

Economic order quantity for a perishable product with random yield

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Abstract: This paper presents an economic order quantity (EOQ) model for a product with a deterministic shelf-life, after which it becomes unfit for use. An order is placed as soon as the inventory level reaches the reordered point, and the ordered lot arrives in stock after a positive, deterministic lead time. While demand is deterministic, the actual quantity received in inventory does not necessarily match the ordered amount. In particular, the received quantity – the *yield* – is random. A cost model is developed, and an optimization problem is formulated. Finally, numerical experiments are conducted to analyze system behavior.

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1. INTRODUCTION

Lot sizing involves determining both the order quantity and timing of replenishments, whether for procurement or internal production. Over the years, this problem has been extensively studied under various conditions, starting with the well-known economic order quantity (EOQ) model introduced by Harris (1913).

In practice, the number of items received from suppliers or internally produced often deviates from the ordered quantity. The amount received (or produced) is referred to as the *yield*. This discrepancy may arise due to various factors, including supplier stockouts, machine failures, product spoilage during transportation, or defects in production (Parlar and Wang, 1993; Snyder and Shen, 2019). While, in some case, yield can be deterministic, it is commonly uncertain (Snyder and Shen, 2019), which is the focus of this paper.

Extensive research has integrated yield uncertainty into lot sizing models. We can cite, for example, Song and Wang (2017), who analyzed stochastic (random) inventory control problems under periodic review, incorporating fixed order cost and uniformly distributed random yields to derive optimal order policies. He et al. (2018) studied a decentralized supply chain with a supplier and a manufacturer, modeling decision-making sequences through game theory, while incorporating random production yields and service level constraints. Yu et al. (2019) proposed a newsvendor model for a firm that needs to make joint inventory and pricing decisions, including random yield and demand. They assumed that a secondary market exists for the firm to acquire or dispose of products, and that the unit prices of the secondary emergency purchase and sale are random functions of the yield. Zhang and Chu (2021) investigated, via game theory, the effect of random yield on decision-making under pull and push contracts within a decentralized supply chain. Jeon et al. (2021) developed an inventory model incorporating supply uncertainty, random yield, and partial product substitution. Finally, Xiao and Wang (2023) studied a multi-location newsvendor problem under additive or multiplicative random yield, exploring expected

costs and variances under centralized and decentralized inventory systems.

A related and extensively studied topic in inventory management is inventory deterioration, which occurs due to damage, spoilage, evaporation, or degradation over time. The literature classifies deteriorating inventory into two main types (Nahmias, 2011): a *decaying inventory* is observed when a fraction of the stock is lost every unit of time, while a *perishable inventory* occurs if the usability of items is constant up to a deterministic or uncertain expiration date (which determines the item’s shelf-life), after which the products lose their fitness for use. Examples of decaying materials are gasoline, alcohol or radioactive substances, while human blood, food or photographic films are examples of perishable items.

Many studies have addressed inventory models with deterioration. Notable recent contributions include: Jain et al. (2023), who analyzed a three-echelon supply chain for a decaying item, considering imperfect production and carbon emissions regulation. Lok et al. (2023), who examined a non-instantaneous deteriorating product, incorporating preservation technology and environmental costs. Bhuniya et al. (2023), who investigated a remanufacturing supply chain for a decaying product, assuming a controllable decay rate. Castellano and Glock (2024), who extended the economic production quantity (EPQ) model for a decaying item under stochastic demand to consider a positive lead time. San-José et al. (2024), who developed a decaying inventory model under power-time-dependent demand, factoring in both shortages and environmental costs. Finally, Khanlarzade and Farughi (2024), who introduced a bounded rationality (limited decision-making ability) in a two-level decaying product supply chain, using game theory.

Interestingly, no prior research has jointly considered both yield uncertainty and product deterioration. This paper seeks to bridge this gap by extending the classical EOQ model to incorporate both product perishability and random yield.

2. SYSTEM DESCRIPTION, NOTATION, AND ASSUMPTIONS

We consider a single-item, single-location inventory system, analyzed in continuous time. Demand is constant and deterministic. The product has a fixed, known shelf-life (i.e., a maximum usable duration), meaning that any unit held in inventory beyond this period must be discarded. Inventory is continuously reviewed, and an order is issued as soon as the inventory level hits, or falls below, the reorder point. The ordered lot arrives in stock after a constant and deterministic lead time. At each replenishment, the number of units received – that is, the yield – may not equal the number of units ordered. In particular, the yield is uncertain, but it cannot exceed the order quantity. Stockout events might happen, and unmet demand is fully lost.

Based on the system structure, we develop the long-run expected total cost rate and formulate an optimization problem to determine the optimal order quantity.

2.1 Notation

The following notation is used in the model:

Decision variables:

Q Order quantity [quantity unit]

Parameters:

D Demand rate [quantity unit/time unit]

A Unit ordering cost [\$/order]

h Stockholding cost rate [\$/quantity unit/time unit]

v Unit purchase cost [\$/quantity unit]

u Unit disposal cost [\$/quantity unit]

π Cost per unit lost sale [\$/quantity unit]

L Lead time [time unit]

m Unit product lifetime [time unit]

p Probability that exactly Q units are received with a replenishment

Random variables:

\bar{Z} Ratio between the yield and the order quantity

Functions and operators:

$\mathbb{E}[\cdot]$ Mathematical expectation

\mathbb{V} Variance

x^+ Maximum between 0 and x , i.e., $x^+ \equiv \max\{0, x\}$

\mathbb{P} Probability

$\mathbf{1}_{\{\cdot\}}$ Indicator function

Sets:

\mathcal{R} Real numbers

Additional notation will be introduced as needed.

2.2 Assumptions

We adopt the following main assumptions to develop the model:

1. The demand rate is constant and deterministic.
2. Each product unit not used and held in stock for more than m time units is discarded.
3. Product lifetime, m , is fixed and deterministic.

4. Product aging starts upon replenishment (not upon ordering).
5. Lead time is constant, deterministic, and does not exceed product lifetime (i.e., $L \leq m$).
6. Inventory is continuously monitored, and Q units are ordered as soon as the inventory level hits, or falls below, the reorder point, r . Then, the order arrives after L time units. The reorder point is $r = DL$, i.e., the safety stock is 0.
7. The number of units received with a replenishment (i.e., the yield), Y , is (probabilistically) uncertain and is given by $Y = Q$, with probability p , and $Y = QZ$, with probability $q \equiv 1 - p$, where Z is a continuous, nonnegative random variable (r.v.), with support $\{z \in \mathcal{R}: 0 \leq z \leq 1\}$. Hence, according to the definition of \bar{Z} , we have $\bar{Z} = 1$ with probability p , and $\bar{Z} = Z$ with probability q . We also observe that the support of Y is $\{y \in \mathcal{R}: 0 \leq y \leq Q\}$.
8. Stockouts result in lost sales.
9. Procurement costs depend on the actual received quantity.
10. An infinite planning horizon is considered.

3. MODEL DEVELOPMENT AND PROBLEM FORMULATION

The inventory level evolves stochastically due to random yield, with probabilistic repetition at the beginning of each cycle. Consequently, both the cost per cycle (K_C) and the cycle length (T) are random variables.

A cycle is defined as the time between two consecutive replenishments. According to the renewal reward theorem (Ross, 1983), the long-run expected total cost rate is given by the ratio between the expected cost per cycle and the expected cycle length.

To analyze the inventory system, we introduce the following key quantities (see Figure 1):

$$I_1 \equiv r + (m - L)D = mD, \quad (1)$$

$$I_2 \equiv mD + r = (m + L)D = I_1 + DL, \quad (2)$$

where I_1 is the demand over the maximum lifetime and I_2 is the demand in the period $m + L$. From (2), we note that $I_2 - I_1 = DL = r$. Evidently, with $Q \leq I_1$, any realization of the yield makes the system behave like if $m = +\infty$, i.e., the finiteness of the item useful life is not observed. Yield realizations in $]I_1, I_2]$ are such that the maximum useful life is reached when the order has been already issued. Finally, yield realizations above I_2 are such that the age limit is reached before the order is issued.

We identify two mutually exclusive scenarios in which a stockout may occur within a cycle:

1. A realization of the yield falls below the reorder point.
2. The maximum useful life is reached before the beginning of the subsequent cycle (yield realizations above I_1).

It is important to note that in case (1), where the yield is insufficient to reach the reorder point, the inventory cycle is exactly equal to the lead time. Additionally, no stockouts occur for yield realizations falling within a specific intermediate range $[r, I_1]$, where neither depletion nor expiration leads to shortages.

According to the previous discussion, it is practical to develop the cost model analyzing four different cases, depending on the domain of Q . The cases are the following: (1) $Q > I_2$, (2) $I_1 < Q \leq I_2$, (3) $r < Q \leq I_1$, and (4) $0 \leq Q \leq r$. Note that case (4) includes the condition (i.e., $Q = 0$) in which the inventory system is permanently out of stock.

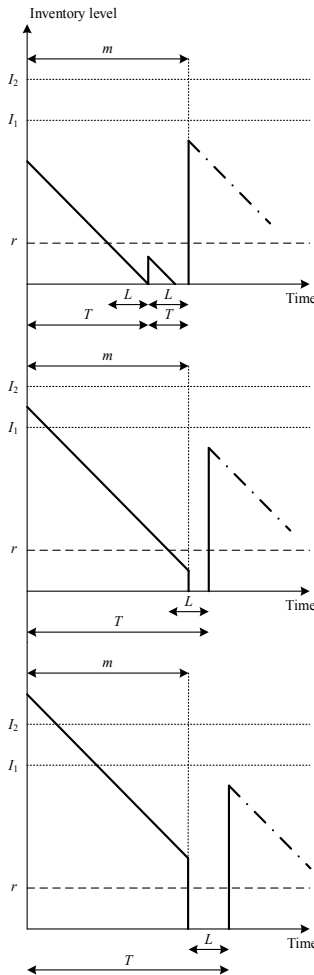


Figure 1. Inventory-time plots for different realization of the yield.

3.1 Case $Q > I_2$

If we let I , W , and U be, respectively, the area under the inventory curve in a cycle, the number of product units perished in a cycle, and the demand lost in a cycle, then the cost per cycle, K_C (a random variable), is given by:

$$K_C \equiv K_C(Q) = A + vY + hI + uW + \pi U,$$

where A is the unit ordering cost, vY is the purchase cost, hI is the stockholding cost, uW is the disposal cost of perished units, and πU is the shortage cost. The expected cost per cycle, $\mathbb{E}K_C$, is

$$\mathbb{E}[K_C] = A + v\mathbb{E}[Y] + h\mathbb{E}[I] + u\mathbb{E}[W] + \pi\mathbb{E}[U]. \quad (3)$$

We now calculate the expectations in the right-hand side of (3), beginning with $\mathbb{E}[Y]$. According to assumption (7), we can write $Y = Q\bar{Z}$, where \bar{Z} is a r.v. with probability function \bar{f} given by

$$\bar{f} \equiv \bar{f}(\bar{z}) = \begin{cases} 0 & \bar{z} \notin [0,1], \\ p & \bar{z} = 1, \\ qf(\bar{z}) & \bar{z} \in [0,1], \end{cases}$$

where f is the probability density function (p.d.f.) of Z . Hence, the expected value of \bar{Z} is

$$\mathbb{E}[\bar{Z}] = p + q \int_0^1 \bar{z}f(\bar{z})d\bar{z} = p + q\mathbb{E}[Z].$$

According to the relationship between Y and \bar{Z} , the probability function of Y is given by:

$$g \equiv g(y) = \begin{cases} 0 & y \notin [0, Q], \\ p & y = Q, \\ qf(y/Q)/Q & y \in [0, Q]. \end{cases}$$

We prove the expression of $g(y)$ for $y \in [0, Q]$. Let G and F be the cumulative distribution function (c.d.f.) of Y and Z , respectively. Since $G(y) \rightarrow q$ as $y \uparrow Q$, and $F(1) = 1$, we have:

$$G(y) \equiv \mathbb{P}(Y \leq y) = q\mathbb{P}(ZQ \leq y) = q\mathbb{P}(Z \leq y/Q) \equiv qF(y/Q).$$

Then:

$$g(y) = dG/dy = qdF(y/Q)/dy = qf(y/Q)/Q.$$

The expectation of Y is thus given by:

$$\mathbb{E}[Y] = Q\mathbb{E}[\bar{Z}] = Q(p + q\mathbb{E}[Z]). \quad (4)$$

We now evaluate $\mathbb{E}[W]$. If we recall the definition of I_1 (see (1)), the units perished in a cycle are $W = (Y - I_1)^+$. Then, using the relation $y = zQ$, which implies $dy = Qdz$, we have:

$$\begin{aligned} \mathbb{E}[W] &= p(Q - I_1) + q/Q \int_0^Q (y - I_1)^+ f(y/Q)dy = \\ &= p(Q - I_1) + q/Q \int_{I_1/Q}^1 (z - I_1/Q) f(z) dz \equiv \\ &= p(Q - I_1) + qQ\psi(I_1/Q), \end{aligned} \quad (5)$$

where ψ is the first-order loss function of Z . Now, we calculate $\mathbb{E}[U]$. According to the above discussion regarding stockout occasions, we have $U = \min\{W, r\} + (r - Y)^+$, where:

$$\begin{aligned} \min\{W, r\} &= \min\{(Y - I_1)^+, r\} \\ &= \begin{cases} 0 & Y \leq I_1, \\ Y - I_1 & I_1 < Y \leq I_2, \\ r & I_2 < Y \leq Q. \end{cases} \end{aligned}$$

Hence, if we let \bar{F} be the complementary c.d.f. of Z , we can write:

$$\begin{aligned} \mathbb{E}[U] &= \frac{q}{Q} \int_{I_1}^{I_2} (y - I_1) f\left(\frac{y}{Q}\right) dy + \\ &r \left(p + \frac{q}{Q} \int_{I_2}^Q f\left(\frac{y}{Q}\right) dy \right) + \frac{q}{Q} \int_0^r (r - y) f\left(\frac{y}{Q}\right) dy = \\ &qQ \int_{I_1/Q}^{I_2/Q} (z - I_1/Q) f(z) dz + r \left(p + q \int_{I_2/Q}^1 f(z) dz \right) + \\ &qQ \int_0^{r/Q} (r/Q - z) f(z) dz = qQ(\psi(I_1/Q) + \\ &-\psi(I_2/Q) + r/Q\bar{F}(I_2/Q)) + r(p + q\bar{F}(I_2/Q)) + \\ &qQ(r/Q - \mathbb{E}[Z] + \psi(r/Q)). \end{aligned}$$

For what concerns the calculation of $\mathbb{E}[I]$, we can write (see Figure 1):

$$I = (Y^2 - W^2)/(2D) \Rightarrow \mathbb{E}[I] = (\mathbb{E}[Y^2] - \mathbb{E}[W^2])/(2D).$$

Since

$$\begin{aligned} \mathbb{E}[\bar{Z}^2] &= p + q \int_0^1 \bar{z}^2 f(\bar{z}) d\bar{z} = p + q\mathbb{E}[Z^2] = \\ &p + q(\mathbb{V}(Z) + \mathbb{E}[Z]^2), \end{aligned}$$

then

$$\mathbb{E}[Y^2] = Q^2\mathbb{E}[\bar{Z}^2] = Q^2[p + q(\mathbb{V}(Z) + \mathbb{E}[Z]^2)].$$

We then have:

$$\begin{aligned} \mathbb{E}[W^2] &= p(Q - I_1)^2 + \frac{q}{Q} \int_{I_1}^Q (y - I_1)^2 f\left(\frac{y}{Q}\right) dy = \\ &p(Q - I_1)^2 + qQ^2 \int_{I_1/Q}^1 (z - I_1/Q)^2 f(z) dz \equiv \\ &p(Q - I_1)^2 + 2qQ^2\psi_2(I_1/Q), \end{aligned}$$

where ψ_2 is the second-order loss function of Z . Hence, we obtain

$$\begin{aligned} \mathbb{E}[I] &= \{Q^2[p + q(\mathbb{V}(Z) + \mathbb{E}[Z]^2)] + \\ &-p(Q - I_1)^2 + 2qQ^2\psi_2(I_1/Q)\}/(2D). \end{aligned} \quad (6)$$

We thus derived all terms in (3). To evaluate the long-run expected total cost rate, we finally need to calculate the expected cycle length, $\mathbb{E}[T]$. From Figure 1, it can be observed that

$$T = \begin{cases} m + L & I_2 < Y \leq Q, \\ m + (Y - I_1)/D & I_1 < Y \leq I_2, \\ Y/D & r < Y \leq I_1, \\ L & Y \leq r. \end{cases}$$

Hence, using directly z as integration variable, we get

$$\begin{aligned} \mathbb{E}[T] &= Lq \int_0^{r/Q} f(z) dz + q \frac{Q}{D} \int_{r/Q}^{I_1/Q} zf(z) dz + \\ &mq \int_{I_1/Q}^{I_2/Q} f(z) dz + qQ/D \int_{I_1/Q}^{I_2/Q} (z - I_1/Q) f(z) dz + \\ &(m + L) \left(p + q \int_{I_2/Q}^1 f(z) dz \right) = \\ &LqF\left(\frac{r}{Q}\right) + q \frac{Q}{D} \int_{r/Q}^{I_1/Q} zf(z) dz + mq(F(I_2/Q) - F(I_1/Q)) + \\ &qQ/D(\psi(I_1/Q) - \psi(I_2/Q) + r/Q\bar{F}(I_2/Q)) + \\ &(m + L)(p + q\bar{F}(I_2/Q)). \end{aligned}$$

To conclude, the long-run expected total cost rate for the case $Q > I_2$, \mathfrak{K}_1 , is given by

$$\mathfrak{K}_1 \equiv \mathfrak{K}_1(Q) = \frac{A + v\mathbb{E}[Y] + h\mathbb{E}[I] + u\mathbb{E}[W] + \pi\mathbb{E}[U]}{\mathbb{E}[T]}, \quad (7)$$

in which all terms have been previously derived.

3.2 Case $I_1 < Q \leq I_2$

The long-run expected total cost rate is formulated as (7), but the calculation of $\mathbb{E}[U]$ and $\mathbb{E}[T]$ differs from the case $Q > I_2$. In fact, now we have $U = W + (r - Y)^+$ and

$$T = \begin{cases} m + (Y - I_1)/D & I_1 < Y \leq Q, \\ Y/D & r < Y \leq I_1, \\ L & Y \leq r. \end{cases}$$

Hence, the expressions of $\mathbb{E}[U]$ and $\mathbb{E}[T]$ are readily adjusted as follows:

$$\mathbb{E}[U] = \mathbb{E}[W] + \mathbb{E}[(r - Y)^+] = p(Q - I_1) + qQ\psi(I_1/Q) + qQ(r/Q - \mathbb{E}[Z] + \psi(r/Q)), \quad (8)$$

$$\begin{aligned} \mathbb{E}[T] &= LqF\left(\frac{r}{Q}\right) + q \frac{Q}{D} \int_{r/Q}^{I_1/Q} zf(z) dz + p \left(m + \frac{Q - I_1}{D} \right) + \\ &mq(F(I_2/Q) - F(I_1/Q)) + qQ/D(\psi(I_1/Q) - \psi(I_2/Q) + \\ &\bar{F}(I_2/Q)r/Q). \end{aligned} \quad (9)$$

In conclusion, the long-run expected total cost rate for the case $I_1 < Q \leq I_2$, \mathfrak{K}_2 , is given by

$$\mathfrak{K}_2 \equiv \mathfrak{K}_2(Q) = \frac{A + v\mathbb{E}[Y] + h\mathbb{E}[I] + u\mathbb{E}[W] + \pi\mathbb{E}[U]}{\mathbb{E}[T]},$$

where $\mathbb{E}[Y]$, $\mathbb{E}[W]$, $\mathbb{E}[I]$, $\mathbb{E}[U]$, and $\mathbb{E}[T]$ are respectively given by (4)–(6), (8), and (9).

3.3 Case $r < Q \leq I_1$

In this case, as already observed, the product useful life does not affect the inventory dynamics. Hence, $W = 0$ deterministically. Moreover, the following additional adjustments, compared to the other cases, are to be made.

The lost sales per cycle are $U = (r - Y)^+$. Therefore, recalling the calculations we previously made, we have

$$\mathbb{E}[U] = qQ(r/Q - \mathbb{E}[Z] + \psi(r/Q)). \quad (10)$$

We then have $I = Y^2/(2D)$, which gives:

$$\mathbb{E}[I] = \frac{\mathbb{E}[Y^2]}{2D} = \frac{Q^2[p + q(\mathbb{V}(Z) + \mathbb{E}[Z]^2)]}{2D}. \quad (11)$$

Moreover, since the cycle length is given by

$$T = \begin{cases} Y/D & r < Y \leq Q, \\ L & Y \leq r, \end{cases}$$

the expected cycle length is

$$\mathbb{E}[T] = LqF\left(\frac{r}{Q}\right) + \frac{Q}{D}\left(p + q \int_{r/Q}^1 zf(z)dz\right). \quad (12)$$

The long-run expected total cost rate for the case $r < Q \leq I_1$, \mathfrak{K}_3 , is given by

$$\mathfrak{K}_3 \equiv \mathfrak{K}_3(Q) = (A + v\mathbb{E}[Y] + h\mathbb{E}[I] + \pi\mathbb{E}[U])/\mathbb{E}[T], \quad (13)$$

where $\mathbb{E}[Y]$, $\mathbb{E}[U]$, $\mathbb{E}[I]$, and $\mathbb{E}[T]$ are respectively given by (4), and (10)–(12).

3.4 Case $0 \leq Q \leq r$

The long-run expected total cost rate is formulated as (13), but the terms $\mathbb{E}[U]$ and $\mathbb{E}[T]$ are different. In fact, $U = r - Y$ and $T = L$ deterministically, i.e., once an order arrives, another one is immediately issued. Then, we have $\mathbb{E}[T] = L$, and

$$\mathbb{E}[U] = r - \mathbb{E}[Y] = r - Q(p + q\mathbb{E}[Z]). \quad (14)$$

Hence, if we let $Q_0 \equiv \{Q \in \mathcal{R}: Q > 0\}$ and $\mathbf{1}_{Q_0} \equiv \mathbf{1}_{Q_0}(Q)$, then the long-run expected total cost rate for the case $0 \leq Q \leq r$, \mathfrak{K}_4 , is given by

$$\mathfrak{K}_4 \equiv \mathfrak{K}_4(Q) = (A\mathbf{1}_{Q_0} + v\mathbb{E}[Y] + h\mathbb{E}[I] + \pi\mathbb{E}[U])/L, \quad (15)$$

where $\mathbb{E}[Y]$, $\mathbb{E}[I]$, and $\mathbb{E}[U]$ are respectively given by (4), (11), and (14). Note that when $Q = 0$ (15) reduces to $\mathfrak{K}_4(0) = \pi r/L = \pi D$.

3.5 Problem formulation

Let $Q_1 \equiv \{Q \in \mathcal{R}: Q > I_2\}$, $Q_2 \equiv \{Q \in \mathcal{R}: I_1 < Q \leq I_2\}$, $Q_3 \equiv \{Q \in \mathcal{R}: r < Q \leq I_1\}$, and $Q_4 \equiv \{Q \in \mathcal{R}: 0 \leq Q \leq r\}$, and let

$$\mathfrak{K} \equiv \mathfrak{K}(Q) = \mathfrak{K}_1\mathbf{1}_{Q_1} + \mathfrak{K}_2\mathbf{1}_{Q_2} + \mathfrak{K}_3\mathbf{1}_{Q_3} + \mathfrak{K}_4\mathbf{1}_{Q_4}, \quad (16)$$

where $\mathbf{1}_{Q_i} \equiv \mathbf{1}_{Q_i}(Q)$, for $i = 1, 2, 3, 4$. Then, the problem consists in minimizing (16) in Q over $Q \equiv \cup_{i=1}^4 Q_i$.

4. NUMERICAL EXPERIMENTS

Due to the complex structure of the cost function, solving the optimization problem analytically is challenging. Therefore, we employ numerical methods to determine the optimal order quantity.

In this study, numerical experiments were conducted using Python and the SciPy library, specifically leveraging the Brent method (Press et al., 2002) for local minimum search. The objective of these experiments is to analyze the impact of different parameter values on the optimal order quantity and key performance metrics, including \mathfrak{K} , $\mathbb{E}[Y]$, $\mathbb{E}[I]$, $\mathbb{E}[W]$, and $\mathbb{E}[U]$. We evaluate four distinct problem instances, with

parameter values shown in Table 1. The parameter values presented in Table 1 are intended for illustrative purposes and are chosen to represent four different optimization conditions. In all instances, the demand rate (D), lead time (L), and product lifetime (m) remain constant, and the corresponding values of I_1 and I_2 are 57.99 units and 76.92 units, respectively. Additionally, in each instance the yield ratio (Z) follows a beta distribution with parameters $\alpha = 2$ and $\beta = 2$. This choice reflects a relatively moderate level of variability in the actual quantity received compared to the size of the order.

The results corresponding to the optimal order quantity for each instance are summarized in Table 2. To further illustrate the behavior of the cost function, Figures 2 and 3 depict its variation for instance 1 and 2, respectively. These experiments provide quantitative insights into how different parametric conditions influence the optimal inventory decisions.

Table 1. Problem instances considered in experiments.

	D	A	h	v	u	π	L	m	p	
Instance	1	1,000	100	25	100	100	150	1/52	3/52	0.75
	2	1,000	10,000	25	100	100	500	1/52	3/52	0.01
	3	1,000	1,000	25	100	100	150	1/52	3/52	0.01
	4	1,000	100	25	1000	100	150	1/52	3/52	0.75

Table 2. Results related to the optimal solution in all instances.

	Q	\mathfrak{K}	$\mathbb{E}[Y]$	$\mathbb{E}[I]$	$\mathbb{E}[W]$	$\mathbb{E}[U]$	
Instance	1	57.69	103,074	50.48	1.37	0	0.45
	2	84.65	375,056	42.75	1.11	2.55	4.17
	3	74.62	134,151	37.69	0.86	0.93	2.04
	4	0	150,000	0	0	0	19.23

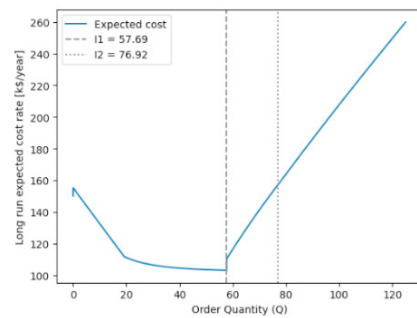


Figure 2. Cost function for instance 1.

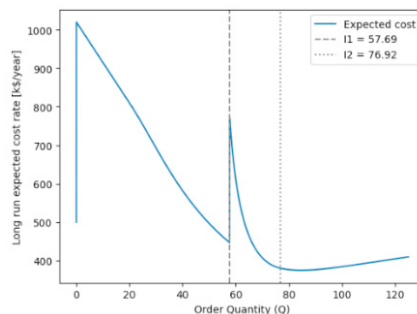


Figure 3. Cost function for instance 2.

We use instance 1 as the baseline scenario. Here, the optimal order quantity aligns exactly with the reorder point. Since the expected yield is slightly lower than the order quantity, the model prioritizes higher stockholding over disposal costs, ensuring that no inventory expires and waste is completely avoided.

Instance 2 has higher order and stockout costs, and a low probability of exactly receiving the ordered quantity, resulting in an optimal order quantity that exceeds I_2 . Compared to instance 1, the minimum cost for instance 2 more than triples. Despite the rise in optimal order quantity by 1.5 times, $\mathbb{E}[Y]$ decreases significantly due to the smaller p value in instance 2. $\mathbb{E}[W]$ increases substantially because of the larger lot size, while $\mathbb{E}[U]$ shows almost a tenfold rise, also linked to the smaller p value. Although the optimal lot size increases, $\mathbb{E}[I]$ decreases, justified by the smaller p and consequently smaller $\mathbb{E}[Y]$, leading to a larger $\mathbb{E}[U]$.

Compared to instance 1, the behavior in instance 3 is like instance 2. Larger A and smaller p push the system towards a larger optimal lot size. Meanwhile, $\mathbb{E}[W]$ and $\mathbb{E}[U]$ increase, but less than in instance 2. Additionally, both $\mathbb{E}[Y]$ and $\mathbb{E}[I]$ are smaller than the values in instance 2.

As the results of instance 4 indicate, when the unit purchase cost increases, the system avoids placing orders. Consequently, only the stockout cost is incurred, which corresponds to the cost of not meeting the entire demand. In this situation, both the inventory level and the quantity of perished product are zero.

5. CONCLUSIONS

This paper examined an EOQ model incorporating product perishability and random yield. A mathematical model was formulated, and numerical experiments were conducted to analyze the model's behavior in some sample scenarios. Future research should explore the cost function's analytical properties to develop an optimization procedure. Further numerical experiments are needed to investigate model sensitivity, offering valuable managerial insights. A case study may also demonstrate the model's practical application.

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