it is not subject to any of the idiosyncrasies of IRR, such as multiple solutions, non-additivity, etc. Most importantly, it uses the benchmark portfolio's true invested capital amounts only and so, unlike IRR, it does not invalidate itself with a second, contradictory set of implied capital amounts.

Concluding remarks

It is very likely that most users of IRR do not fully appreciate the fact that, for an investment represented by a given sequence of cash flows, IRR is computed as a constant rate of return that has, associated with it, internally implied beginning-of-period capital values upon which its rate of return is earned. The recognition of implied interim values is a critical component of IRR in proving that its use in an ICM-style analysis is illegitimate. So, the next question is: "Is there another Money-Weighted Rate of Return metric that can replace IRR for this style of analysis?" Fortunately, the answer is "yes", and one particularly simple and intuitive candidate is a MWRR named Aggregate Return on Investment (AROI), introduced in Magni (2011) and later developed for capital asset investments in Magni (2015). The AROI is a variant of the AIRR approach presented in Magni (2010, 2013). It is a simple ratio, and can use the very same capital values as insightfully derived by Long & Nickels in their ground-breaking 1996 ICM analysis. Furthermore, it is an additivity-preserving measure in that ΔAROI correctly expresses the incremental return of the PEI over the benchmark portfolio, not to mention that it does not incur the problem of no or multiple rate of return solutions known to be possibilities when using IRR-based approaches.

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Appendix

The AROI preserves additivity. As evidence of such, consider that, as we have seen, the difference between the whole dollar profit of the benchmark portfolio, as compared to that of the PEI, is equal to the terminal value of the benchmark portfolio, C_n^*, and such a value measures by how much the PEI has outperformed (if negative) or underperformed (if positive) the benchmark portfolio. That is, it expresses the opportunity cost of investing in the PEI. It can be easily shown that it is equal to the value added of the PEI (the future accumulated value of the PEI's cash flows):

$VA = \sum_{t=0}^{n} F_t \prod_{k=1}^{t}(1 + r_k)$ where $r_k = (C_k^* + F_k - C_{k-1}^*)/C_{k-1}^*$ is the benchmark portfolio's period return. Consider now the algebraic sum of the benchmark portfolio's net cash flows. It can be shown that it coincides with the sum of the period returns:

$$\sum_{t=0}^{n} F_t + C_n^* = \sum_{t=1}^{n} r_t C_{t-1}^* \tag{A.1}$$

(see also Magni 2015, eq. (6)). This implies that the value added is equal to the difference between the whole dollar profit and the reference portfolio's return:

$$VA = \sum_{t=0}^{n} F_t - \left(\sum_{t=1}^{n} r_t C_{t-1}^* \right) \tag{A.2}$$

Dividing by $C^* = C_0 + C_1^* + \cdots + C_{n-1}^*$ (total capital invested),

$$\frac{VA}{C^*} = \frac{\sum_{t=0}^{n} F_t}{C^*} - \frac{\sum_{t=1}^{n} r_t C_{t-1}^*}{C^*} \tag{A.3}$$

Using (A.1) and (5a)-(5b), one gets

$$\frac{VA}{C^*} + AROI_{ICM} = AROI_{PEI} \tag{A.4}$$

(see also Magni 2015, eq. (7)). Equation (A.4) can also be written

$$AROI_{PEI} = \Delta AROI + AROI_{ICM} \tag{A.5}$$

where $\Delta AROI = VA/C^*$ is the active AROI. Equation (A.5) shows that the AROIs are additive measures and that $\Delta AROI$ is an appropriate measure of the incremental return over the benchmark. In our example, $\Delta AROI = 1.05\%$; multiplied by the total capital, $C^* = 1286.92$, it produces the value added by the PEI, $VA = 13.49$, which, as we know, is the terminal value of the benchmark portfolio (changed in sign).

Exhibit 1. The Private Equity Investment (PEI)

Year	PEI's Cash Flow*
1988	($25.00)
1989	($25.00)
1990	($25.00)
1991	($25.00)
1992	($10.00)
1993	$5.00
1994	$20.00
1995	$35.00
1996	$50.00
1997	$65.00
1998	$80.00
1999	$125.00
Whole Dollar Profit (sum of cash flows)	$270.00
IRR	17.91%

*Cash flow occurs at the end of the year

Exhibit 2. The IRR's Implied Values of the PEI

Year	IRR's Implied Invested Capital	Internal Rate of Return	IRR's Implied Ending Balance	PEI's Cash Flow (from Exhibit 1)	IRR's Implied Residual Balance [1]
1988				($25.00)	$25.00
1989	$25.00	17.91%	$29.48	($25.00)	$54.48
1990	$54.48	17.91%	$64.24	($25.00)	$89.24
1991	$89.24	17.91%	$105.23	($25.00)	$130.23
1992	$130.23	17.91%	$153.55	($10.00)	$163.55
1993	$163.55	17.91%	$192.86	$5.00	$187.86
1994	$187.86	17.91%	$221.51	$20.00	$201.51
1995	$201.51	17.91%	$237.61	$35.00	$202.61
1996	$202.61	17.91%	$238.91	$50.00	$188.91
1997	$188.91	17.91%	$222.75	$65.00	$157.75
1998	$157.75	17.91%	$186.01	$80.00	$106.01
1999	$106.01	17.91%	$125.00	$125.00	$0.00
A = Prior E	**B**	**C=A·(1+B)**	**D**	**E = (C) - (D)**

[1] Mathematically, each Residual Balance can be derived more directly by computing the Net Present Value, at the IRR rate, of all remaining cash flows. By definition of IRR, the Implied IRR Residual Value at the end of 1999 will always be zero.

Exhibit 3. The ICM Equivalent Benchmark Portfolio (S&P 500)

Year	ICM Invested Capital	Index's Rate Of Return	ICM Ending Balance	PEI's Cash Flow (from Exhibit 1)	ICM Residual Balance
1988				($25.00)	$25.00
1989	$25.00	31.69%	$32.92	($25.00)	$57.92
1990	$57.92	-3.10%	$56.13	($25.00)	$81.13
1991	$81.13	30.47%	$105.85	($25.00)	$130.85
1992	$130.85	7.62%	$140.82	($10.00)	$150.82
1993	$150.82	10.08%	$166.02	$5.00	$161.02
1994	$161.02	1.32%	$163.14	$20.00	$143.14
1995	$143.14	37.58%	$196.94	$35.00	$161.94
1996	$161.94	22.96%	$199.12	$50.00	$149.12
1997	$149.12	33.36%	$198.87	$65.00	$133.87
1998	$133.87	28.58%	$172.12	$80.00	$92.12
1999	$92.12	21.04%	$111.51	$125.00	($13.49)

Exhibit 4a. Cash Flows of ICM's Equivalent Investment

Year	ICM's Cash Flow
1988	($25.00)
1989	($25.00)
1990	($25.00)
1991	($25.00)
1992	($10.00)
1993	$5.00
1994	$20.00
1995	$35.00
1996	$50.00
1997	$65.00
1998	$80.00
1999	**$111.51**
IRR	17.49%

Exhibit 4b. The ICM Equivalent Investment Thru An IRR Lens

Year	IRR's Implied Invested Capital	Rate of Return	Ending Balance	ICM's Cash Flow	Residual Balance
1988				($25.00)	$25.00
1989	$25.00	17.49%	$29.37	($25.00)	$54.37
1990	$54.37	17.49%	$63.88	($25.00)	$88.88
1991	$88.88	17.49%	$104.43	($25.00)	$129.43
1992	$129.43	17.49%	$152.08	($10.00)	$162.08
1993	$162.08	17.49%	$190.43	$5.00	$185.43
1994	$185.43	17.49%	$217.86	$20.00	$197.86
1995	$197.86	17.49%	$232.48	$35.00	$197.48
1996	$197.48	17.49%	$232.02	$50.00	$182.02
1997	$182.02	17.49%	$213.86	$65.00	$148.86
1998	$148.86	17.49%	$174.91	$80.00	$94.91
1999	$94.91	17.49%	$111.51	$111.51	$0.00

**Exhibit 5. The ICM Equivalent Benchmark Portfolio -
Showing Two Sets Of Contradictory Values**

Year	ICM Invested Capital	IRR's Implied Invested Capital
1989	$25.00	$25.00
1990	$57.92	$54.37
1991	$81.13	$88.88
1992	$130.85	$129.43
1993	$150.82	$162.08
1994	$161.02	$185.43
1995	$143.14	$197.86
1996	$161.94	$197.48
1997	$149.12	$182.02
1998	$133.87	$148.86
1999	$92.12	$94.91
Total	$1,286.92	$1,466.33

Exhibit 6. AROI Active Return Analysis

PEI - AROI ANALYSIS			ICM PORTFOLIO - AROI ANALYSIS		
Year	Invested Capital (beginning of yr)	PEI Cash Flows (end of yr)	Year	Invested Capital (beginning of yr)	ICM Cash Flows (end of yr)
1988		($25.00)	1988		($25.00)
1989	$25.00	($25.00)	1989	$25.00	($25.00)
1990	$57.92	($25.00)	1990	$57.92	($25.00)
1991	$81.13	($25.00)	1991	$81.13	($25.00)
1992	$130.85	($10.00)	1992	$130.85	($10.00)
1993	$150.82	$5.00	1993	$150.82	$5.00
1994	$161.02	$20.00	1994	$161.02	$20.00
1995	$143.14	$35.00	1995	$143.14	$35.00
1996	$161.94	$50.00	1996	$161.94	$50.00
1997	$149.12	$65.00	1997	$149.12	$65.00
1998	$133.87	$80.00	1998	$133.87	$80.00
1999	$92.12	$125.00	1999	$92.12	$111.51
Totals	$1286.92 (A)	$270.00 (B)	Totals	$1286.92 (A)	$256.51 (C)
PEI AROI (B/A):		20.98%	ICM AROI (C/A):		19.93%
AROI Active Return		1.05%	Value Added		$13.49

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