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Direct numerical simulation of the flow around a rectangular cylinder at a moderately high Reynolds number

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Abstract

We report a Direct Numerical Simulation (DNS) of the flow around a rectangular cylinder with a chord-to-thickness ratio $B/D = 5$ and Reynolds number $Re = 3000$. Global and single-point statistics are analysed with particular attention to those relevant for industrial applications such as the behaviour of the mean pressure coefficient and of its variance. The mean and turbulent flow is also assessed. Three main recirculating regions are found and their dimensions and turbulence levels are characterized. The analysis extends also to the asymptotic recovery of the equilibrium conditions for self-similarity in the fully developed wake. Finally, by means of two-point statistics, the main unsteadinesses and the strong anisotropy of the flow are highlighted. The overall aim is to shed light on the main physical mechanisms driving the complex behaviour of separating and reattaching flows. Furthermore, we provide well-converged statistics not affected by turbulence modelling and mesh resolution issues. Hence, the present results can also be used to quantify the influence of numerical and modelling inaccuracies on relevant statistics for the applications.

Keywords: Flow benchmark; Rectangular cylinder; Flow reattachment; Direct Numerical Simulation;

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1. Introduction

The flow around bluff bodies with sharp corners is known to be of overwhelming interest for several wind engineering applications [1]. The case of a rectangular cylinder encompasses the range of bluff bodies from a flat plate normal to the flow, to a square cylinder, and, finally, to a flat plate parallel to the flow, as its chord-to-thickness ratio is varied from zero to infinity. For these reasons, these kind of flows have been the subject of several numerical and experimental studies. Of particular interest for civil engineering applications is the case of slender bodies typical of buildings and structures. A peculiarity of these shapes resides in the fact that the flow exhibits a large-scale separation at the leading-edge and also a reattachment before the definitive separation at the trailing-edge. Indeed, while the shedding instability in the wake is observed in all bluff bodies, only long bluff bodies present further instabilities which are due to the separating and reattaching leading-edge shear layer. This leads to the formation of an additional shedding of large-scale vortices before the trailing edge. A detailed investigation into the nature of this separating and reattaching flow is found in Cherry *et al.* [2]. Despite the fact that these kind of flows have been the subject of several numerical and experimental studies, the topic is still attractive, as highlighted in a recent work by Bruno *et al.* [3]. From an applicative point of view, the interest is given by the fact that both experimental and numerical techniques appear to be unable to tackle the problem in a unequivocal way. Indeed, a large variability of results is found in the literature, even for global or first order statistics, see again the review of Bruno *et al.* [3]. The reason of these discrepancies is the high sensitivity of the flow on the test boundary conditions and measurement accuracy in experiments and on the turbulence model, numerical schemes and mesh properties in CFD analysis. Here, we focus on the numerical approach.

For low Reynolds numbers, $10^2 < Re < 10^3$ where $Re = U_\infty D/\nu$, U_∞ the free-stream velocity, D the rectangular cylinder thickness and ν the kinematic viscosity, the flow around rectangular cylinders has been studied in several

31 works, see e.g. Nakamura *et al.* [4], Ohya *et al.* [5], Hourigan *et al.* [6], and
32 Tan *et al.* [7]. The main aim of the above mentioned works is the assessment of
33 the main instabilities of the flow and of the self-sustaining mechanisms which
34 generate them. Concerning the high Reynolds number regime, $Re > 10^4$, it is
35 worth mentioning the works of Shimada and Ishihara [8] and of Yu and Ka-
36 reem [9] where RANS (Reynolds Average Navier-Stokes) and LES (Large Eddy
37 Simulation) techniques are respectively used. In this context, it is important to
38 point out a benchmark activity on the aerodynamics of rectangular cylinders at
39 Reynolds numbers of the order of 10^4 , i.e. the BARC project (Benchmark on
40 the Aerodynamics of a Rectangular 5:1 Cylinder) [10]. Within this framework,
41 a series of experiments and simulations have been conducted aiming at estab-
42 lishing reliable standards for the simulation and measurement of such a flow
43 configuration, see e.g. Bruno *et al.* [11], Mannini *et al.* [12], Ricci *et al.* [13]
44 and Patruno *et al.* [14] for its extension to non-null angles of attack.

45 As summarized in Bruno *et al.* [3], the recent results within the BARC
46 project are still characterized by a large scatter, thus highlighting that a clear
47 picture of the combined influence of mesh resolution, turbulence model and
48 boundary conditions on the flow statistics is still missing. One of the possible
49 reasons is that, up to now, no reference data are available in the literature, i.e.
50 experimental data obtained under well-defined boundary conditions (e.g. free-
51 stream turbulence level) and unaffected by measurement errors, or numerical
52 data not influenced by modelling and mesh resolution issues. Indeed, to the
53 best of the authors' knowledge, no Direct Numerical Simulation (DNS) for suf-
54 ficiently high Reynolds number has been performed in such a flow configuration.
55 We found only two attempts in the literature. In the first one, Tamura *et al.*
56 [15] approached the problem by means of a finite difference technique at high
57 Reynolds number, $Re = 10^4$. However, the grid resolution adopted was not
58 fine enough to capture the smallest scales of motion and, hence, the simulation
59 reported appears to be more an implicit LES than a DNS. More recently Hour-
60 igan *et al.* [6] proposed a more accurate analysis through a spectral-element
61 method. However, the DNS data reported refer to very low Reynolds numbers,

62 namely from $Re = 350$ to $Re = 500$, and a fully developed turbulent state is
63 not achieved.

64 In the present work we produce, for the first time, high-fidelity data of
65 the flow around a rectangular cylinder with chord-to-thickness ratio $B/D =$
66 5 and Reynolds number $Re = 3000$. The study is aimed at understanding
67 the main physical mechanisms driving the flow and at providing statistics, not
68 affected by numerical issues, to be used for the validation and calibration of
69 CFD techniques. For obvious computational cost reasons, the Reynolds number
70 considered, $Re = 3 \cdot 10^3$, is smaller than the ones considered in the recent
71 literature. However, let us point out that as shown by Sasaki and Kiya [16], the
72 flow develops the main turbulent structures typical of larger Reynolds numbers
73 already for $Re > 380$. By further increasing Re , it is also found that the bubble
74 length does not increase significantly anymore. It is also worth mentioning that,
75 based on spectral arguments, Nakamura *et al.* [17] argue that an asymptotic
76 large Reynolds number regime is attained for $Re = 3000$ since for $Re > 3000$
77 the Strouhal number of the spectral peak does not increase anymore. Based on
78 these results, we argue that the considered Reynolds number is sufficiently large
79 to capture the main physical features observed at larger Reynolds numbers. As
80 an example, the two main unsteadinesses observed by Kiya and Sasaki [18, 19]
81 for very large Reynolds numbers and consisting of a shedding of vortices from
82 the separation bubble and of a large scale oscillation encompassing the entire
83 flow field, are found to be reproduced both qualitatively and quantitatively at
84 the present Reynolds number (see section §3 for the details).

85 The paper is organized as follows. A description of the numerical simulation
86 and of the statistical procedure is reported in section §2. The main statistical
87 properties of the flow, with particular attention to those mostly debated in
88 the BARC project, are shown in section §3. In order to rigorously assess the
89 physical features characterizing the flow, single-point and two-point statistics
90 are analysed in detail in sections §4 and §5. The paper is then closed by final
91 remarks in section §6.

92 2. Direct Numerical Simulation and statistical convergence

93 A Direct Numerical Simulation has been performed to study the flow around
94 a rectangular cylinder. The evolution of the flow is governed by the continuity
95 and momentum equations,

$$\begin{aligned} \frac{\partial u_i}{\partial x_i} &= 0 \\ \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \end{aligned} \quad (1)$$

96 where $x = x_1$ ($u = u_1$), $y = x_2$ ($v = u_2$), $z = x_3$ ($w = u_3$) are the stream-
97 wise, vertical and spanwise directions (velocity components), p is the pressure
98 field and $Re = U_\infty D/\nu$ is the Reynolds number where U_∞ is the free-stream
99 velocity, D is the thickness of the rectangular cylinder and ν is the kinematic
100 viscosity. In accordance with the above equations, all the variables presented in
101 the following will be reported in a dimensionless form by using D as length scale
102 and D/U_∞ as time scale. A cell-centered finite volume method has been chosen
103 to discretize the equations by means of the OpenFOAM[®] open source code
104 [20]. Time integration is performed by means of a second-order backward Euler
105 implicit scheme while convective and diffusive fluxes at the volume faces are
106 evaluated through a second-order central difference scheme. Finally, a pressure-
107 implicit split-operator algorithm [21] is used to numerically solve the pressure-
108 velocity coupling. Given the simple geometry of the problem, a block-structured
109 Cartesian grid is adopted. Inlet-outlet boundary conditions are imposed in the
110 streamwise direction. The inlet condition is a simple unperturbed flat velocity
111 profile. The outlet boundary condition combines a Neumann/Dirichlet condi-
112 tion. In particular, a stress-free (zero gradient) condition is enforced when the
113 flow exits the boundary, while a zero velocity vector is imposed when an in-
114 ward flow is detected. The same kind of boundary condition is imposed in the
115 vertical direction, the only difference being that in case of inward flow the im-
116 posed Dirichlet condition equals the free-stream inlet velocity. Finally, periodic
117 conditions are imposed in the spanwise direction.

118 The flow case consists of a rectangular cylinder whose dimensions are $(L_x, L_y) =$

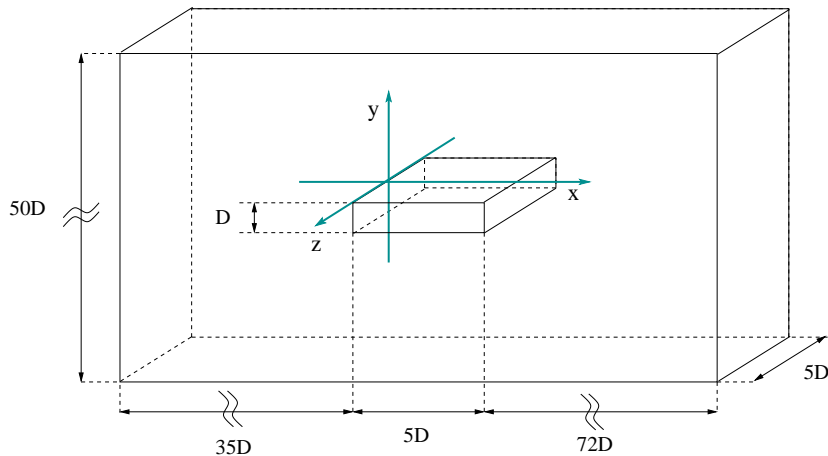


Figure 1: Configuration of the system.

119 $(5D, D)$. The Reynolds number considered is $Re = 3000$. The extent of the nu-
 120 merical domain is $(\mathcal{D}_x, \mathcal{D}_y, \mathcal{D}_z) = (112D, 50D, 5D)$ and is found large enough to
 121 not interfere with the flow dynamics, see the analysis of the spanwise correla-
 122 tion function shown in section §5. A sketch of the system configuration and of
 123 the reference coordinate system is reported in figure 1. The structured Carte-
 124 sian grid employed is composed by $1.5 \cdot 10^7$ volumes. A multi block-structured
 125 approach is used by employing 5 main blocks characterized by a stepwise vari-
 126 ation of the number of volumes in the spanwise direction. In particular, in
 127 the inner block, the number of volumes in the (x, z) -plane above the rectangle
 128 is $(Nx, Nz) = (128, 144)$ in the streamwise and spanwise direction, respec-
 129 tively. The volume distribution is homogeneous in the spanwise direction while
 130 in the streamwise and vertical directions a geometric progression is adopted,
 131 $\Delta x_i = k_x^{i-1} \Delta x_1$ and $\Delta y_j = k_y^{j-1} \Delta y_1$ with $k_x = 1.06$, $k_y = 1.04$, $\Delta x_1 = 0.004$
 132 and $\Delta y_1 = 0.004$. This approach is used to obtain higher resolution levels in the
 133 near-wall leading- and trailing-edge regions. Such a practice leads to a mean
 134 wall resolution of $(\overline{\Delta x^+}, \overline{\Delta y^+}, \overline{\Delta z^+}) = (6.1, 0.31, 5.41)$, where $\overline{(\cdot)}$ denotes the
 135 streamwise average along the rectangle length and the superscript $+$ implies
 136 normalization with friction units. The time step varies during simulation to
 137 obtain a condition $CFL < 1$ in each point of the domain, the resulting average

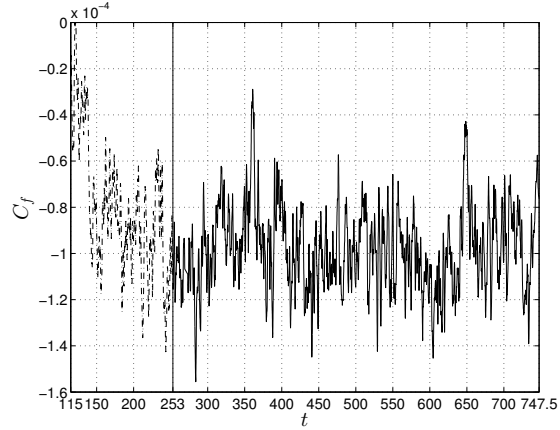


Figure 2: Time-behaviour of the friction drag coefficient C_f . The vertical line denotes the end of the initial transient and the start of the fully developed state used for the computation of the statistics. To note that the plot starts from $t = 115$. Indeed, from $t = 0$ to $t = 115$ a precursory coarser simulation has been used to reduce the computational time for reaching a flow state close to the statistical equilibrium.

138 time step being $\Delta t = 0.0023$.

139 In the present flow case, the computational demand for well-converged statis-
 140 tics denoted as $\langle \cdot \rangle$, is mitigated by the statistical stationarity of the flow field
 141 and by the statistical homogeneity in the spanwise direction. Furthermore, the
 142 flow exhibits certain statistical symmetries in the vertical direction which are
 143 better expressed by shifting the origin of the vertical coordinate to the centre of
 144 the prism, $\tilde{y} = y - D/2$. Indeed, the transformation $\tilde{y} \rightarrow -\tilde{y}$ leaves quantities
 145 like $U = \langle u \rangle$, $\langle u_i u_i \rangle$ statistically invariant while reversing the sign of quantities
 146 like $V = \langle v \rangle$, $\langle uv \rangle$ and $\partial \langle \cdot \rangle / \partial \tilde{y}$. In conclusion, the average of a generic quantity
 147 β is defined as

$$\langle \beta \rangle(x, \tilde{y}) = \frac{1}{N} \sum_{i=1}^N \frac{1}{2} \left(\frac{1}{L_z} \int_{-L_z/2}^{L_z/2} \beta(x, \tilde{y}, z, t) dz \pm \frac{1}{L_z} \int_{-L_z/2}^{L_z/2} \beta(x, -\tilde{y}, z, t) dz \right), \quad (2)$$

148 where the sum and difference of the two integrals is given by the symmetric or
 149 antisymmetric property of the considered variable, respectively. The number of
 150 fields sampled during time is $N = 317$. These samples are taken once the initial

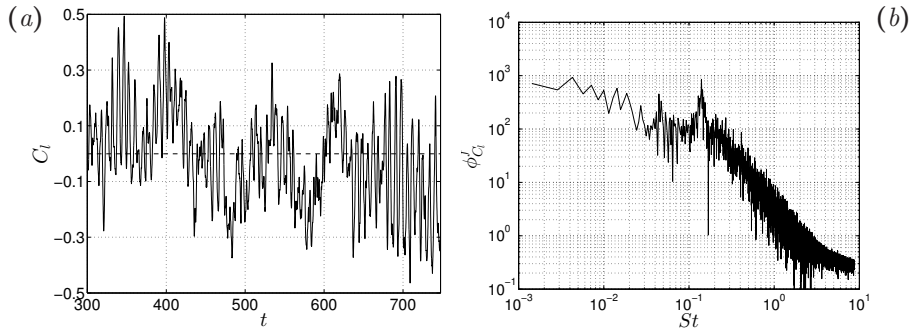


Figure 3: (a) Time evolution of the lift coefficient C_l . (b) Frequency spectrum of the lift coefficient, $\phi_{C_l}^f$, as a function of the Strouhal number St .

151 transition of the flow field is washed out, see figure 2, and are taken with a time
 152 separation $\Delta T = T$ where $T = D/U_\infty$ is the characteristic time scale of the
 153 flow. In the following, the customary Reynolds decomposition of the velocity
 154 field is adopted, $u_i = U_i + u'_i$, where U_i and u'_i denote the mean and fluctuating
 155 velocity.

156 3. Main properties of the flow

157 We start the analysis by considering the main integral parameters of the flow,
 158 i.e. the lift, C_l , and drag, C_d , coefficients. Obviously, for symmetry reasons,
 159 the average lift coefficient is null, $\tilde{C}_l = 0$ where $\tilde{\cdot}$ denotes the time average.
 160 As shown in figure 3(a), instantaneously the lift coefficient is not zero, but it
 161 fluctuates in time. Different time scales are recognized and can be studied by
 162 considering the frequency spectrum defined as

$$\phi_{C_l}^f(St) = \hat{C}_l \hat{C}_l^* \quad (3)$$

163 where $\hat{\cdot}$ denotes the Fourier transform, $*$ the complex conjugate and St the
 164 dimensionless frequency, i.e. the so-called Strouhal number. As shown in figure
 165 3(b), the frequency spectrum confirms the presence of different temporal scales.
 166 In particular, a clear peak for $St \approx 0.14$ is present and will be shown in the
 167 following to be related with the frequency of the large scale vortex detachment

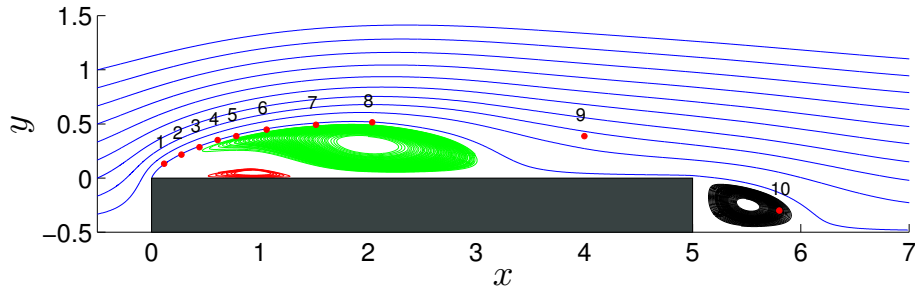


Figure 4: Streamlines of the mean velocity field $(U, V)(x, y)$. The green lines show the *primary vortex*, the red lines mark the *secondary vortex* and the black lines denote the *wake vortex*. The red dots denote the locations of the probes used for the computation of time spectra in section §5.

168 in the wake. A second peak is recognizable at a lower frequency, namely $St \approx$
 169 0.042 . This very low frequency is responsible for the long term fluctuations of C_l
 170 observed in figure 3(a) and could explain the scatter of the data of \tilde{C}_l obtained
 171 in different works, see the review of Bruno *et al.* [3], as a result of statistical
 172 convergence problems. The frequencies of the two spectral peaks detected here
 173 quantitatively agree with those observed in Kiya and Sasaki [18, 19] for very
 174 large Reynolds numbers. In accordance with these works, we argue that the
 175 peak at $St \approx 0.14$ is related with a large scale shedding of vortices from the
 176 main recirculating region while the very slow peak at $St \approx 0.042$ is due to
 177 the presence of a low-frequency unsteadiness encompassing the entire flow field.
 178 Concerning the drag coefficient, we measure that $\tilde{C}_d = 0.96$.

179 Let us now consider the topology of the mean flow field. As shown in figure
 180 4, the streamlines of the mean flow highlight the presence of a large scale recir-
 181 culation extending from the leading edge up to $x_r \approx 3.65$, see the green lines in
 182 figure 4. The centre of rotation, defined as the singularity point within the recir-
 183 culating region where the mean velocity field vanishes $(U, V) = (0, 0)$, is located
 184 at $(x_c^{pv}, y_c^{pv}) = (2.04, 0.35)$. This separation bubble will be hereafter called the
 185 *primary vortex*. As shown by the isocontours of mean pressure in figure 5, this
 186 separated region is associated with a large area of low pressure with a mini-

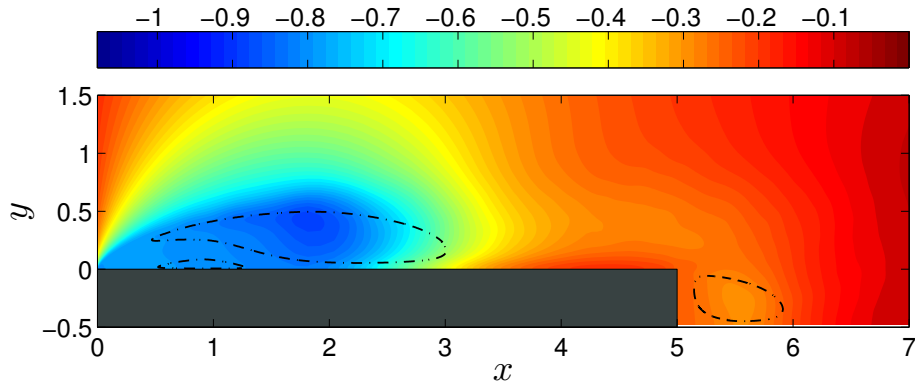


Figure 5: Isocontours of the mean pressure field $P(x, y)$. The dashed lines report the location of the *primary vortex*, *secondary vortex* and *wake vortex*.

187 mum located at $(x_p^{pv}, y_p^{pv}) = (1.81, 0.42)$. It is worth noting that the location
 188 of this minimum does not coincide with the centre of the *primary vortex*, thus
 189 highlighting the strongly inhomogeneous non-axisymmetric shape taken by the
 190 recirculating flow. Actually, a second recirculating region is present and high-
 191 lighted with red lines in figure 4. This *secondary vortex* takes place below the
 192 *primary vortex*. Indeed, the reverse flow induced in the near-wall region by the
 193 *primary vortex* creates a boundary layer moving upstream. As shown by the
 194 isocontours of the mean pressure field in figure 5, the induced boundary layer
 195 undergoes an adverse pressure gradient, hence it decelerates, becomes thicker
 196 and, finally, breaks down leading to a further separation. Hence, the *secondary*
 197 *vortex*, being induced by the *primary vortex*, is counter-rotating with respect
 198 to the *primary vortex* and its characteristic length and time scales are smaller
 199 than those of the *primary vortex*. The *secondary vortex* is found to extend for
 200 $0.4 < x < 1.4$. The intensity of the mean flow within this region is very low,
 201 thus highlighting how this object is difficult to capture from a statistical point
 202 of view, see Bruno *et al.* [3] for a review of the different results in literature.
 203 After the reattachment point, for $x > x_r$, the flow evolves in a downstream
 204 boundary layer and finally detaches at the trailing edge where it develops into
 205 the wake. As shown with black lines in figure 4, the separated wake highlights

206 a *wake vortex* extending from the trailing edge down to $x \approx 6.2$ and centered at
 207 $(x_c^{wv}, y_c^{wv}) = (5.5, -0.25)$. As for the *primary vortex*, the *wake vortex* sets a low
 208 pressure region centered at $(x_p^{wv}, y_p^{wv}) = (5.52, -0.27)$, see figure 5. The main
 209 difference with respect to the leading-edge recirculation is given by the fact that
 210 the extent and intensity of the low pressure area are smaller.

211 Let us now study the streamwise behaviour of the skin friction and pressure
 212 coefficients along the horizontal surface of the rectangular cylinder. As shown
 213 in figure 6(a), the mean skin friction coefficient $\langle c_f \rangle(x)$ exhibits strong negative
 214 values in the very first part of the rectangular cylinder, close to the leading-edge
 215 corner. Then, $\langle c_f \rangle$ increases and reaches small but positive values in the region
 216 $0.4 < x < 1.4$. This region of positive shear is the near-wall footprint of the
 217 *secondary vortex*. Moving downstream, for $1.4 < x < 3.65$, the average skin
 218 friction coefficient becomes negative again and shows a minimum at $x \approx 2.37$.
 219 This region of negative shear is the near-wall footprint of the *primary vortex*.
 220 Actually, the *primary vortex* is responsible also for the previously observed
 221 strong negative values of $\langle c_f \rangle(x)$ in the leading-edge region thus indicating that
 222 the *primary vortex* reattaches upstream the *secondary vortex*. Finally, in the
 223 last part of the rectangular cylinder, for $x > 3.65$, a forward attached boundary
 224 layer takes place. Indeed, the skin friction is positive and assumes increasingly
 225 large values moving downstream.

226 Let us now considering the streamwise behaviour of the pressure coefficient
 227 $\langle c_p \rangle(x)$ and of its variance $\langle c_p'^2 \rangle(x)$ shown in figure 6(b). These two observables
 228 are of paramount relevance for the applications since they carry information
 229 about the aerodynamic loads and their fluctuations. However, as pointed out
 230 in Bruno *et. al* [3], the prediction of these two statistical observables is very
 231 challenging for numerical simulations, as highlighted by the variability of the
 232 results obtained within the BARC project. The reason is that the behaviour
 233 of $\langle c_p \rangle(x)$ and $\langle c_p'^2 \rangle(x)$ is strongly influenced by the shape and extent of the re-
 234 circulating regions of the flow, which in turn are determined by the turbulence
 235 levels therein. Hence, the turbulence modelling and mesh resolution adopted
 236 in numerical simulations strongly impacts the resolved turbulent dynamics and,

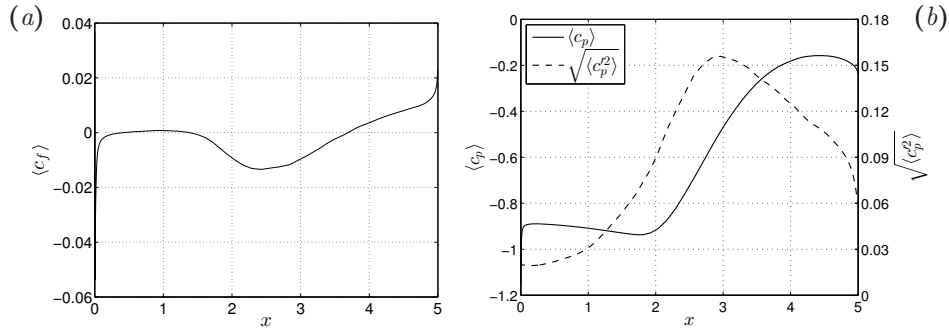


Figure 6: (a) Streamwise behaviour of the average skin friction coefficient $\langle c_f \rangle(x)$. (b) Streamwise evolution of the average pressure coefficient $\langle c_p \rangle(x)$ and of its standard deviation $\sqrt{\langle c_p'^2 \rangle}(x)$.

237 hence, the predicted extent of the recirculation regions. Accordingly, the present
 238 data should help to clarify the behaviour of mean and fluctuating pressure field
 239 at the wall, since they are not affected by turbulence modelling and mesh reso-
 240 lution issues.

241 Figure 6(b) shows that the mean pressure coefficient $\langle c_p \rangle(x)$ is always nega-
 242 tive on the rectangular cylinder. It starts from its minimum at the leading-edge
 243 corner and shows a sharp increase in a very small region corresponding to the
 244 region of upstream reattachment of the *primary vortex*. Then, for $x > 0.13$,
 245 a weak decrease is observed up to $x \approx 1.81$. This streamwise location is the
 246 wall footprint of the low-pressure levels associated with the *primary vortex* core.
 247 Further downstream, the wall pressure shows a significant increase. This pres-
 248 sure rise is maintained up to $x \approx 4.39$ where it forms a maximum, since a slight
 249 pressure decrease follows up to the trailing edge corner.

250 As shown again in figure 6(b), the variance of the wall pressure fluctuations is
 251 very low in the very first part of the rectangular cylinder. However, a monotonic
 252 increase with the streamwise location is observed, leading to a maximum inten-
 253 sity for $x \approx 2.96$. Downstream this maximum, an almost equivalent decrease of
 254 the intensity of the wall pressure fluctuations is observed and maintained up to
 255 the trailing edge corner.

256 4. Single-point statistics

257 In figure 7, the behaviour of the turbulent intensities and of the fluctuating
258 pressure is reported in the (x, y) -plane. These plots show that the separation at
259 the leading-edge gives rise to a free-shear layer which is essentially laminar in its
260 first portion. The instabilities associated with the shear layer are then amplified
261 moving downstream and, for $x > 1$, a three dimensional turbulent pattern is
262 observed. Indeed, all the three components of turbulence are different from
263 zero. By following the streamline of the *primary vortex*, shown with dashed
264 line in figure 7, it is evident that the most amplified fluctuations in the leading
265 edge free shear layer are the streamwise ones, while the vertical and spanwise
266 fluctuations are still significant but remain weaker.

267 Let us analyze in detail the behaviour of each component of turbulent fluctu-
268 ation. By considering first the intensity of the streamwise fluctuations, figure
269 7(a), a well-defined region of maximum turbulent intensity can be identified.
270 This region crosses the external paths of the average recirculating bubble for
271 $1.5 < x < 3.5$ with a maximum reached at $(x, y) = (2.57, 0.44)$. The iso-levels of
272 streamwise fluctuations are stretched in the streamwise direction. Apparently,
273 no deflection of the isocontours towards the wall is observed, indicating that
274 detached streamwise fluctuations are mostly convected from the *primary vortex*
275 to the wake region without interacting with the wall. A second weaker region
276 of streamwise fluctuations is observed in the wake with a relative maximum lo-
277 cated at $(x, y) = (6.05, -0.05)$. The vertical location suggests that this relative
278 maximum is a result of the amplification, throughout the trailing edge shear
279 layer, of fluctuations produced in the attached forward boundary layer.

280 The vertical fluctuations show a substantially different behaviour with re-
281 spect to the streamwise ones. As already mentioned, the typical intensity of
282 vertical fluctuations is smaller than the streamwise ones. However, as shown
283 in figure 7(b), significant differences are observed also from a topological point
284 of view. The region of maximum intensity is located further downstream and
285 closer to the wall with a maximum at $(x, y) = (2.96, 0.31)$. This aspect could be

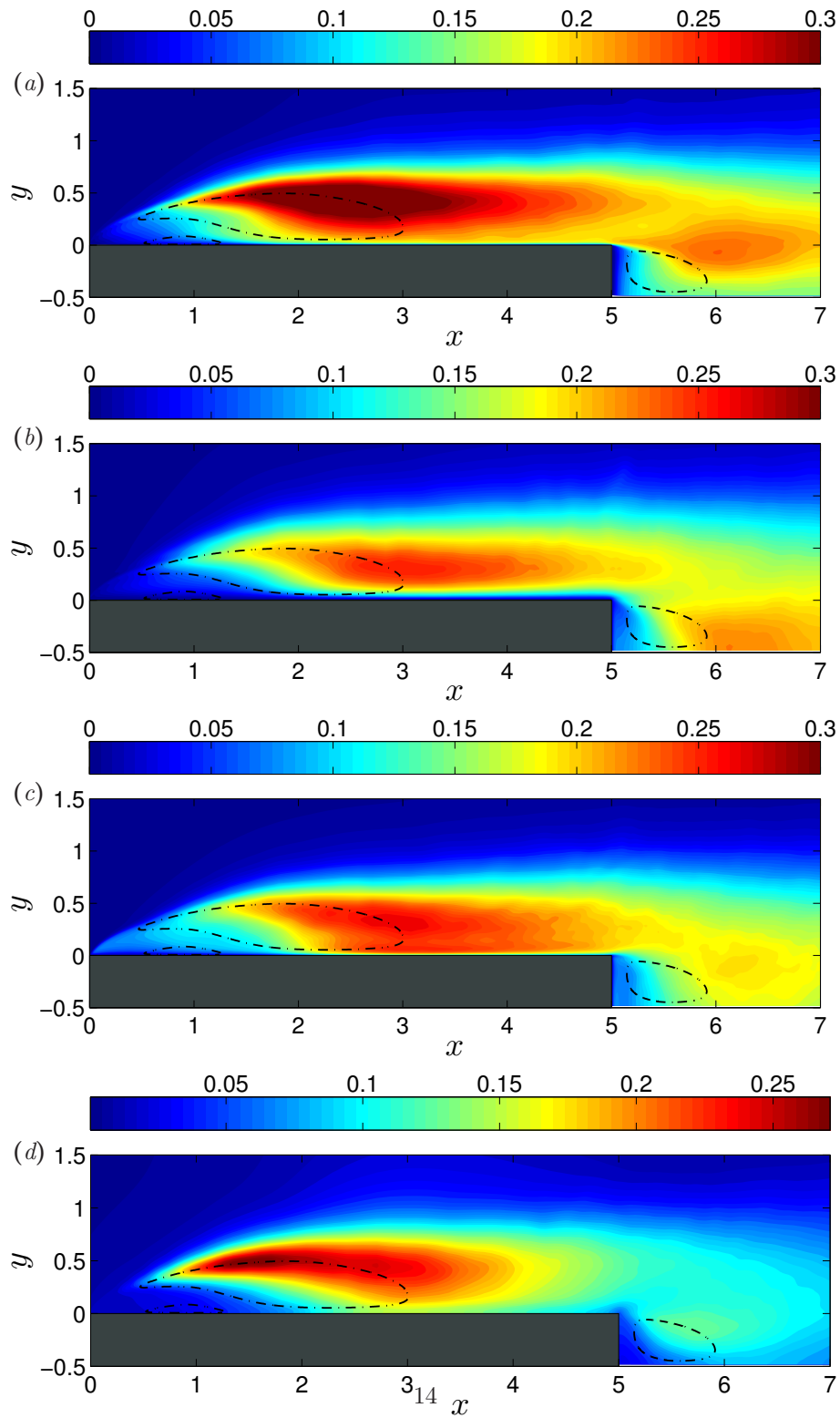


Figure 7: Isocontours of the turbulent intensities and of the fluctuating pressure in the (x, y) -plane: $\sqrt{\langle u'^2 \rangle}$, $\sqrt{\langle v'^2 \rangle}$, $\sqrt{\langle w'^2 \rangle}$ and $\sqrt{\langle p'^2 \rangle}$ in (a), (b), (c) and (d) respectively. The dashed lines report the location of the *primary vortex*, *secondary vortex* and *wake vortex*.

286 partially ascribed to the mean velocity paths associated with the downstream
287 part of the *primary vortex*. Indeed, downstream the peak of intensity of the
288 streamwise fluctuations, the mean velocity field bends towards the wall before
289 reattaching. Hence, the structures associated with these intense streamwise
290 fluctuations, being advected and stretched by the mean velocity field, undergo
291 a deflection towards the wall thus leading to a partial reorientation of stream-
292 wise fluctuations in vertical ones. Also in this case, the shape of the isocontours
293 of vertical fluctuations is found to be essentially elongated in the streamwise
294 direction. Hence, as for the streamwise ones, the produced vertical fluctua-
295 tions do not seem to interact with the wall, but rather they appear to be freely
296 advected downstream towards the free flow. The second region of activity of
297 vertical fluctuations occurs in the wake region. Contrarily to what happens for
298 streamwise fluctuations, this region is centered in the symmetry plane of the
299 wake at $y = -0.5$ for $x \approx 6.15$. This is a clear footprint of the periodic shedding
300 of large scale vortices in the separated wake.

301 The most interesting aspect of the spanwise fluctuations shown in figure 7(c)
302 is that the region of high turbulent intensity, located at $(x, y) = (2.96, 0.28)$ and
303 associated with the *primary vortex*, extends also to the wall region and forms a
304 thin layer of turbulent activity centered at $(x, y) = (2.96, 0.06)$. A possible ex-
305 planation for the high intensity of spanwise fluctuations in the near-wall region is
306 the following. Turbulent fluctuations produced through the *primary vortex* are,
307 on average, transported towards the wall due to the deflection of the streamline
308 paths of the mean velocity field. The consequent impingement gives rise to hor-
309 izontal fluctuations, since the impermeability constraint of the wall leads to a
310 dumping of the vertical fluctuations in the near-wall region. Actually, as shown
311 here in quantitative terms, the impingement essentially gives rise to spanwise
312 fluctuations. Indeed, we observe a near-wall layer in the reattachment region
313 characterized by intense spanwise and weak streamwise fluctuations. From the
314 reattachment region, a reverse flow takes origin, transporting turbulent fluctu-
315 ations towards the leading-edge shear layer. Since the higher levels of turbulent
316 intensities in the reattachment region are detected in the spanwise direction,

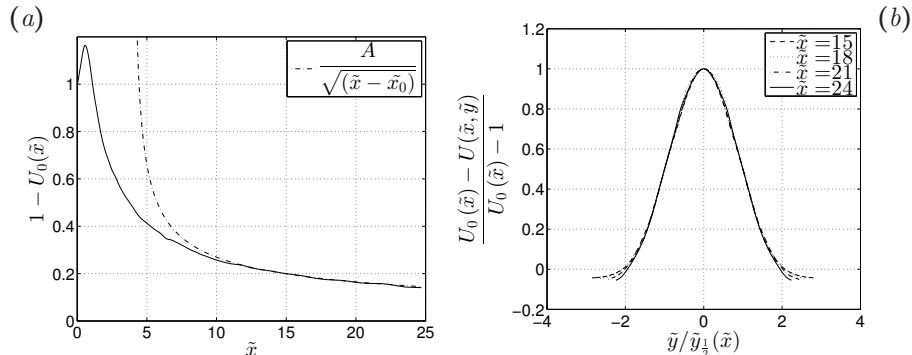


Figure 8: (a) Streamwise behaviour of the centerline velocity defect $1 - U_0(\tilde{x})$ (solid line). The self-similar power law decay (4) is reported in dashed line where $A = 0.66$ and $\tilde{x}_0 = 4$ are considered. (b) Vertical profiles of velocity defect $[U_0(\tilde{x}) - U(\tilde{x}, \tilde{y})]/[U_0(\tilde{x}) - 1]$ evaluated at different streamwise locations and reported as function of $\tilde{y}/\tilde{y}_{1/2}(\tilde{x})$ where $\tilde{y}_{1/2}(\tilde{x})$ is the wake half-width defined such that $U(\tilde{x}, \pm\tilde{y}_{1/2}) = (1 + U_0)/2$.

317 these fluctuations result to be the most intense also in the reverse flow region,
 318 thus forming the observed near-wall layer of intense spanwise fluctuations.

319 Let us now analyse the behaviour of the fluctuating pressure field shown in
 320 figure 7(d). The most intense region of fluctuations is again the leading-edge
 321 shear layer and its consequent evolution along the *primary vortex*. In accordance
 322 with the behaviour of the pressure coefficient shown in figure 6(b), the region of
 323 reverse flow of the *primary vortex* is essentially unperturbed. Only for $x > 2$ the
 324 high levels of pressure fluctuations above the *primary vortex* are felt in the near-
 325 wall region. Interestingly, the variance of the pressure fluctuations associated
 326 with the vortex shedding in the separated wake forms a second region of activity,
 327 which however appears to be much weaker with respect to the region associated
 328 with the leading-edge shear layer.

329 To conclude this section, we consider the evolution of the wake also in the
 330 far field. For this analysis it is useful to consider a shifted reference frame
 331 $(\tilde{x}, \tilde{y}) = (x + 5D, y - D/2)$. In figure 8(a) the streamwise behaviour of the
 332 wake centerline velocity defect, $1 - U_0(\tilde{x})$ with $U_0(\tilde{x}) = U(\tilde{x}, 0)$, is shown. As

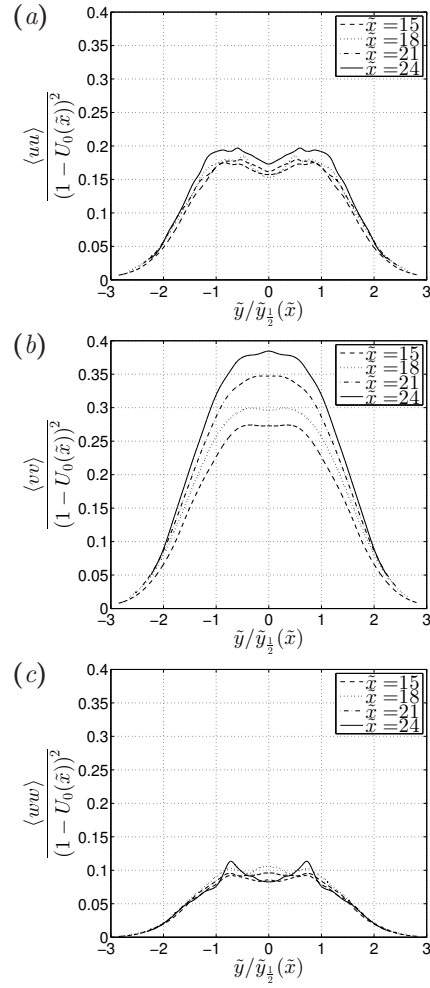


Figure 9: Vertical profiles of turbulent intensities evaluated at different streamwise locations and scaled in similarity variables. (a) streamwise, (b) vertical and (c) spanwise turbulent intensities.

333 apparent, the wake centerline velocity approaches the self-similar decay,

$$1 - U_0(\tilde{x}) \approx \frac{A}{\sqrt{\tilde{x} - \tilde{x}_0}} \quad (4)$$

334 for $\tilde{x} > 10$. The self-similarity of the mean streamwise velocity is confirmed in
 335 figure 8(b) where the vertical profiles of the velocity defect,

$$\frac{U_0(\tilde{x}) - U(\tilde{x}, \tilde{y})}{U_0(\tilde{x}) - 1} \quad (5)$$

336 evaluated at different streamwise locations for $\tilde{x} > 10$ are reported as a function
 337 of the similarity variable $\tilde{y}/\tilde{y}_{1/2}(\tilde{x})$ where $\tilde{y}_{1/2}(\tilde{x})$ is the wake half-width defined
 338 such that,

$$U(\tilde{x}, \pm\tilde{y}_{1/2}) = (1 + U_0)/2. \quad (6)$$

339 It is worth noting that the above self-similar behaviour implies [22] that

$$\frac{1}{1 - U_0} \frac{d\tilde{y}_{1/2}}{d\tilde{x}} \approx const \quad (7)$$

340 and, hence, that the wake spreads as a power law, i.e. $\tilde{y}_{1/2} \sim \tilde{x}^{1/2}$.

341 To complete the analysis of the wake in the far field, let us analyse the
 342 behaviour of the turbulent intensities shown in figure 9. Similarity variables
 343 are again utilized and a good degree of scaling is observed for the streamwise
 344 and spanwise components of turbulent fluctuations. In contrast, the vertical
 345 fluctuations do not exhibit self-similarity, at least at the considered streamwise
 346 locations. This aspect denotes a slower asymptotic recovery of the equilibrium
 347 conditions needed for self-similarity, i.e. the vertical fluctuations are found to
 348 maintain memory of the shedding mechanisms of separating and reattaching
 349 flow over a longer distance.

350 5. The structure of turbulence and two-point statistics

351 In this section, the main unsteadiness of the flow and the inherent multiscale
 352 nature of the turbulent fluctuations are analysed by means of two-point statis-
 353 tics. Before that, we start by addressing the topological pattern taken by the
 354 main turbulent fluctuations populating the flow. To this aim we consider the

355 eduction scheme proposed by Jeong *et al.* [23] and based on the second largest
 356 eigenvalue (λ_2) of the tensor

$$S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj} \quad (8)$$

357 where

$$\begin{aligned} S_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \\ \Omega_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \end{aligned} \quad (9)$$

358 are the symmetric and antisymmetric part of the velocity gradient tensor. This
 359 eduction scheme is known to accurately extract the three-dimensional pattern
 360 of vortical structures, see e.g. Cucitore *et al.* [24] and Dubief *et al.* [25].

361 In figure 10, the instantaneous pattern taken by $\lambda_2 = -2$ is shown with
 362 iso-surfaces colored by \tilde{y} . From the perspective and top views, it is evident that
 363 in the very first part of the leading-edge shear layer, for $x < 0.3$, the flow is lam-
 364 inar, in accordance with the statistical analysis reported so far. Then, almost
 365 two-dimensional spanwise rolls appear as a result of the well-known Kelvin-
 366 Helmholtz instability. Under the effect of the mean shear, perturbations of the
 367 flow field lead to the lift up and stretching of these spanwise vortices, thus
 368 forming hairpin-like structures [6, 26, 27]. Hence, the flow motion develops
 369 streamwise vortices [19, 28, 29], the legs of the hairpin vortices, which in turn
 370 induce high- and low-speed streaks in between them. The statistical evidence of
 371 the presence of streamwise vortices and streaks is reported in the following anal-
 372 ysis of the two-point spanwise correlation function. These sparsely distributed
 373 structures, grow, burst and decay thus giving rise to a fully turbulent flow for
 374 $x > 1$. Most of this complex multiscale pattern, particularly the large scale
 375 structures, is shed downstream in the free flow. On the other hand, a por-
 376 tion of the structures, in particular small scale fluctuations, are pushed towards
 377 the wall. The consequent impingement gives rise to both forward and reverse
 378 boundary layers characterized by small scale motions. By following the branch
 379 moving downstream, we observe that turbulent fluctuations become more and

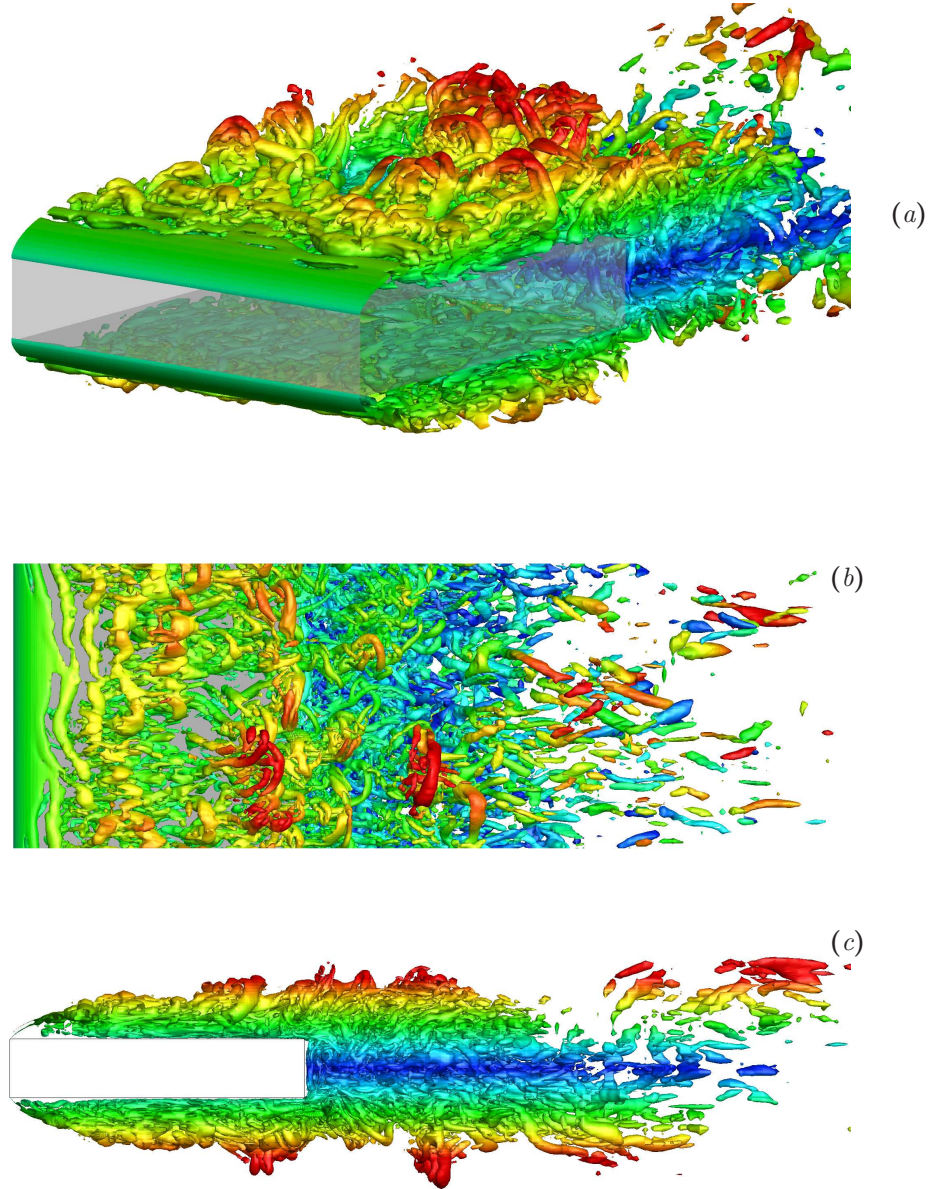


Figure 10: Instantaneous isosurfaces of $\lambda_2 = -2$ colored with \tilde{y} . Perspective, top and lateral views in (a), (b) and (c) plots, respectively.

380 more aligned in the streamwise direction. Finally, by reaching the trailing edge,
 381 these relatively small fluctuations are encompassed by an oscillatory large scale
 382 motion reminiscent of the laminar von Kármán instability of the separated wake.

383 The statistical signature of the above mentioned turbulent motion can be
 384 highlighted by means of two-point statistics, such as the correlation function in
 385 physical space and the energy spectrum in wavenumber/frequency space. While
 386 the spatial correlation function is a measure of how the velocity fluctuations
 387 are coherent in space, the energy spectrum allows us to analyse how turbulent
 388 kinetic energy is distributed across the different scales of motion. Let us start
 389 with the spatial correlation function. Due to the symmetry of the flow, the
 390 only statistical homogeneous direction where it is possible to define a space
 391 of homogeneous scales not affected by the inhomogeneity of the flow, is the
 392 spanwise direction (see Cimarelli *et al.* [30] for an example of the complexity
 393 emerging from the study of the space of inhomogeneous scales). In this setting,
 394 the spatial spanwise correlation function is rigorously defined and for a generic
 395 quantity β can be written as,

$$R_{\beta\beta}(x, y, r_z) = \frac{\langle \beta'(x, y, z + r_z/2, t) \beta'(x, y, z - r_z/2, t) \rangle}{\langle \beta' \beta' \rangle(x, y)}. \quad (10)$$

396 In figure 11, the spanwise correlation function of the three components of
 397 velocity and pressure fluctuations evaluated in the shedding region of the *pri-*
 398 *mary vortex*, is shown. This region of the flow is fully turbulent and relatively
 399 small spanwise correlation lengths are observed. In particular, the fluctuating
 400 velocity field shows a zero value of correlation at $r_z \approx 0.3$ for the vertical and
 401 streamwise fluctuations and at $r_z \approx 0.5$ for the spanwise ones. Interestingly,
 402 for larger values of the spanwise increment r_z , a degree of negative correlation
 403 is observed. The peak of anticorrelation occurs at $r_z \approx 0.45, 0.5$ and 0.9 for
 404 the vertical, streamwise and spanwise fluctuations, respectively. These negative
 405 correlation peaks are in accordance with the presence of streamwise vortices
 406 and streamwise velocity streaks as a result of motions induced by the hairpin
 407 vortices observed in figure 10. Indeed, the peak of negative correlation of ver-
 408 tical and spanwise fluctuations can be understood as a statistical evidence of

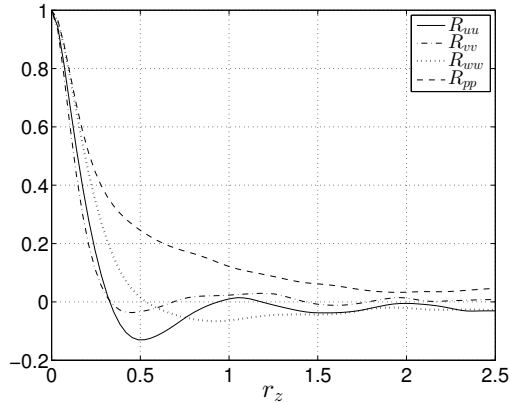


Figure 11: Spanwise correlation function of streamwise, vertical, spanwise and pressure fluctuations evaluated in the shedding region of the *primary vortex* at a (x, y) -position corresponding to probe 9. The specific location of the probe within the flow is shown in figure 4.

409 the presence of counter-rotating pairs of streamwise vortices. In particular, the
 410 negative peak of R_{vv} at $r_z \approx 0.45$ is a measure of the statistical diameter of
 411 streamwise vortices while the negative peak of R_{ww} at $r_z \approx 0.9$ is indicative of
 412 the mean spacing between counter-rotating vortex pairs. On the other hand,
 413 the negative peak of R_{uu} is an evidence of the presence of streamwise velocity
 414 streaks. In particular, the negative peak of R_{uu} at $r_z \approx 0.5$ is a measure of
 415 the mean spanwise spacing between high and low streamwise velocity streaks.
 416 See Kim *et al.* [31] for well-established and analogous considerations in wall-
 417 turbulent flows. The pressure fluctuations show a wider correlation length and
 418 no negative correlation peak is observed. **Finally, it is worth pointing out that**
 419 **the behaviour shown here by the two-point spatial correlation function supports**
 420 **the choice of the numerical domain width, $\mathcal{D}_z = 5$. Indeed, figure 11 shows that**
 421 **the velocity and pressure fields are uncorrelated for spanwise lengths that are**
 422 **significantly shorter than the domain width.**

423 Let us consider now the scale-space distribution of kinetic energy by means
 424 of a spectral analysis. In particular, we consider the multiscale features of the
 425 flow both in time and space by means of one-dimensional spectra of turbulent
 426 kinetic energy. By taking advantage of the statistical homogeneity of the flow

427 in the spanwise direction and in time, the spectrum of turbulent kinetic energy
 428 $q = u_i u_i / 2$ can be defined as

$$\Phi_{qq}(k_z, St, x, y) = \frac{1}{2} \langle \tilde{u}_i(k_z, St, x, y) \tilde{u}_i^*(k_z, St, x, y) \rangle \quad (11)$$

429 where k_z and St are the spanwise wavenumber and frequency, while $\langle \cdot \rangle$ denotes
 430 the two-dimensional Fourier transform with respect to the spanwise direction
 431 and time. To simplify the analysis, in the following we consider separately
 432 the one-dimensional wavenumber spectrum and the one-dimensional frequency
 433 spectrum. The one-dimensional wavenumber spectrum is derived from the two-
 434 dimensional one by integration with respect to St ,

$$\Phi_{qq}^{k_z}(k_z, x, y) = \int \Phi_{qq}(k_z, St, x, y) dSt \quad (12)$$

435 and, analogously, the one-dimensional frequency spectrum is computed by inte-
 436 grating with respect to k_z ,

$$\Phi_{qq}^f(St, x, y) = \int \Phi_{qq}(k_z, St, x, y) dk_z \quad (13)$$

437 By using the Taylor's hypothesis of frozen turbulence we also address the puta-
 438 tive wavenumber spectrum in the streamwise direction defined as

$$\Phi_{qq}^{k_x}(k_x, x, y) = \Phi_{qq}^f(St/U(x, y), x, y) . \quad (14)$$

439 The main unsteadinesses of the flow are analysed in figure 12 by means of
 440 frequency spectra of turbulent kinetic energy evaluated in the leading-edge shear
 441 layer and in the wake. As shown in figure 12(a), the leading-edge shear layer is
 442 characterized by well-defined peaks of spectral energy at a relatively large range
 443 of frequencies. In particular, moving from probe P1 to P7 along the shear layer,
 444 the peaks move from $St \approx 1.8$ to $St \approx 0.9$. This range of frequencies represents
 445 the temporal scales of the fluctuations amplified by the transitional mechanisms
 446 of the shear layer. By looking at the frequency spectrum in the wake 12(b), a
 447 clear separation of scales is evinced between the vortex detachment in the sepa-
 448 rated wake and the amplified fluctuations through the leading-edge shear layer.
 449 In fact, the frequency spectrum in the wake highlights a well-defined peak at

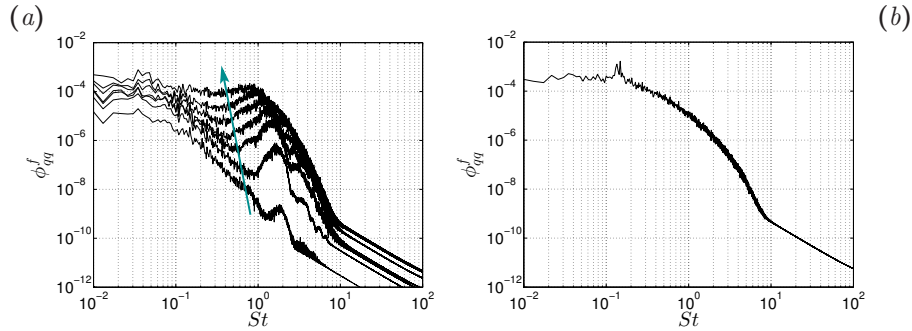


Figure 12: Frequency spectrum of turbulent kinetic energy, $\phi_{qq}^f(St)$, evaluated along the leading-edge shear layer at probes 1 to 7 (a) and in the wake at probe 10 (b). The arrow in (a) indicates probes from 1 to 7. The specific location of probes within the flow is shown in figure 4.

450 relatively larger temporal scales for $St \approx 0.14$. This peak is a clear statistical
 451 evidence of the oscillatory large scale motion reminiscent of the laminar von
 452 Kármán instability. Being at much larger temporal scales, the vortex shedding
 453 mechanisms in the wake are found to be not directly connected with the for-
 454 mation processes of fluctuations in the transitional region of the leading-edge
 455 shear layer. To appreciate this, one may compare the spectral peak at frequency
 456 $St \approx 0.14$ shown in figure 12(b) with the amplified spectral peak frequencies
 457 from $St \approx 1.8$ to $St \approx 0.9$ in figure 12(a). On the contrary, we find a clear
 458 matching of scales between the slow vortex detachment in the separated wake
 459 and the fluctuation of the lift coefficient, which appear evidently by comparing
 460 the spectral peaks for $St \approx 0.14$ in figures 3(b) and 12(b). Being driven by
 461 the pressure differences between the top and bottom sides of the rectangular
 462 cylinder, the temporal fluctuations of the lift coefficient are essentially given by
 463 the instantaneous difference in the extent of the low pressure regions settled by
 464 the separated recirculating flow in the opposite sides of the rectangle. Hence,
 465 we argue that the spectral peak frequency $St \approx 0.14$ shown by the lift coefficient
 466 is representative of the time scale of the large scale phenomenon of alternative
 467 enlargement and shrinking of the separation bubble in the top and bottom sides
 468 of the rectangle [18, 19]. Accordingly, the clear matching of scales between the

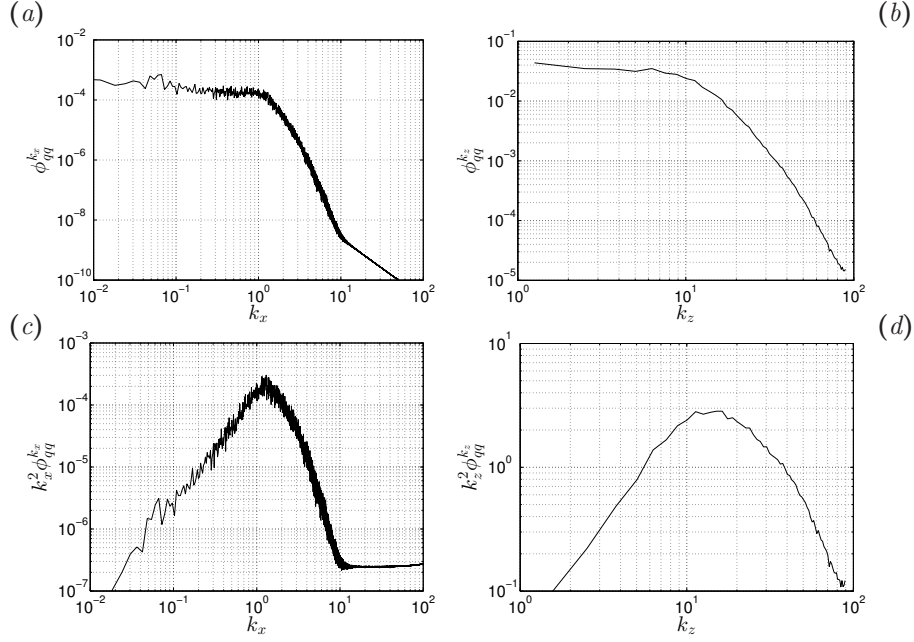


Figure 13: Streamwise (a) and spanwise (b) spectrum of turbulent kinetic energy, $\phi_{qq}^{k_x}(k_x)$ and $\phi_{qq}^{k_z}(k_z)$, respectively, evaluated at probe 8. Streamwise (c) and spanwise (d) dissipation spectrum of turbulent kinetic energy, $k_x^2 \phi_{qq}^{k_x}(k_x)$ and $k_z^2 \phi_{qq}^{k_z}(k_z)$, respectively, evaluated again at probe 8.

469 vortex detachment in the wake and the fluctuation of the lift coefficient suggests
 470 a locking of the vortex shedding phenomena in the wake with the alternative
 471 shedding of large scale vortices from the top and bottom *primary vortices* and,
 472 hence, with their enlargement and shrinking [2].

473 Let us consider now the spatial scales of the flow. In figure 13(a) and (b),
 474 the streamwise and spanwise spectra of turbulent kinetic energy in the fully
 475 developed part of the leading-edge shear layer are shown. The two plots re-
 476 veal a markedly anisotropic behaviour, consisting of turbulent fluctuations elon-
 477 gated in the streamwise direction. Indeed, the energy-containing fluctuations
 478 are found to fill the streamwise spectrum up to $k_x \approx 1$ while, in the spanwise
 479 one, up to $k_z \approx 10$. In other words, the most energetic fluctuations are stretched
 480 in the streamwise direction and their size is of the order of $O(D)$ and $O(10^{-1}D)$
 481 in the streamwise and spanwise direction, respectively. This anisotropy is re-

482 tained up to the small dissipative scales as shown by the spectra of turbulent
483 dissipation shown in figure 13(c) and (d). Indeed, the maximum rate of dissi-
484 pation is achieved for $k_x \approx 1.4$ and $k_z \approx 15$, respectively in the streamwise and
485 spanwise direction. These aspects need to be taken carefully into account when
486 CFD techniques such as LES (Large Eddy Simulation) are applied for the simu-
487 lation of the flow. Accordingly with Bruno *et al.* [32], the spanwise resolution is
488 a central object when setting up a CFD simulation being the site of the smallest
489 but energy containing scales of the flow as shown here in quantitative terms.

490 6. Conclusions

491 The flow around rectangular cylinders is recognized to be an extremely in-
492 teresting case both for fundamental and applicative studies. Despite the simple
493 geometry, this category of flows contains basic phenomena characterizing the
494 behaviour of more complex flows typical of the applications. In this respect, it
495 is still difficult to achieve reliable results both from a numerical and experimen-
496 tal point of view. The reason is the high sensitivity of the different phenomena
497 driving the flow on the experimental conditions from one side and on the tur-
498 bulence modelling and mesh properties from the other. Here, we report for
499 the first time a Direct Numerical Simulation of such a flow for a moderately
500 high Reynolds number. The main goal is to shed light on the main physical
501 mechanisms driving the complex behaviour of the flow and to provide well con-
502 verged statistics not affected by uncertainties on the boundary conditions and
503 by inaccuracies related to turbulence modelling and mesh resolution.

504 Global and single-point statistics are reported aiming at defining the exact
505 behaviour of relevant statistical quantities. As an example, we characterize the
506 behaviour of the pressure coefficient and of its variance which are known to be
507 very important quantities for civil engineering applications aiming at predict-
508 ing wind loads over buildings. In this respect, also the behaviour of the lift
509 coefficient in the frequency space is reported, highlighting a well defined peak
510 for $St \approx 0.14$. The mean and turbulent flow is then assessed. Three main

511 recirculating regions are found and their dimensions and turbulence levels are
512 characterized. The first one is a large scale bubble originating from the flow
513 separation at the leading edge corner and it is herein called *primary vortex*.
514 The related shear layer is the locus of the instability and transitional processes
515 giving rise to turbulence as shown here in quantitative terms by the increasing
516 levels of intensity of the fluctuations along its development. This *primary vortex*
517 is found to shed vortices downstream giving rise to a region of high turbulence
518 levels. The *primary vortex* is responsible for the observed second recirculating
519 region, here called *secondary vortex*. Indeed, it induces a reverse flow at the
520 wall which experiences an adverse pressure gradient and thus separates giving
521 rise to the second near-wall recirculating region. Finally, the third recirculat-
522 ing region, here called *wake vortex*, takes place in the separated flow at the
523 trailing edge. As for the *primary vortex*, the trailing-edge shear layer is respon-
524 sible for additional instability mechanisms, thus giving rise to a second region
525 of high turbulence intensity. Finally, the behaviour of the fully developed wake
526 is also analysed. A rigorous assessment of the wake flow features is reported
527 in a simple way by making use of self-similarity variables. The analysis reveals
528 a slower asymptotic recovery of the equilibrium conditions for self-similarity of
529 the vertical fluctuations with respect to the streamwise and spanwise ones.

530 To complete the study of the flow, two-point statistics are also computed,
531 namely the spanwise correlation function and the energy spectrum. The study
532 of the two-point spatial correlation function of the fluctuating velocity and pres-
533 sure fields allows us first to prove that the width of the numerical domain is large
534 enough to reproduce the main flow features. On the other hand, we found a
535 clear statistical evidence of the presence of streamwise vortices and high- and
536 low- streamwise streaks within the flow. In particular, we found that stream-
537 wise vortices statistically occur as pairs of counter-rotating vortices. The spac-
538 ing between counter-rotating vortex pairs is 0.9 while the diameter of a single
539 streamwise vortex is 0.45. Regarding the velocity streaks we statistically prove
540 their presence and we measure that the spanwise spacing between positive and
541 negative streamwise velocity streaks is 0.9. We argue that both streamwise

542 vortices and streaks are a result of the flow motion induced by the presence of
543 hairpin-like turbulent structures here detected by analysing the instantaneous
544 pattern taken by λ_2 .

545 The spectral analysis in the frequency space allows us to identify and quan-
546 tify the main unsteadinesses of the flow. In particular, a separation of scales be-
547 tween the amplified fluctuations in the leading-edge shear layer for $St = O(10^0)$
548 and the large scale vortex detachment at the wake for $St = O(10^{-1})$ is observed.
549 Indeed, this low frequency is found to be actually locked with the shedding of
550 vortices from the *primary vortex* and, in particular, with the alternative enlarg-
551 ment and shrinking of the recirculating regions in the top and bottom sides of
552 the rectangle. On the other hand, the analysis in the wavenumber space allows
553 us to study the anisotropy of the flow in the space of scales. It is found that
554 both the energy-containing and dissipative fluctuations are anisotropic, being
555 elongated in the streamwise direction and thin in the spanwise one. This infor-
556 mation should be taken carefully into account when designing meshes in CFD
557 applications.

558 It is finally worth pointing out that the present results, besides reporting a
559 detailed statistical analysis of the flow, are also intended to be a reference for
560 CFD studies. Indeed, the statistical objects here reported would be useful to
561 quantify and understand the effects of modelling and mesh resolution issues by
562 means of a comparison with the same statistics obtained with CFD simulations
563 at the same Reynolds number. Hence, they would be useful for developing a
564 *best practice* for CFD.

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