Decisions in Economics and Finance
https://doi.org/10.1007/s10203-025-00552-7

Financial mathematics and its unexpected connections to accounting and corporate finance

Carlo Alberto Magni[1]

Received: 22 April 2025 / Accepted: 11 November 2025

Abstract

Investment analysis requires frameworks that combine theoretical rigor with real-world complexity. While financial mathematics offers powerful tools, its reliance on simplifying assumptions may limit practical applicability. Conversely, accounting and corporate finance provide transaction-level detail and managerial insight, but their integration with financial mathematics remains underexplored. This paper introduces a unifying framework, presenting foundational insights and architectural principles that integrate these disciplines through two universal concepts: the law of motion (governing capital dynamics) and the law of conservation (ensuring equilibrium). These principles, foundational in economics, converge in the Split Screen Matrix, a novel architecture that reveals connections across financial mathematics, accounting, and corporate finance. The Split Screen Matrix uncovers the mathematical linkage between financial statements, integrating balance sheets, income statements, and cash flow statements (traditionally treated in isolation) and shows how these connections mirror market-traded portfolios to capture economic profitability. It also acts as a diagnostic device, allowing validation of models and detection of internal inconsistencies. This interdisciplinary approach demonstrates that cash flows, incomes, and capital amounts are interdependent variables governed by these laws and must be modeled jointly. This integration unifies financial planning, valuation, and decision making, resolving theoretical gaps. Researchers can align their models with accounting and corporate finance principles, clarifying interdisciplinary relationships and exploring new research avenues. Pedagogically, the framework supports student understanding by reducing cognitive load and integrating traditionally fragmented areas of financial analysis into a unified paradigm that mirrors real-world transactions. This approach advances theory and practice, offering practitioners advanced tools, researchers new exploration avenues, and educators innovative teaching strategies, thereby reflecting AMASES's enduring mission to bridge theory and practice, dissolve disciplinary boundaries, and illuminate economic realities through mathematics.

✉ Carlo Alberto Magni
magni@unimo.it

1 Department of Economics "Marco Biagi" and School of Doctorate E4E (Engineering for Economics - Economics for Engineering), University of Modena and Reggio Emilia, Reggio Emilia, Italy

Published online: 12 February 2026

1 Introduction

Investment analysis poses a unique intellectual challenge, requiring the convergence of different disciplines. Financial mathematics, accounting, and corporate finance have traditionally operated as distinct disciplines, each with its own methodologies, terminologies, and objectives. This separation creates barriers to integrated financial analysis and decision making. While financial mathematics provides rigorous tools for valuation and decision making, (e.g., the time-value-of-money principle and the no-arbitrage principle), its theoretical models often rely on simplified assumptions that overlook real-world complexities and limit its practical applicability. Accounting, with its focus on recording and systematizing financial transactions, offers granularity but is often perceived as backward-looking, creating a disconnect from forward-looking financial analysis and decision making. Corporate finance addresses real-world financial management, yet its integration with financial mathematics remains underdeveloped.

Despite these apparent divergences, the three disciplines share foundational principles that reveal an unexpected symmetry. This paper serves as a roadmap to harmonizing these disciplines by unearthing their shared-but-hidden foundations and providing unifying frameworks for financial planning, evaluation, and decision making. In particular, it aims to

- map the conceptual and formal architecture of their integration,
- highlight key results bridging these fields, and
- provide references to foundational works, methodological advances, and practical applications for researchers and practitioners.

The synthesis is grounded in a vast body of academic work, spanning corporate finance, accounting, and financial mathematics, which collectively reveals a deep structural harmony. Two universal principles underpin this integration:

1 the *law of motion*, governing capital evolution through income and cash flows,
2 the *law of conservation*, ensuring equilibrium between investments and financings.

These principles are universal, governing everything from simple loans to complex corporate projects and firms. They are foundational in economics (see Appendix A) and formalizing them reveals the deep interconnections across disciplines. For instance, the law of motion manifests

- in financial mathematics as the time-value-of-money principle,
- in accounting as the clean surplus relation, and
- in corporate finance as the dynamics of asset-debt-equity and market-traded portfolios.

Similarly, the law of conservation manifests

- in financial mathematics as the no-arbitrage principle,
- in corporate finance as the *Law of One Price*, and
- in accounting as the balance-sheet identity.

These shared principles can be merged into a single framework, expressed by the Split Screen Matrix (SSM), a structured, scalable model that visually integrates financial statements (balance sheet, income statement, and cash flow statement) and aligns them with the quantitative rigor of financial mathematics and corporate finance metrics. This *split-screen* approach serves three core functions:

- **Financial planning**. By tracking the interdependencies between capital, income, and cash flow, the split-screen approach (i) supports optimal resource allocation, (ii) systematizes cash flow classifications (FCF, CFE, CCF), (iii) quantifies how payout policies (dividends/retention) drive reinvestment dynamics and (iv) how financing choices (debt/equity) propagate through the financial system.
- **Evaluation**. The split-screen approach (i) bridges accounting and economic valuation by reconciling absolute (NPV, residual income) and relative performance measures (e.g., ROA, ROE, ROI), (ii) integrates valuation models (APV, FCF-WACC, CCF, FCFE) to isolate operating efficiency from capital structure (debt/equity mix) and tax effects (e.g., tax shields, pre-tax and after-tax performance), (iii) resolves long-standing methodological flaws by correcting traditional measures such as IRR and MIRR through coherent capital depreciation, (iv) restores residual-income additivity through market-value adjustments, and (v) quantifies the contribution of scale and financial efficiency to value creation, enabling both period-by-period and aggregate performance analysis.
- **Decision making**. The split-screen approach (i) harmonizes competing investment criteria by resolving conflicts between accounting-based and cash-flow-based approaches, while optimizing capital structure (debt/equity), payout allocation (dividends/reinvestments), and (ii) enhances decision models with granular linkages between operational metrics and financial indicators.

In all three domains – planning, evaluation, and decision making – the split-screen approach also plays a diagnostic role. Because all quantities are governed by the same laws and embedded in the same architecture, internal inconsistencies and structural anomalies can be automatically detected. This built-in validation capacity enhances the model's robustness and reinforces its methodological soundness in both academic analysis and practical application.

These principles also provide a pedagogical framework for teaching finance, accounting, and corporate finance as interconnected systems rather than isolated disciplines. By treating capital, income, and cash flow as interdependent variables governed by universal laws, financial planning, evaluation, and decision making are unified into a coherent analytical framework, fostering deeper conceptual understanding and more integrated learning.

By unifying these perspectives, our framework offers practitioners a robust methodology for modeling, valuation, and value-creating decision making. For academics, it clarifies the symbiotic relationship between disciplines and provides a roadmap for further research.

The paper is a synthesizing framework with key insights: rather than providing detailed proofs or exhaustive discussion of full results (which would exceed the scope of this work), it distills essential coordinates and architectural principles from foundational works (cited in each section), focusing on core interdisciplinary connections. By demonstrating how disparate theories converge into a unified analytical framework, it bridges and advances theory and practice, providing practitioners with spreadsheet-friendly, software-ready tools while offering researchers novel pathways for exploration.

The remainder of the paper proceeds as follows. Section 2 presents two core principles in financial mathematics, the time-value-of-money principle and the no-arbitrage principle, which form the basis for this integrated framework. Section 3 introduces the laws of motion and conservation, demonstrating their universal application across financial mathematics, accounting, and corporate finance. Section 4 combines these laws into the Split Screen Matrix (SSM), creating a unified, scalable structure for financial planning and dynamic accounting integration. Section 5 connects the Law of One Price to the SSM, outlining the reconciliation of NPV variants (APV, FCF-WACC, CCF), market value added, and residual income. Section 6 shows how IRR's and MIRR's methodological flaws can be resolved by linking accounting rates (ROA, ROE) to NPV and analyzing reinvestments dynamics (e.g., feedback loops between retained earnings, liquid assets, and future cash flows). Section 7 concludes the paper, summarizing key insights and implications for theory and practice. Appendix A is devoted to elucidating the foundations of the laws of motion and conservation underlying the paper's framework. Appendix B outlines the financial plan of a capital budgeting project and its financial analysis using spreadsheet modeling.

2 Time value of money and no-arbitrage principle

Financial mathematics provides essential tools for evaluation and decision problems in multi-period economic transactions involving time-dependent, two-party monetary exchanges. Among these, the loan, embodying the concepts of borrowing and lending, is the most elemental and historically significant transaction. The term "capital", used to designate the principal of a loan, originates from the medieval *capital (pars)* (from *caput*, head), originally referring to livestock as a measure of wealth (Fetter 1937). The associated notion of "interest", representing the growth of loaned capital, also has ancient roots (Van de Mieroop 2005). Together, capital (C), interest (I), and cash flow (F) form the three core components of a loan, which are dynamically linked by the intertemporal recurrence relation

$$C_t = C_{t-1} + I_t - F_t \tag{1a}$$

for $t = 0, 1, \ldots n$, with the boundary conditions $C_{-1} = C_n = 0$, indicating that before the transaction begins and after the loan has been fully repaid, the capital loaned is zero. This fundamental equation governs loan dynamics. Interest is obtained by multiplying interest rate by the outstanding debt, $I_t = i_t C_{t-1}$. We may then write

$$C_t = C_{t-1}(1 + i_t) - F_t \tag{1b}$$

from which we derive a backward induction formula:

$$C_{t-1} = \frac{C_t + F_t}{1 + i_t} \tag{1c}$$

illustrating the principle of *time value of money*, a cornerstone of financial mathematics. This principle implies that current capital is the discounted value of future cash flows:

$$C_0 = \sum_{t=1}^{n} \frac{F_t}{\prod_{k=1}^{t}(1+i_k)}. \tag{2}$$

Over time, financial assets have evolved from basic loans to include promissory notes, bonds, shares, and other tradable securities, all adhering to eq. (1a-c).

The *no-arbitrage principle* complements the time-value-of-money principle as a key concept in financial mathematics, stating that risk-free profit opportunities (arbitrage) cannot persist in competitive markets, as they are quickly exploited, restoring price equilibrium. This principle underpins rational choice theory as its most fundamental axiom of economic rationality. Its formalization dates back to de Finetti's (de Finetti 1937) coherence principle, or the Dutch Book Theorem, which guarantees that rational decision makers eliminate arbitrage opportunities (see also de Finetti 1974; Ross 1978; Varian 1987; Nau 1999, 2025). In financial mathematics, the no-arbitrage principle serves as the foundation for pricing models, derivative valuation, equilibrium theory, and decision making under uncertainty. Key results across these areas stem from this unifying principle and are derived using essentially a single mathematical framework (Nau 2025).

The time value of money and the no-arbitrage principle drive the evaluation of financial and real assets, with financial mathematics providing useful theoretical constructs. However, real-world decisions involve complex transactions with multiple agents. A firm's investment spans transactions with suppliers, customers, employees, workers, tax authorities, shareholders, and lenders; similarly, a bank's financial activities engage depositors, borrowers, investors, regulators, shareholders, and suppliers. These interactions require considering factors often overlooked in financial mathematics. For example, in financial planning for projects or firms, the cash flow is not exogenously determined, but it depends on prices, quantities, supply, labor costs, credit terms, inventory policy, financing policy, payout policy, tax regulations, and other business transactions triggered by the project itself. These factors are more thoroughly addressed in other disciplines, such as accounting and corporate finance. Accounting focuses on the recording and summarizing of financial transactions through financial statements such as balance sheets, income statements, and cash flow statements. Corporate finance deals with managing a company's financial resources, including financial planning, investment, and risk management. To fully understand and manage the subtleties of financial decision making, it is essential to combine financial mathematics with accounting and corporate finance into a cohesive approach that addresses the practical complexities of business operations.

Analysts evaluating projects, firms, or portfolios must possess expertise in accounting and corporate finance concepts to build consistent quantitative models. These models should rely on input data that are processed and systematically incorporated into financial plans, which include accounting variables with subtle interconnections. These variables should then be assessed for economic profitability using various metrics that express values, profits, or rates of return, guiding toward economically rational decisions (e.g., see Damodaran 1999, 2006; Hartman 2007; Welch 2009; Newnan et al. 2009; Park 2013; Ross et al. 2013; Benninga 2014; Berk and DeMarzo 2017; Benninga and Mofkadi 2018; Brealey et al. 2020).

This leads to the question: how can we effectively integrate financial mathematics with accounting and corporate finance? Is it possible to harmonize their differing perspectives, terminologies, and methodologies to develop practical principles and models for financial planning and decision making? The answer is yes. Financial mathematics shares the same foundational principles with accounting and corporate finance, all grounded in two basic laws: a *law of motion* and a *law of conservation*. These laws create a cohesive framework for analysis and decision making. The following sections will explain how these laws bring these disciplines together, providing a unified approach for tackling complex real-world financial problems.

3 Shared foundations: law of motion and law of conservation

All economic entities, including firms, projects, or securities, can be viewed as dynamic systems characterized by three fundamental elements: capital, income (or profit), and cash flow. Capital represents monetary resources, income reflects its growth, and cash flow tracks inflows/outflows to/from the entity. These entities follow two fundamental laws: the law of motion (dynamics) and the law of conservation (statics). The *law of motion* describes how capital evolves over time due to income and cash flow:

$$\text{Capital}_t = \text{Capital}_{t-1} + \text{Income}_t - \text{Cash Flow}_t. \tag{3}$$

This equation generalizes eq. (1a). While financial mathematicians typically refer to it in the context of borrowing and lending, it is a more general dynamic relation that applies to any entity, including firms, projects, savings accounts, securities, and more. For example, eq. (3) is interpreted for a firm, a project, a bank account, and a public debt, respectively, as

C_t	$= C_{t-1}$	$+I_t$	$-F_t$
Equity_t	$= \text{Equity}_{t-1}$	$+ \text{Net Income}_t$	$- \text{Cash Flow to Equity}_t$
$\text{Invested Capital}_t$	$= \text{Invested Capital}_{t-1}$	$+ \text{Return}_t$	$- \text{Net Cash Flow}_t$
Account Balance_t	$= \text{Account Balance}_{t-1}$	$+ \text{Interest}_t$	$- \text{Net Withdrawal}_t$
Public Debt_t	$= \text{Public Debt}_{t-1}$	$+ \text{Interest}_t$	$- \text{Primary Balance}_t$

If capital is nonzero, then eq. (3) can be expressed in terms of the rate of return, as in eq. (1b). The key difference is that for a firm or project, the rate i_t is not fixed a priori, but derived from estimates of income, I_t, and capital, C_{t-1}, which are first-order variables: $i_t = I_t/C_{t-1}$.

Furthermore, economic entities have two sides. Specifically, the entity raises capital from capital providers (financing side) and injects resources into assets (investment side). Investments and financings follow their own laws of motion:

$$\begin{array}{ll}\text{Investments:} & C_t^{\text{inv}} = C_{t-1}^{\text{inv}} + I_t^{\text{inv}} - F_t^{\text{inv}} \\ \text{Financings:} & C_t^{\text{fin}} = C_{t-1}^{\text{fin}} + I_t^{\text{fin}} - F_t^{\text{fin}}\end{array} \tag{4}$$

with obvious meanings of the symbols. At each point in time t, the investment side and the financing side balance in terms of capital, income, and cash flow, reflecting a *law of conservation*:

$C_t^{\text{inv}} = C_t^{\text{fin}}$: investments equal financings
$I_t^{\text{inv}} = I_t^{\text{fin}}$: income from investments equals income to capital providers
$F_t^{\text{inv}} = F_t^{\text{fin}}$: cash flow from investments equals cash flow to capital providers

These two laws manifest across disciplines:

- in financial mathematics, as the time-value-of-money and no-arbitrage principles (Section 2)
- in accounting, as the *clean surplus relation* and balance-sheet identity (Horngren et al. 2012; Weygandt et al. 2012) (Section 4)
- in corporate finance, as the dynamics of asset-debt-equity and market-traded portfolios, and the Law of One Price (Section 5).

The law of motion and the law of conservation can be conveniently combined into a two-dimensional format (eq. (5)):

Capital$_t$	**Capital**$_{t-1}$	$+$**Income**$_t$	$-$**Cash Flow**$_t$
$+C_t^{\text{inv}}$	$+C_{t-1}^{\text{inv}}$	$+I_t^{\text{inv}}$	$-F_t^{\text{inv}}$
$+C_t^{\text{fin}}$	$+C_{t-1}^{\text{fin}}$	$+I_t^{\text{fin}}$	$-F_t^{\text{fin}}$

(5)

Here, the red horizontal and vertical bars act as equality signs, enforcing the law of motion (horizontally) and the law of conservation (vertically). This subdivision splits up the matrix into four partitions (quadrants), giving rise to a *Split Screen Matrix* (SSM) (Magni 2020. See also this theoretical account). Reading by row, one observes the dynamic evolution of both capital invested and capital raised for the economic entity under scrutiny; reading by column, one observes the static equilibrium between

- capital invested and capital raised from fund providers at times $t-1$ and t (columns 1-2)
- income from investments and income to fund providers at time t (column 3)
- cash flow from investments and cash flow to fund providers at time t (column 4).

As we will see in Section 4, the SSM operationalizes the law of motion and the law of conservation into a unified analytical framework, offering a concise visual representation of accounting and finance foundations. It captures both the dynamic evolution and static equilibrium of an economic entity across investments and financings. By summarizing these principles in a simple yet comprehensive format, the SSM serves as a powerful tool for analyzing economic assets and supporting financial decision making in financial planning, investment appraisal and decision making.

4 Financial planning with accounting principles

At any fixed date $t = 0, 1, \ldots, n$, capital raised from capital providers can be broken down into debt (e.g., loans) and equity (e.g., shareholders' investments). Correspondingly, income is allocated to interest expenses (for debtholders) and net income (for equityholders), and cash flow is divided into cash flow to debt (CFD) and cash flow to equity (CFE). For a given project or firm,

$$\begin{aligned}
\overbrace{C_t^{\text{inv}}}^{\text{capital invested}} &= \overbrace{C_t^d + C_t^e}^{\text{capital raised from capital providers}} \\
\overbrace{I_t^{\text{inv}}}^{\text{income from investments}} &= \overbrace{I_t^d + I_t^e}^{\text{income to capital providers}} \\
\overbrace{F_t^{\text{inv}}}^{\text{cash flow from investments}} &= \overbrace{F_t^d + F_t^e}^{\text{cash flow to capital providers}}
\end{aligned} \tag{6}$$

where the superscripts d and e refer to debt and equity, respectively.

Thus, the SSM in (5) expands as shown in eq. (7):

Capital$_t$	**Capital**$_{t-1}$	$+$**Income**$_t$	$-$**Cash Flow**$_t$
$+C_t^{\text{inv}}$	$+C_{t-1}^{\text{inv}}$	$+I_t^{\text{inv}}$	$-F_t^{\text{inv}}$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

(7)

This three-area matrix is partitioned by red horizontal and vertical bars, acting as equality signs: for any given row or column, the elements of the opposite partitions sum up to the same amount. The rows show the dynamic evolution (law of motion) and the columns reflect the static equilibrium (law of conservation). Columns 1-2 describe the static equilibrium between investments and financings at $t-1$ and t: $C_t^{\text{inv}} = C_t^d + C_t^e$. Column 3 shows the equilibrium of incomes: $I_t^{\text{inv}} = I_t^d + I_t^e$. Column 4 establishes the cash flow equilibrium: $F_t^{\text{inv}} = F_t^d + F_t^e$.

Each area can be further disaggregated into several classes to suit specific decision-making contexts, with the number and types of classes depending on the nature of the problem. For example, in project planning, the investment area can be disaggregated into categories corresponding to the various project's operating transactions. These include fixed capital, such as property, plant, and equipment, and net working capital, comprising accounts receivable and inventory, net of operating liabilities (e.g., accounts payable, salaries and wages payable, and taxes payable). Similarly, debt area can be categorized into different types of financing (e.g., loans, bonds, notes payable), each with its own income and cash flow components, interconnected by the law of motion. Figure 1 illustrates the corresponding expanded SSM.[1] The equilibrium in columns 1-2 represents what accountants call the Balance Sheet (BS), while column 3

[1] Notably, cash flow from inventory is always zero, which implies, by the law of motion, that the inventory income is equal to Δ Inventory (i.e., change in inventory from the beginning to the end of the period).

BALANCE SHEET$_t$	BALANCE SHEET$_{t-1}$	+ INCOME STATEMENT$_t$	- CASH FLOW STATEMENT$_t$
+ Accounts receivable$_t$	+ Accounts receivable$_{t-1}$	+ Sales$_t$	- Payments from customers$_t$
+ Inventory$_t$	+ Inventory$_{t-1}$	+ Inventory income$_t$	- Cash flow from inventory$_t$
- Accounts payable$_t$	- Accounts payable$_{t-1}$	- Cost of purchases$_t$	+ Payments to suppliers$_t$
- Salaries and wages payable$_t$	- Salaries and wages payable$_{t-1}$	- Labor cost$_t$	+ Payments to employees$_t$
+ Net Fixed Assets$_t$	+ Net Fixed Assets$_{t-1}$	- Depreciation$_t$	- Net disposals of assets$_t$
- Taxes payable$_t$	- Taxes payable$_{t-1}$	- Taxes$_t$	+ Tax payments$_t$
+ Loans$_t$	+ Loans$_{t-1}$	+ Interest on loans$_t$	- Loans instalments$_t$
+ Bonds$_t$	+ Bonds$_{t-1}$	+ Interest on bonds$_t$	- Payments to bondholders$_t$
+ Equity$_t$	+ Equity$_{t-1}$	+ Net income$_t$	- Cash flow to equity$_t$

Fig. 1 Split Screen Matrix - expanded. This architecture captures the intricate interdependencies among a project's or firm's cash flows, incomes, and capital amounts, and their various constituents. This integrated view simplifies the modeling process, providing a robust foundation for financial planning

reflects the Income Statement (IS), and column 4 represents the Cash Flow Statement (CFS). This structure is summarized by the dynamic accounting equation below:

$$\overbrace{\text{Balance Sheet}_t}^{C_t} = \overbrace{\text{Balance Sheet}_{t-1}}^{C_{t-1}} + \overbrace{\text{Income Statement}_t}^{I_t} - \overbrace{\text{Cash Flow Statement}_t}^{F_t} \tag{8}$$

Logical coherence is maintained by the laws of motion and conservation. For example,

– Equity can be derived as a residual from the BS, based on the law of conservation

– Payments from customers can be estimated using the second row, based on the law of motion, after estimating sales and accounts receivable.

– Net income can be obtained from column 3 via the law of conservation, after estimating the other components of IS.

– Taxes payable can be derived from the sixth row using the law of motion, based on tax estimates and tax payments, which depend on deadlines set by the Treasury.

This integration creates a dynamic balance sheet, enabling period-by-period tracing of financial analysis within the matrix's invariant structure, where accounting relationships are simultaneously visible and causally linked. In addition to its integrative and representational functions, the SSM also embeds a diagnostic structure: by aligning all quantities with the laws of motion and conservation, the matrix enables automatic detection of inconsistencies and modeling errors. Unlike traditional financial modeling — where diagnostic checks are often neglected — the SSM incorporates coherence conditions intrinsically, allowing validation to emerge directly from the model's logical architecture (see Magni 2025 for a comprehensive set of model verification formulas, Appendix B for additional details, and the accompanying Excel file for implementation).

Equation (8) dramatically extends eq. (1a) from financial mathematics to accounting, presenting a foundational advance by explicitly modeling the dynamic and static interconnections among the three financial statements, a novel integration that is absent from standard accounting frameworks. This reveals the common ground between the two disciplines, showing that their discrepancies are more apparent than real when viewed through the unifying lens of the law of motion. This synergy is profound: financial mathematics, rooted in the time-value-of-money principle, and accounting, focused on systematic construction of financial statements, are integrated by eq. (8). This equation illustrates that they are the two sides of the same coin, both derived from a general law of motion that governs all economic entities.

This architecture simplifies mathematical processing of real-world financial transactions and offers a clear view of a project's financials, laying the foundations for both financial planning and evaluation. The SSM's scalability is evident in its expandable structure: new rows (e.g., assets, liabilities, or tax layers) can be added without disrupting the logical flow, enforced by the invariant laws of motion and conservation. Its bidirectional design (vertical and horizontal reading) preserves clarity at any scale, from simple transactions to complex multilayer projects or firms.

For an n-period project, there are $n+1$ SSMs, generating a *split-screen strip* (i.e., a sequence of $n+1$ interconnected matrices). The strip unifies all periods into a single row per area enabling end-to-end analysis. This collapses multiperiod complexity into a bidirectional framework, combining planning, valuation, and decision making in one tool and one visualization (see Appendix B for details).

5 Valuation and decision-making principles in corporate finance

Modern corporate financial theory traces its roots to the foundational work of Modigliani and Miller (1958) and Miller and Modigliani (1961). Their theorems on capital structure and dividend irrelevance are grounded in the no-arbitrage principle. From this principle, the *Law of One Price* emerged. Sometimes referred to as the law of *conservation of value* (e.g., Brealey et al. 2020, p. 910, Berk and DeMarzo 2017, p. 499), it states that identical goods or assets must trade at the same price, assuming no transaction costs or barriers. Price discrepancies create arbitrage opportunities that quickly eliminate the gap. Nau and McCardle (1991) and Nau (1999) show that the no-arbitrage principle represents the primal characterization of economic rationality, with the Law of One Price as its dual characterization. The Law of One Price unifies valuation and financial decision making. It is the building block of corporate finance, as emphasized by Berk and De Marzo (2017, pp. 23-24):

> "We present corporate finance as an application of a set of simple, powerful ideas. At the heart is the principle of the absence of arbitrage opportunities, or Law of One Price …This simple concept is a powerful and important tool in financial decision making …We use the Law of One Price as a compass; it keeps financial decision makers on the right track and is the backbone of the entire book …Modern finance theory and practice is grounded in the idea of the absence of arbitrage – or the Law of One Price – as the unifying concept in valuation. We introduce the Law of One Price concept as the basis for NPV and the time value of money …we relate major concepts to the Law of One Price"

5.1 Net Present Value: Benchmark matrix, intrinsic value, and additivity

To assess a project, consider an alternative scenario with investments in competitive financial markets. The matrix architecture from eq. (7) can be applied to benchmark portfolios, traded in the financial markets, that replicate project, debt, and equity cash flows from $t = 1$ to $t = n$. Let r_t, r_t^d, and r_t^e denote the expected rates of return of these portfolios, respectively. These portfolios follow the law of motion, and their market values can be written with either backward or forward induction as

$$\begin{array}{llll} \text{Backward Induction} & & \text{Forward Induction} & \\ V_t &= \dfrac{V_{t+1} + F_{t+1}}{1 + r_{t+1}} & V_t &= V_{t-1}(1 + r_t) - F_t \\ V_t^e &= \dfrac{V_{t+1}^e + F_{t+1}^e}{1 + r_{t+1}^e} & V_t^e &= V_{t-1}^e(1 + r_t^e) - F_t^e \\ V_t^d &= \dfrac{V_{t+1}^d + F_{t+1}^d}{1 + r_{t+1}^d} & V_t^d &= V_{t-1}^d(1 + r_t^d) - F_t^d \end{array} \tag{9}$$

where $t = 0, 1, \ldots, n-1$ and $V_n = V_n^e = V_n^d = 0$. These represent the market (or *economic* or *intrinsic*) value of the project, debt, and equity, respectively.[2] The rates r_t, r_t^e, r_t^d represent risk-adjusted (opportunity) *costs of capital* and reflect the minimum returns demanded by investors to compensate for risk, where r_t^e is the return foregone by the equityholders (cost of equity), r_t^d is the return foregone by the debtholders (cost of debt), and r_t is the return foregone by both capital providers collectively (overall cost of capital).

The benchmark matrix, shown in eq. (10), represents a conceptual extension of the SSM in eq. (7), linking accounting to finance. The key difference is that, in this benchmark scenario, column-wise equality is driven by the Law of One Price, not by accounting identities. The Law of One Price (for market values) and the accounting identity (for accounting values) are two sides of the same coin, reflecting dual interpretations of a general law of conservation – one applied to market values and the other to accounting values (eq. (10)).

$$\begin{array}{|c|ccc|} \hline +V_t & +V_{t-1} & +r_t V_{t-1} & -F_t \\ \hline +V_t^d & +V_{t-1}^d & +r_t^d V_{t-1}^d & -F_t^d \\ +V_t^e & +V_{t-1}^e & +r_t^e V_{t-1}^e & -F_t^e \\ \hline \end{array} \tag{10}$$

While accounting values (C_t) track historical transactions, market values (V_t) incorporate forward-looking investor expectations. The SSM bridges this gap: its structure remains identical, but accounting income (I_t) is replaced by market-driven profit ($r_t V_{t-1}$).

From column 3, the well-known concept of *Weighted Average Cost of Capital* (WACC) emerges. Specifically, from $r_t V_{t-1} = r_t^d V_{t-1}^d + r_t^e V_{t-1}^e$, it follows

$$r_t = \frac{r_t^e V_{t-1}^e + r_t^d V_{t-1}^d}{V_{t-1}^e + V_{t-1}^d}. \tag{11}$$

[2] The intrinsic value of a project can be thought of as "the price the project would have if it were traded" (Mason and Merton 1985, pp. 38–39).

This formula shows that the project's cost of capital is a value-weighted mean of equity and debt costs of capital.[3] In practice, to calculate the cost of capital, the most widespread model is the Capital Asset Pricing Model (CAPM) (see Sharpe 1964; Lintner 1965; Mossin 1966, 1969; Tuttle and Litzenberger 1968; Hamada 1969; Stapleton 1971; Bierman and Hass 1973; Rubinstein 1973; Bogue and Roll 1974, Lewellen 1977, Senbet and Thompson 1978; Ang and Lewellen 1982), derived from Markowitz's (1952) mean-variance portfolio theory,[4] although domain-specificity and subjectivity can also play a role in real-world cost-of-capital determination (see here for a discussion of heuristic NPV and the interplay between bounded and unbounded rationality).

The *Net Present Value* (NPV) notion arises when comparing the equilibrium (i.e., market) scenario to the disequilibrium (i.e., project) scenario.[5] The basic idea behind NPV is that a project is worth undertaking if it creates value for its owners, meaning that its market value is higher than the cost to undertake it. Specifically, for each area, the NPV is calculated as the difference between market and accounting values:[6]

- the project NPV is $\text{NPV} = V_0 - C_0$
- the debt NPV is $\text{NPV}^d = V_0^d - C_0^d$
- the equity NPV is $\text{NPV}^e = V_0^e - C_0^e$.

More generally, subtracting column 1 of the project matrix from column 1 of the benchmark matrix yields, for each area, the so-called *Market Value Added* (MVA) (Grant 1996; Hartman 2000; Fernández 2002), which measures the difference between market value and accounting value. The NPV is simply the MVA at time $t = 0$.

In competitive markets, any investment or financing is value-neutral (NPV = 0). A positive NPV indicates disequilibrium, signaling that the project is economically profitable. Shareholder wealth increases (i.e., value is created) if $\text{NPV}^e > 0$, meaning that the project is worth undertaking. Assuming a firm's shares are traded in the financial market, the equity NPV is positive if and only if the change in price following the announcement of the project undertaking is positive. It can be shown that the equity NPV measures the product of the change in share price and the number of shares outstanding:

[3] Note that r_t in eq. (11) is not equal to $(r_t^e C_{t-1}^e + r_t^d C_{t-1}^d)/(C_{t-1}^e + C_{t-1}^d)$, where the economic values V_{t-1} are replaced by the accounting values C_t (e.g., see Robichek and Myers 1965, Ch. 7, and Magni 2021).

[4] Markowitz's work was anticipated by de Finetti (1940), as recently acknowledged (see Markowitz 2006; Rubinstein 2006; Pressacco and Serafini 2007. See also Barone 2008). See Dybvig and Ingersoll (1982) for inconsistencies between CAPM and no-arbitrage pricing (further relevant discussions can be found here and here).

[5] Intriguingly, the first to theoretically envisage the method of present values for comparing alternative courses of action was the Italian mathematician Leonardo [da Pisa] Fibonacci in his Liber Abaci [1202] (Goetzmann 2004; Goetzmann and Rouwenhorst 2005). Subsequent contributions from mathematicians, actuarial scientists, and engineers were driven by real-world applications in legal, banking, and business contexts (Wing 1965; Edwards and Warman 1981; Miller and Napier 1993; Scorgie 1996; Brackenborough et al. 2001). See Parker (1968) for a detailed account of the roles played by actuaries, political economists, engineers, and accounting scholars in the development and dissemination of discounted-cash-flow methods.

[6] More precisely, the NPV is the algebraic sum of market value V_0 and initial cash flow F_0. Only if $I_0 = 0$, then $F_0 = -C_0$.

$$\text{NPV}^e = \text{change in share price} \times \text{number of shares outstanding}$$

(Robichek and Myers 1965, p. 11; Magni 2014, p. 199).

Owing to the Law of One Price, the NPV is additive: NPV $=$ NPV^e $+$ NPV^d.[7] This means that shareholder value creation equals the portion of the project's NPV that is not captured by debtholders: NPV^e $=$ NPV $-$ NPV^d. If the cost of debt, r_t^d, equals the interest rate on debt, i_t^d (a common assumption in corporate finance), the debt is fairly valued ($C_t^d = V_t^d$, whence NPV$^d = 0$), allowing shareholders to retain the entire value generated by the project: NPV^e $=$ NPV.

5.2 Disaggregating NPV: Operating, non-operating, and reinvestment dynamics

The SSM can be expanded to accommodate the distinction between operating assets (C_t^o), which generate core project cash flows (F_t^o), and non-operating (typically, liquid) assets (C_t^l), which absorb reinvestments of retained, undistribtued funds (F_t^l), so that $x_t^{\text{inv}} = x_t^o + x_t^l$, with $x = C, I, F$. This decomposition is represented in the 4-area SSM (eq. 12):

$$\begin{array}{|c|ccc|}\hline +C_t^{\text{inv}} & +C_{t-1}^{\text{inv}} & +I_t^{\text{inv}} & -F_t^{\text{inv}} \\ \hline +C_t^d & +C_{t-1}^d & +I_t^d & -F_t^d \\ +C_t^e & +C_{t-1}^e & +I_t^e & -F_t^e \\ \hline \end{array} \xrightarrow{\text{expands to}} \begin{array}{|c|ccc|}\hline +C_t^o & +C_{t-1}^o & +I_t^o & -F_t^o \\ +C_t^l & +C_{t-1}^l & +I_t^l & -F_t^l \\ \hline +C_t^d & +C_{t-1}^d & +I_t^d & -F_t^d \\ +C_t^e & +C_{t-1}^e & +I_t^e & -F_t^e \\ \hline \end{array} \tag{12}$$

This reveals how financial policy governs the allocation of operating cash flows F_t^o among three competing uses: (i) debt obligations (F_t^d), (ii) dividend distributions (F_t^e), and (iii) reinvestments of retained funds (F_t^l).

This framework also accommodates for liquid assets as a third source of financing beyond equity and debt: when $F_t^l > 0$, disinvestment from liquid assets (cash, banks, or marketable securities) provides the necessary funding.[8] In corporate projects, eq. (12) expands to Figure 2. Further expansions of the SSM are possible, tailored to the specific project or firm. Figure 3 presents a condensed sequence illustrating the progressive expansion of the SSM. Starting from a compact law of motion, the structure is gradually unpacked into a more detailed representation. In the final expansion, liquid assets are split into financial assets and excess cash, while accounts payable and salaries and wages payable are broken down into *manufacturing* (m) and *non-manufacturing* (nm) components. This layered expansion highlights the model's flexibility and facilitates understanding of its modular design. (See also the CAD Inc. project in Appendix B for an illustration.)

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[7] Brealey et al. (2020) list the seven most important ideas in finance, including value additivity, NPV, CAPM, and the law of conservation of value.

[8] The 4-area SSM endogenizes reinvestments through dynamic feedback (equity cash flow → liquid assets → income → equity cash flow), while maintaining structural simplicity via its invariant laws (see Section 6 for comparison with static, nontransparent methods like the MIRR).

The 4-area SSM is subsequently associated with the corresponding 4-area benchmark matrix, which reports the intrinsic value of the 4 cash flow streams, where r^o is the (opportunity) cost of operating asset and r^l is the (opportunity) cost of non-operating assets (eq. (13)):

$$\begin{array}{|c|ccc|}\hline +V_t^o & +V_{t-1}^o & +r_t^o V_{t-1}^o & -F_t^o \\ +V_t^l & +V_{t-1}^l & +r_t^l V_{t-1}^l & -F_t^l \\ \hline +V_t^d & +V_{t-1}^d & +r_t^d V_{t-1}^d & -F_t^d \\ +V_t^e & +V_{t-1}^e & +r_t^e V_{t-1}^e & -F_t^e \\ \hline \end{array} \tag{13}$$

This generates four NPV components, with the total project NPV comprising:

$$\text{Project NPV} = \underbrace{\text{Operating NPV}}_{\text{core business}} + \underbrace{\text{Non-operating NPV}}_{\text{reinvestments}}$$

where Operating NPV, equal to the sum of F_t^o discounted at r_t^o, measures value creation from the core business activities (e.g., production, sales), and Non-operating NPV, equal to the sum of F_t^l discounted at r_t^l, captures value from reinvestments (e.g., interest income from liquid assets). Likewise, by NPV additivity,

$$\text{Project NPV} = \text{Equity NPV} + \text{Debt NPV}.$$

Equity NPV is then equal to

$$\text{Equity NPV} = \text{Operating NPV} + \text{Non-operating NPV} - \text{Debt NPV} \tag{14}$$

where the Debt NPV is found by discounting F_t^d at r_t^d.

5.3 Reconciling valuation methods: CCF, FCF, APV, and CFE

Several NPV variants have emerged in corporate finance, each stemming from different perspectives, cash flow variants and measurements of tax effects. These include methods such as the Capital Cash Flow (CCF), Free Cash Flow (or FCF-WACC),

	BALANCE SHEET$_t$	BALANCE SHEET$_{t-1}$	+ INCOME STATEMENT$_t$	- CASH FLOW STATEMENT$_t$
Operating capital	+ Accounts receivable$_t$	+ Accounts receivable$_{t-1}$	+ Sales$_t$	- Payments from customers$_t$
	+ Inventory$_t$	+ Inventory$_{t-1}$	+ Inventory income$_t$	- Cash flow from inventory$_t$
	- Accounts payable$_t$	- Accounts payable$_{t-1}$	- Cost of purchases$_t$	+ Payments to suppliers$_t$
	- Salaries and wages payable$_t$	- Salaries and wages payable$_{t-1}$	- Labor cost$_t$	+ Payments to employees$_t$
	+ Net Fixed Assets$_t$	+ Net Fixed Assets$_{t-1}$	- Depreciation$_t$	- Net disposals of assets$_t$
	- Taxes payable$_t$	- Taxes payable$_{t-1}$	- Taxes$_t$	+ Tax payments$_t$
	+ Liquid assets$_t$	+ Liquid assets$_{t-1}$	+ Interest income$_t$	- Cash flow from liquid assets$_t$
	+ Loans$_t$	+ Loans$_{t-1}$	+ Interest on loans$_t$	- Loans instalments$_t$
	+ Bonds$_t$	+ Bonds$_{t-1}$	+ Interest on bonds$_t$	- Payments to bondholders$_t$
	+ Equity$_t$	+ Equity$_{t-1}$	+ Net income$_t$	- Cash flow to equity$_t$

Fig. 2 Split Screen Matrix - expanded to include liquid assets (or non-operating assets, or any other asset where cash flows are reinvested)

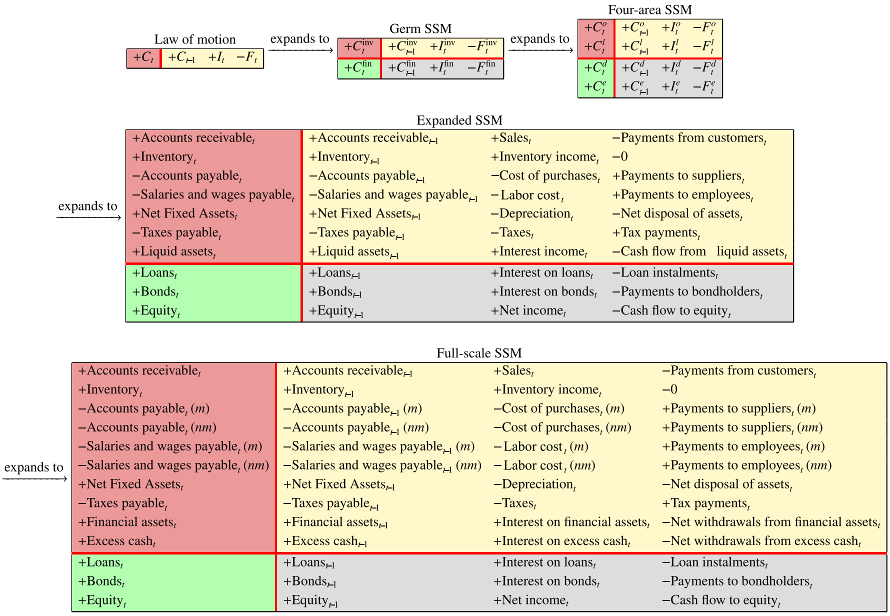


Fig. 3 Stepwise expansion of the Split-Screen Matrix (SSM), from the law of motion to its full-scale disaggregated form

Adjusted Present Value (APV), Cash Flow to Equity (CFE), and others. While these approaches may appear divergent, they are easily reconciled by the two fundamental principles of motion and conservation, which model core operating activities, reinvestment policies, and financing decisions explicitly.

To formalize this connection concisely, define:

- $C^D_t = C^d_t - C^l_t$: net debt
- $I^D_t = I^d_t - I^l_t$: net interest expense
- τ: corporate tax rate
- EBIT_t: Earnings Before Interest and Taxes (i.e., pre-tax operating income)
- $\Delta C^o_t = C^o_t - C^o_{t-1}$: change in operating assets
- $\text{T}_t = \tau \cdot (\text{EBIT}_t - I^D_t)$: income taxes

The after-tax operating cash flow, F^o_t, also known as Capital Cash Flow (CCF), is obtained through the law of motion as

$$F^o_t = \overbrace{\text{EBIT}_t - \text{T}_t}^{\text{operating income after income taxes}} - \Delta C^o_t \tag{15}$$

This formulation allows to decompose CCF into two components: Free Cash Flow (FCF) and the interest tax shield, τI^D_t, expressing the tax benefits from net debt:

$$F^o_t = \text{FCF}_t + \tau I^D_t = \text{EBIT}_t(1-\tau) - \Delta C^o_t + \tau I^D_t. \tag{16}$$

Thus, FCF thus represents the portion of CCF that is independent of the firm's financial policy. From the law of conservation (i.e., equality of cash flows), we get

$$\overbrace{\text{FCF} + \tau I_t^D}^{F_t^o} = F^e + F_t^D \tag{17}$$

where $F_t^D = F_t^d - F_t^l$ is the cash flow to net debt (see also Tham and Vélez-Pareja 2004; Fernández 2004). This equation shows that FCF can also be viewed as *unlevered CFE*, representing what would be distributed to shareholders if the project were unlevered (i.e., equity-financed).

From the third column in eq. (13), we obtain an expression for the operating cost of capital, also known as *pre-tax WACC*:

$$r_t^o = \frac{r^e V_{t-1}^e + r_t^l V_{t-1}^l - r_t^d V_{t-1}^d}{V_{t-1}^e + V_{t-1}^l - V_{t-1}^d} \tag{18}$$

or, more compactly,

$$r_t^o = \frac{r^e V_{t-1}^e + r_t^D V_{t-1}^D}{V_{t-1}^e + V_{t-1}^D} \tag{19}$$

where $V_{t-1}^D = V_{t-1}^d - V_{t-1}^e$ is the market value of net debt and $r_t^D = (r_t^d V_{t-1}^d - r_t^l V_{t-1}^l)/(V_{t-1}^d - V_{t-1}^l)$ is the cost of capital for (net) debt.

The law of motion for operating assets,

$$V_t^o = V_{t-1}^o(1 + r_t^o) - F_t^o \quad \text{or, equivalently} \quad V_{t-1}^o = \frac{F_t^o + V_t^o}{1 + r_t^o}$$

can be restated algebraically in terms of FCF as

$$V_t^o = V_{t-1}^o(1 + r_t^{wacc}) - \text{FCF}_t \quad \text{or, equivalently,} \quad V_{t-1}^o = \frac{\text{FCF}_t + V_t^o}{1 + r_t^{wacc}} \tag{20}$$

where, r_t^{wacc} is the *after-tax* WACC, defined as

$$r_t^{wacc} = \overbrace{r_t^o}^{\text{pre-tax WACC}} - \frac{\tau I_t^D}{V_{t-1}^o} \tag{21}$$

or, equivalently,

$$r_t^{wacc} = \frac{r^e V_{t-1}^e + r_t^D V_{t-1}^D - \tau I_t^D}{V_{t-1}^e + V_{t-1}^D}. \tag{22}$$

This reformulation demonstrates that the operating NPV can be computed by either

- discounting CCF at the pre-tax WACC (CCF method), or

- discounting FCF at the after-tax WACC (WACC method).[9]

An alternative and widely used valuation approach is the Adjusted Present Value (APV) method, which decomposes the operating NPV into two distinct components:

- the *Unlevered NPV*, obtained by discounting FCFs at the *unlevered cost of assets*, k^u
- the *Tax-shield NPV*, obtained by discounting tax shields at a suitable risk-adjusted rate of return r^τ.

The unlevered NPV represents the value of operating assets independently of the firm's financial policy. In contrast, the tax shield NPV captures the value generated by the tax deductibility of net financial expenses and is therefore shaped by the firm's financing choices. Therefore,

$$\text{Operating NPV} = \underbrace{\text{Unlevered NPV}}_{\text{value of investment policy}} + \underbrace{\text{Tax shield NPV}}_{\text{net debt benefits}} \tag{23}$$

When integrating this decomposition with eq. (14), we obtain a comprehensive valuation expression for the Equity NPV:

$$\text{Equity NPV} = \underbrace{\text{Unlevered NPV}}_{\text{value of investment policy}} + \underbrace{\text{Tax shield NPV} + \text{Non-operating NPV} - \text{Debt NPV}}_{\text{value of financial policy}} \tag{24}$$

This decomposition reveals distinct sources of value:

(1) Value of investment policy, derived from core operations (measured via FCFs)
(2) Value of financial policy, derived from

  (2a) Tax benefits from net debt (tax shields, τI^D)
  (2b) Reinvestment activities (non-operating cash flows, F^l)
  (2c) Debt management (debt cash flows, F^d).

A notable special case is when $r^\tau = k^u$, implying $r^o = k^u$. In this case, the APV method becomes mathematically equivalent to the CCF method. This is commonly referred to as *compressed APV* or *conventional CCF* approach.[10]

Finally, in the CFE method, the CFEs are discounted at the cost of equity to determine the equity value:

$$V^e_{t-1} = \frac{V^e_t + F^e_t}{1 + r^e_t}.$$

[9] The standard assumption in corporate finance for the WACC method is that (i) the firm rebalances net debt to maintain a constant, prespecified leverage ratio: $\theta_t = V^D_t/(V^e_{t-1} + V^D_{t-1}) = \theta$, (ii) r^e_t and r^D_t are constant, and (iii) r^D is equal to the interest rate on net debt, $r^D = i^D$, so the NPV of net debt is zero and the equity NPV equals the operating NPV. In this case, the WACC in eq. (22) is given by the textbook formula $r^{wacc} = \theta(1-\tau)r^D + (1-\theta)r^e$.

[10] For detailed treatments of the relationships among the APV, CCF, and WACC method and the valuation of tax shields, see Modigliani and Miller (1963), Myers (1974), Arditti and Levy (1977), Miles and Ezzell (1980, 1985), Harris and Pringle (1985), Lewellen and Emery (1986), Taggart (1991), Ruback (1994, 2002), Inselbag and Kaufold (1997), Luehrman (1997), Fernández (2002, 2004), Arzac and Glosten (2005), Tham and Vélez-Pareja (2004), Booth (2007), Cooper and Nyborg (2006, 2007), Titman and Martin (2016), Berk and DeMarzo (2017), Jagannathan et al. (2017), Magni (2020, 2025).

The equity NPV is then derived as $\text{NPV}^e = V_0^e - C_0^e$. Reconciliation with the other methods is ensured by the following relationships, which express the cost of equity in terms of the pre-tax WACC, after-tax WACC, and the unlevered cost of assets:

$$\begin{aligned} r^e &= r^o + \frac{V^D}{V^e}(r^o - r^D) \\ r^e &= r^{wacc} + \frac{V^D}{V^e}(r^{wacc} - r^D) + \frac{\tau I^D}{V^e} \\ r^e &= k^u + \frac{V^D}{V^e}(k^u - r^D) + \frac{V^\tau}{V^e}(r^\tau - k^u). \end{aligned} \tag{25}$$

Subscripts are omitted for convenience (see Magni 2020, Section 6 for details. See, in particular, Table 6.31). Note that the third formula reduces to the first if $r^\tau = k^u$ (since k^u reduces to r^o).

5.4 Residual income: linking accounting profit and value creation

While cash flows determine NPV, income gives rise to the concept of *residual income* (or *excess profit*). Widely used in accounting and finance, residual income measures the project's income that exceeds what would be earned by investing the capital amount C_{t-1} at the cost of capital r_t, Formally, it is defined as

$$\text{RI}_t = I_t - r_t C_{t-1}$$

or, equivalently, $\text{RI}_t = C_{t-1}(i_t - r_t)$ when $C_{t-1} \neq 0$. The concept was first anticipated by Hamilton (1777) and Marshall (1890) and later developed within accounting and finance (see Mepham 1980 and Arnold 2000). From a different perspective, it was explored within financial mathematics through the work of Lorenzo Peccati, who first extended the loan dynamics in eq. (1b) to projects. At the 1987 AMASES conference and later in a 1991 monograph, Peccati applied accounting concepts by reinterpreting i_t as an accounting rate derived from pre-determined capital amounts C_{t-1} and C_t, in order to decompose the equity NPV into period margins and demonstrate the intrinsic symmetry between accounting and financial performance metrics (Peccati 1987, 1991).

A notable feature of this model is that the sum of the residual incomes, discounted at r_t, yields the project's NPV (see Preinreich 1938; Edwards and Bell 1961; Bodenhorn 1964; Peasnell 1981, 1982; Peccati 1987, 1989, 1991; Ohlson 1989; Gallo and Peccati 1993; Luciano and Peccati 1993; Lundholm and O'Keefe 2001; Feltham and Ohlson 1995; Penman 1998, 2001. See also Lohmann 1988, eq. (43). For a critical review of residual income theories, see Magni 2009). From the capital providers' perspective, residual incomes are

$$\text{RI}_t^e = I_t^e - r_t^e C_{t-1}^e \quad \text{and} \quad \text{RI}_t^d = I_t^d - r_t^d C_{t-1}^d,$$

Table 1 NPV and Residual-Income additivity in the Split-Screen Framework (red bar signifies equality)

	Accounting Value	Market Value	NPV = Total ERI
Project	$+C_0$	$+V_0$	$+V_0 - C_0 = \sum_{t=1}^{n}(I_t - r_t V_{t-1})$
Debt	$+C_0^d$	$+V_0^d$	$+V_0^d - C_0^d = \sum_{t=1}^{n}(I_t^d - r_t^d V_{t-1}^d)$
Equity	$+C_0^e$	$+V_0^e$	$+V_0^e - C_0^e = \sum_{t=1}^{n}(I_t^e - r_t^e V_{t-1}^e)$

sometimes referred to as *residual* (or *abnormal*) *earnings* and *residual financial expense* (Feltham and Ohlson 1995; Ohlson 1995, 2003, 2005; Penman 2010). A modified version, *Economic Value Added* (EVA), popularized by Stewart (1991), is widely used in applied corporate finance and management accounting (Stern et al. 1995; O'Byrne 1996; Biddle et al. 1999; Martin et al. 2003; O'Hanlon and Peasnell 1998, 2014). Residual income, however, suffers from non-additivity: the project's residual income, $\text{RI}_t = I_t - r_t C_{t-1}$, does not equal the sum of residual earnings and residual financial expenses:

$$\begin{aligned} \text{RI}_t^e + \text{RI}_t^d &= (I_t^e - r_t^e C_{t-1}^e) + (I_t^d - r^d C_{t-1}^d) \\ &\neq I_t^e + I_t^d - \frac{r_t^e V_{t-1}^e + r_t^d V_{t-1}^d}{V_{t-1}^e + V_{t-1}^d} \cdot (C_{t-1}^e + C_{t-1}^d) \\ &= I_t - r_t C_{t-1} = \text{RI}_t. \end{aligned} \tag{26}$$

(see Magni 2021, Proposition 1). The paradox is resolved with the described SSM architecture. By subtracting column 3 of the benchmark matrix from column 3 of the project matrix, we obtain the *economic* residual income (ERI), where economic values V_t replace accounting values C_t. Due to the invariance property of the SSM, this ensures additivity: the project's economic residual income,

$$\text{ERI}_t = I_t - r_t V_{t-1}, \tag{27}$$

equals the sum of the economic residual earnings, $\text{ERI}_t^e = I_t^e - r_t^e V_{t-1}^e$, and the economic residual financial expenses, $\text{ERI}_t^d = I_t^d - r_t^d V_{t-1}^d$. Specifically, by applying the law of conservation to income and accounting values, and using equation (11),

$$\begin{aligned} \text{ERI}_t^e + \text{ERI}_t^d &= (I_t^e - r_t^e V_{t-1}^e) + (I_t^d - r^d V_{t-1}^d) \\ &\neq I_t^e + I_t^d - \frac{r_t^e V_{t-1}^e + r_t^d V_{t-1}^d}{V_{t-1}^e + V_{t-1}^d} \cdot (V_{t-1}^e + V_{t-1}^d) \\ &= I_t - r_t V_{t-1} = \text{ERI}_t. \end{aligned} \tag{28}$$

Furthermore, consistency with the project NPV is maintained, as the sum of the economic residual incomes equals the NPV, as shown in Table 1, column 4 (see also Magni 2021).

Thus, economic residual income 'unpacks' NPV into periodic contributions: just as NPV aggregates discounted cash flows, it also equals the sum of undiscounted residual incomes, showing a powerful (additive) link between income-based and cash-flow based valuation.

While residual income decomposes NPV into periodic terms, evaluating financial efficiency requires relative metrics. Section 6 leverages the SSM's architecture to reconcile accounting rates of return (ROA, ROE) with risk-adjusted costs of capital, resolving IRR's shortcomings and linking directly to the additive framework developed here.

6 Relative profitability metrics: From IRR to accounting rates

Absolute metrics, such as the NPV or residual income, fail to capture financial efficiency, defined as return per unit of invested capital over and above the return obtained by a benchmark asset. NPV does not distinguish whether value creation stems from a large amount of capital invested at a low rate of return or from a small amount of capital invested at a high rate of return. In practice, managers and advisors rely on relative metrics such as rates of return to evaluate investments, alongside or even in place of NPV.

6.1 Internal Rate of Return

The *Internal Rate of Return* (IRR) is the most popular relative profitability measure. It is derived from eq. (2) by setting up the equation $\sum_{t=0}^{n} F_t(1+\sigma)^{-t} = 0$, where the unknown σ replaces the return estimates i_t (Fisher 1930; Boulding 1935; Keynes 1936/1967; Alchian 1955). A project's IRR exceeding the cost of capital is taken as an indication of value creation for shareholders. Unfortunately, IRR has significant limitations, including

(i) potential multiple or no solutions,[11]
(ii) lack of strong NPV-consistency,
(iii) NPV-IRR ranking conflict,
(iv) erroneous signaling of short positions,
(v) built-in capital amounts disconnected from capital estimates
(vi) and more

(see Lorie and Savage 1955; Hirshleifer 1958; Bailey 1959; Karmel 1959; Soper 1959; Arrow and Levhari 1969; Hicks 1970; Flemming and Wright 1971; Nuti 1973; Burmeister 1974; Sen 1975; Aucamp and Eckardt 1976; De Faro 1978; Herbst 1978; Ross et al. 1980; Gronchi 1984/1987; Hazen 2003, 2009; Hartman and Schafrick 2004; Borgonovo and Peccati 2004, 2006; Cuthbert and Cuthbert 2012; Percoco and Borgonovo 2012; Magni 2013; Magni and Marchioni 2020). Despite its popularity, these shortcomings make the IRR unsuitable for evaluating financial assets or capital budgeting projects.

[11] Notably, for equity cash flows, IRR *never* exists in cases where funding comes from internal sources or debt.

6.2 Modified Internal Rate of Return: From arbitrary assumptions to dynamic accounting

The *Modified Internal Rate of Return* (MIRR) addresses IRR's non-uniqueness and non-existence by assuming that interim cash flows are reinvested at some (unspecified) rate, thereby separating outflows and inflows.[12] Under suitable conditions, MIRR yields a unique solution, but it just reflects an internal rate of return "on a modified set of cash flows, not the project's actual cash flows" (Ross et al. 2011, p. 250), failing to capture true economic profitability. This modification problem is fundamental: (i) infinite variations of the cash flow stream produce multiple MIRRs (e.g., see Ross et al. 2011, p. 250), and (ii) as Keane (1979, p. 50) emphasizes, the need for reinvestment assumptions exposes flaws in the original financial plan: "the original cash flow specifications are incorrect because they should have included the effects of the entire programme of interdependent projects".

Complying with Keane's recommendation, the split-screen approach embeds retention policies directly in the cash flow specification through liquid assets and propagating feedback effects. In this way, arbitrary modifications are eliminated (i.e., reinvestment yields emerge from the project's actual financial system). This is operationally implemented through the following dynamic loop:

1. Payout decisions, based on net income (I_t^e), allocate the available cash flow ($F_t^o - F_t^d$) to dividends (F_t^e) or reinvestments (F_t^l)
2. Reinvested funds update liquid assets (C_t^l), generating interest income (I_{t+1}^l)
3. Interest income propagation adjusts net income (I_{t+1}^e), which in turn shapes equity cash flow (F_{t+1}^e) and reactivates the loop.

Figure 4 illustrates the detailed loop.[13] This endogenous cycle eliminates the need for arbitrary post-distribution modifications of cash flows and the use of unspecified *ad hoc* auxiliary rates, such as those employed in the MIRR. Despite the apparent complexity, the algebraic intricacies remain tractable because dynamics is fully captured by the law of motion, statics conforms to the law of conservation, and all interactions emerge from the SSM's invariant architecture.

6.3 Average Accounting Rate of Return: AIRR and IARR

The integrated approach proposed here offers more reliable profitability metrics than IRR or MIRR. Specifically, the rates $i_t = I_t^{\text{inv}}/C_{t-1}^{\text{inv}}$ (Return On Assets, ROA) and $i_t^e = I_t^e/C_{t-1}^e$ (Return On Equity, ROE) provide a framework for estimating a project's economic profitability. Averaging out these rates gives an overall rate of return, as first

[12] MIRR was anticipated by the French actuary Duvillard (1755-1832) (Biondi 2006) and revived in the second half of the 20th century by many authors, including Solomon (1956), Baldwin (1959), Kirshenbaum (1965), Lin (1976), Athanasopoulos (1978), Bernhard (1979), Chang and Owens (1999), Kierulff (2008). See also Peterson and Fabozzi (2002).

[13] Owing to a no-arbitrage argument, if retained funds are reinvested at the cost of equity, payout irrelevance holds (Miller and Modigliani 1961; De Angelo and De Angelo 2006, 2007). Under this condition, forecasting the division between dividends and reinvestments is unnecessary: discounting after-debt cash flows at the cost of equity suffices to determine equity NPV. However, this assumption rarely holds in practice.

Dynamic Reinvestment Loop in SSM:
1. **Payout Policy** sets F_t^e (dividends vs retention)
2. **Retained Funds**: $F_t^l = F_t^e + F_t^d - F_t^o$ (*law of conservation*)
3. **Liquid Assets**: $C_t^l = C_{t-1}^l + I_t^l - F_t^l$ (*law of motion*)
4. **Interest Earned**: $I_{t+1}^l = i_{t+1}^l \cdot C_t^l$
5. **Net Income**: $I_{t+1}^e = I_{t+1}^o + I_{t+1}^l - I_{t+1}^d$ (*law of conservation*)
6. **Next Period**: Payout policy sets F_{t+1}^e as a percentage of I_{t+1}^e (and cycle repeats)

Fig. 4 The self-reinforcing loop of retained funds in SSM. Retentions (F_t^l) update liquid assets (C_t^l), which generate income (I_{t+1}^l) that feeds back into future cash flows. Laws of motion/conservation ensures consistency

advocated by the accounting scholar Vatter (1966, pp. 695-696): "The project rate is an average rate of return over the project term, not an annual one; …The project rate is the average of the book yields."[14] The relevant question is: What kind of average? Using an appropriate invariance function for a Chisini mean (Chisini 1929; de Finetti 1931; Peccati 1991), an average rate of return is generated, termed *Average Internal Rate of Return* (AIRR):

$$i = \frac{\sum_{t=0}^{n} I_t \mathrm{d}_{t,0}}{\sum_{t=0}^{n} C_t \mathrm{d}_{t,0}} \tag{29}$$

where $\mathrm{d}_{t,0} = \prod_{k=1}^{t}(1+r_k)^{-1}$ is the discount factor. If $C_t \neq 0$ for all t, eq. (29) can be easily reframed as a capital-weighted arithmetic mean of the period rates of return $i_t = I_t / C_{t-1}$ (Magni 2010, 2013. See also Magni 2016, eq. (17)). AIRR can also be obtained as the sum of project cost of capital and the NPV per unit of total invested

$$i = r + \frac{\mathrm{NPV}}{\sum_{t=0}^{n} C_t \mathrm{d}_{t,0}}$$

where r here represents a suitable capital-weighted mean of time-adjusted period costs of capital r_t (see Magni 2016, eq. (16), and Magni 2020, eq. (8.33)).[15] The AIRR avoids the pitfalls of IRR and, when compared with the average cost of capital r, correctly reveals financial efficiency (Magni 2010, 2013, 2016).

The IRR itself can be interpreted as a special case of AIRR, specifically associated with the *Hotelling depreciation class*, a particular equivalence class of *ad hoc* capital amounts and time-varying accounting rates. More precisely, the IRR is a capital-weighted arithmetic mean of generally time-varying one-period rates (for details, see Magni 2010, Theorem 3; Magni 2013, *Pars destruens* F13; Magni 2016, Section 7.1).

The AIRR framework has been extended to handle stochastic cash flows (see this discussion) and fuzzy mathematics (see this theoretical treatment), with

[14] Some finance scholars have also concurred: "The rate of return is an average concept. It takes a series of n numbers and converts them into an average value" (Haley and Schall 1979, p. 68. See also Edwards et al. 1987).

[15] Equation (29) defines AIRR as an "instantaneous" rate, so to speak, since it is expressed as a ratio of present values. It was originally introduced in discrete form, referring to a single period, by Magni (2010, 2013); see also Magni (2020, 2025) for further details.

applications spanning diverse fields, including industrial project evaluation, financial portfolio analysis, shareholder value creation, Rotating Savings and Credit Associations, investment behavior analysis (e.g., see Bosch-Badia et al. 2014; Barry and Robison 2014, Mørch et al. 2016, Jiang 2017; Lima e Silva et al. 2018), as well as real estate investment (discussed here) and project financing (e.g., see this contribution in a fuzzy environment). Most recent adaptations have transformed it into an accounting-based framework for estimation and competitive advantage measurement (Danielson 2023, 2024).

A discount-free variant of this average accounting rate, the *Internal Average Rate of Return* (IARR), is directly generated within the split-screen framework by dividing the (undiscounted) sum of the n accounting incomes by the (undiscounted) sum of the n capital amounts:

$$i = \frac{\sum_{t=0}^{n} I_t}{\sum_{t=0}^{n} C_t}. \tag{30}$$

In accounting terms, i is an average accounting rate of return. Unlike IRR and AIRR, the IARR ensures that a portfolio's return equals the weighted average of its constituent assets' rates of return. Specifically, given two assets A and B and their IARRs, i^A, i^B, the IARR of the combined portfolio A+B is

$$i^{A+B} = \frac{i^A C^A + i^B C^B}{C^A + C^B}$$

with $C^j = \sum_{t=0}^{n} C_t^j$, $j = A, B$ (Magni 2021). When compared with a suitable weighted average cost of capital, $\rho = \sum_{t=0}^{n} I_t^V / \sum_{t=0}^{n} C_t$, with $I_t^V = r_t V_{t-1}$, the IARR correctly signals value creation or destruction, in alignment with the NPV.[16]

In this accounting-based approach, an explicit relationship emerges between NPV and the average accounting rate of return, which decomposes the NPV into the two components of scale and efficiency:

$$\text{Project NPV} = \overbrace{\text{Total capital invested}}^{\text{Project scale}} \times \overbrace{(\text{Average ROA} - \text{Average WACC})}^{\text{Project's financial efficiency}} \tag{31}$$

$$\text{Equity NPV} = \overbrace{\text{Total equity invested}}^{\text{Scale of equity investment}} \times \overbrace{(\text{Average ROE} - \text{Average cost of equity})}^{\text{Equity's financial efficiency}} \tag{32}$$

(see also Magni 2015). The project is economically profitable if and only if the average ROA exceeds the average WACC, and shareholder's wealth increases if and only if the average ROE exceeds the average cost of equity (the rule is reversed if the project is a net borrowing).

16 Another variant of AIRR is Aggregate Return On Investment (AROI). Magni (2020, Section 10.6) shows that AIRR, IARR, and AROI are associated, respectively, with RI, NPV, and Net Future Value (see, in particular, Tables 10.8, 10.10, and 10.11).

6.4 The TRM model: Capital dynamics and accounting consistency

The decomposition of NPV into scale and financial efficiency can be applied to the well-known TRM model by Teichroew et al. (1965a, b). Unlike MIRR, this model does not modify cash flows but instead assumes that a pair of rates, a project financing rate, i^N, and a project investment rate, i^P, governs the capital dynamics. The capital C_t evolves, in terms of law of motion, as

$$C_t = \begin{cases} C_{t-1}(1+i^N) - F_t & \text{if } C_{t-1} \leq 0 \quad \text{(borrowing period)} \\ C_{t-1}(1+i^P) - F_t & \text{if } C_{t-1} > 0 \quad \text{(investment period)} \end{cases} \tag{33}$$

where (i^N, i^P) is an *internal pair* , in the sense that they satisfy $C_n(i^N, i^P) = 0$ (see also Gronchi 1984/1987, Pressacco and Stucchi 1997. See also Weingartner 1966 on the notion of *internal vector*). Assuming a constant cost of capital r, setting either $i^N = r$ or $i^P = r$, the authors show that, respectively, $i^P = i^P(r)$ and $i^N = i^N(r)$ are uniquely determined functions of r. They also establish that NPV > 0 if and only if, respectively, $i^P(r) > r$ or $i^N(r) < r$. However, the first scenario ($i^N = r$) restrictively implies that value creation is only captured in the investment periods, whereas the second scenario ($i^P = r$) restrictively implies that value creation is only captured in the borrowing period. Furthermore, the authors neither clarify which assumption is more appropriate nor account for intermediate cases where neither i^N nor i^P equals r. They also do not provide a measure of the project's overall rate of return and the model is inapplicable when the cost of capital is time-varying.

From an accounting-and-finance perspective, we can interpret i^N and i^P as time-invariant ROAs for borrowing and investment periods, respectively. This allows decomposing the NPV into investment and financing NPV, (similar to eq. (31)), each in turn decomposed into scale and financial efficiency:

$$\text{NPV} = \text{NPV}^{\text{inv}} + \text{NPV}^{\text{fin}} = (i^P - r)C^{\text{inv}} + (r - i^N)C^{\text{fin}}, \tag{34}$$

where C^{inv}, C^{fin} are, respectively, the total invested and borrowed capital amounts, such that the difference between them equals the total net invested capital. The project's overall rate of return, here denoted as j, can then be computed, following the average-based approach mentioned above, as the (generalized) capital-weighted mean of i^N and i^P:

$$j = \frac{i^P \cdot C^{\text{inv}} - i^N \cdot C^{\text{fin}}}{C^{\text{inv}} - C^{\text{fin}}}. \tag{35}$$

Here, i^N and i^P are not constrained to equal r but instead represent any valid solution pair of $C_n(i^N, i^P) = 0$. If the project is a net investment (investment positions exceed borrowing positions), then it is acceptable if and only if $i > r$; if the project is a net borrowing (borrowing positions exceed investment positions), then it is acceptable if and only if $i < r$ (see Magni 2014, 2015 for details and extensions to time-varying rates).

These findings reveal a critical limitation of cash-flow-centric paradigms: financial mathematicians cannot reliably compute economic rates of return without first accounting for capital depreciation patterns (Magni 2016). This capital-based approach, incorporating accounting dynamics, is the key for unifying, under the same logical umbrella, diverse profitability measures and models, such as the aforementioned IRR, MIRR, TRM as well as Profitability Index, Benefit-Cost Ratio, Cash Multiple, Simple and Modified Dietz, Time-Weighted Rate of Return, thus resolving their methodological disparities (e.g., see this study).

7 Concluding remarks and future research directions

Financial mathematics, accounting, and corporate finance reveal unexpected interconnections through their shared foundational principles: the laws of motion and conservation. This paper serves as a guide and a unifying framework, demonstrating how their integration supports robust frameworks for investment analysis, valuation, and decision making, addresses complex financial challenges, and delivers practical, impactful solutions. Tools like the Split Screen Matrix (SSM), which combines the two laws, exemplify the potential of interdisciplinary approaches, enabling practitioners to develop real-world financial models that blend theoretical rigor with practical relevance.

The split-screen approach offers practitioners a unified and scalable structure that integrates two traditionally distinct activities: financial planning and investment valuation. This convergence is a crucial innovation: while these tasks are often treated as separate domains with different tools and logics, the split-screen approach shows that both can be conducted using a single algebraic framework. The same structure that governs capital dynamics during the planning phase also supports value creation analysis and performance evaluation in the valuation phase. Significantly, this unified structure also enforces internal consistency by providing a built-in diagnostic mechanism for detecting logical anomalies and internal inconsistencies, an essential capability that traditional financial models typically lack. Practitioners can use this integrated framework to model real-world scenarios – such as changes in payout policies, mixed reinvestment strategies, or impacts of tax regulation – and observe their effects on retained earnings, liquidity, interest income, and equity. The SSM framework ensures consistency between cash flows, capital variations, and incomes, thus avoiding the inconsistencies that often arise in fragmented spreadsheet models found in reeal-world applications. By combining multiple profitability metrics (NPV, IRR, residual income, ROA, ROE, etc.) into a logically coherent system, the split-screen approach supports robust financial decision making and enables a multi-layered analysis of economic performance. The method naturally lends itself to spreadsheet modeling and can be incorporated into professional software platforms, making it both analytically robust and immediately usable in practice. Furthermore, incorporating detailed inputs that accurately describe underlying transactions facilitates the assessment of how individual inputs impact economic profitability and decision making. Significantly, this framework shows that cash flows are just one component: incomes and capital amounts must also be considered to fully capture financial structures and their interdependencies.

For researchers, this integration opens new research directions for exploring interdisciplinary connections. The formal reconciliation of planning models, valuation techniques, and profitability metrics through the split-screen approach invites deeper analysis into how investment and financing NPVs, residual incomes, rates of return, interact under different financial architectures and benchmark systems. The approach enables analytical decompositions that clarify the economic meaning of various profitability measures and resolve long-standing inconsistencies, such as the incompatibility between IRR and NPV or the disconnect between accounting and economic returns. Potential research paths include adapting the model to stochastic settings, incorporating fuzzy logic or risk-based modeling, automating sensitivity analysis, and creating diagnostic tools to validate financial models across disciplines. There is also scope to develop computational extensions of the matrix for use in algorithmic investment planning and software-based early warning diagnostics. Furthermore, the SSM can be reformulated as a quadruple-entry bookkeeping system (Magni 2020, Section 3.6), enhanced by a split-screen technique leveraging its invariance property: shifting rows and columns allows transactions to be recorded while uncovering novel connections among accounting and financial magnitudes. By grounding all these results in a general mechanical system (governed by the laws of motion and conservation), this approach offers a rigorous and extendable foundation for further theoretical innovation.

In educational settings, this framework offers a structured approach to teaching investment analysis and financial decision making as a holistic discipline: by emphasizing the interdependence of accounting, corporate finance, and financial mathematics through the SSM's architecture, students learn to analyze economic entities as dynamic systems, bridging theoretical principles with real-world complexity. The SSM provides a didactic architecture that unites selected elements of accounting, finance, and investment analysis into a coherent conceptual system for understanding, modeling, and evaluating investment decisions. This framework facilitates intuitive teaching advanced concepts through visual manipulation. Its tabular, visual structure makes abstract financial relationships accessible to students from diverse backgrounds, including engineering, business, and economics. Educators can use this approach to incorporate it into existing courses or design interdisciplinary courses – such as *Mathematics of Financial Planning and Decision Making* or *Engineering of Financial Planning and Decision-Making* – that illuminate how changes in assumptions propagate through capital positions, incomes, and cash flows. This approach also offers a natural transition from classical teaching of financial mathematics (centered on cash flow logic) to capital-centric reasoning, helping students grasp how valuation models arise from a structured system of accounting identities and market benchmarks. By shifting the focus from cash-flow-based intuition to capital-centric reasoning, educators can help students connect the dots between accounting rates (e.g., ROA, ROE) and market-consistent valuation (e.g., NPV, residual income, cost of capital). The methodological synthesis presented in the paper can be deployed in case-based teaching and spreadsheet labs, fostering the development of both analytical precision and modeling skills. Excel-based applications, in particular, allow students to simulate complete investment evaluations from planning to profitability diagnostics, using a single, consistent logic.

In this context, the law of motion (with its temporal progression) and the law of conservation (with its point-in-time equilibrium) are to financial decision making what melody and harmony are to music, forming the shared structure of financial mathematics, accounting, and corporate finance. Together, they harmonize fragmented perspectives into an actionable framework for financial decision making, equipping financial mathematicians with interdisciplinary tools derived from accounting and corporate finance, and promoting collaborative innovations across disciplines.

Appendix A. Physical and Mechanical Metaphors in Economics: Laws of Motion and Conservation.

The "law of motion" and "law of conservation" used in this paper borrow metaphors from engineering and physics. The rationale is that any economic entity – whether a firm, project, security, or portfolio – can be modeled as a mechanical system, described in terms of both dynamics and statics. From a dynamic perspective, the economic unit behaves like a dynamical system: its capital position starts at rest (at a zero level), then jumps up and/or down the zero level owing to perturbations caused by incomes and cash flows, and eventually returns to zero level at time n, when the unit is liquidated. Mathematically, this behavior is described by a first-order difference equation. From a static perspective, the entity is subject to balancing forces that keep it in a permanent state of equilibrium. At any point in time, the capital invested equals the capital raised from capital providers; the income generated from investment equals the income accrued to capital providers; and the cash flow extracted from the investments equals the cash flow distributed to capital providers. Mathematically, this is expressed through three equalities that reflect such static equilibrium.

The connection between mechanics and economics, and the use of mechanical analogies in economic sciences has deep academic root dating back more than a century. In his PhD thesis *Mathematical Investigations in the Theory of Value and Prices*, Irving Fisher introduced Chapter II, titled 'Mechanics', by stating

> Scarcely a writer on economics omits to make some comparison between economics and mechanics […] the economist borrows much of his vocabulary from mechanics. Instances are: equilibrium, stability, elasticity, expansion, inflation, contraction, flow, efflux, force, pressure, resistance, reaction, distribution (price), levels, movement, friction. (Fisher 1892, p. 24)

Marginalist economists were fascinated by the principles of physics – especially, mechanics – and were influenced by it. Pareto, for instance, claimed that "this science [of pure economics] does not merely resemble mechanics, it is regarded as a type of mechanics" (Dawson 2021, C436. See also Turk 2012). Mirowski (1984a) highlights that the radical discontinuity brought about by the marginalist revolution – which marked the birth of neoclassical economics – was characterized by importation of metaphors, mathematical techniques, and methodological assumptions from mid-19th century physics. Cross (1995) underscores the "propensity of neoclassical economists to draw their metaphors from what they perceived to be the most prestigious scientific discipline" even though the growing specialization of academic fields in the twentieth century left many scarcely "aware of the fact that they are using metaphors drawn from nineteenth century physics" (p. 128).

The analogy between economics and mechanics was also explicit in Léon Walras (e.g., see Turk 2006). De León and De Diego (1998, p. 335) even maintain that "The fundamental laws of Mechanics and Economic Dynamics are essentially the same", and the use of analogies and metaphors from physics is frequent in economics (Turk 2006). Among them, the concepts of *laws of motion* and *laws of conservation* are pervasive, though their meanings vary depending on the field, context, perspective, and aims.

Law of motion. In economics sciences, the *law of motion* is a well-established term that typically refers to a difference or differential equation describing the evolution of an economic variable over time. While the mathematical form is often standard, its structure and economic interpretation vary significantly depending on the models, analytical perspectives, and object of study. Boyer (2011) notes that the expression "economic laws" can have various meanings and proposes several conjectures about possible *laws of motion in capitalism*.

Many studies illustrate the versatility of this concept. For instance, Garivaltis (2019) investigates a law of motion for the broker call rate, while Costain, Nuño, and Thomas (2025) use distinct laws of motion for bonds and for wealth in their arbitrage-based model of the yield curves in a monetary union exposed to sovereign default risk. Similarly, Rebelo, Wang, and Yang (2022) employ laws of motion for output, spot rate, and financial wealth in their model of sovereing debt.

In a portfolio-based framework, Yang et al. (2024) present laws of motion for investor wealth, net financial savings (including dividend payouts), and U.S. net foreign assets to understand the recent decline in U.S. foreign assets.

The term is also common in fiscal policy and macroeconomic dynamics. Angeletos et al. (2024) incorporate a *law of motion* for public debt to show that deficits can be self-financing by stimulating aggregate demand. Faria-e-Castro (2024) develops a dynamic model of fiscal policy with laws of motion for several variables, including housing stock, banks' earnings and equity, the share of bank equity owned by the government, and the share of guaranteed bank debt.

Krusell and Smith (1998) develop an equilibrium model featuring a law of motion for aggregate capital in an equilibrium model with income and wealth heterogeneity (notably, the term *law of motion* appears no fewer than 23 times in their paper). LeMouel and Schiersch (2024) study the effect of intangible capital on firm productivity by embedding it into the law of motion for productivity.

The law of motion also plays a central role in modeling capital accumulation and growth. El-Borhamy, Medhat, and Ali (2021) propose a model of economic growth based on energy transport network that builds on the law of motion for capital, extending the mathematical frameworks of Bianca et al. (2013) and Dalgaard and Strulik (2011). Auray et al. (2014) use the law of motion for capital to investigate the impact of conflicts on macroeconomic aggregates. Luckstead et al. (2014) incorporate investment-specific technology modifying capital investment into the law of motion of capital, while Glomm (1997) employs a law of motion to develop a simple model describing human capital accumulation. Other examples include Oueslati (2002), Martin and Taddei (2013), who use the phrase "law of motion" no fewer than 38 times, and Lindquist (2004).

More generally, the *law of motion* is explicitly used in graduate and undergraduate textbooks (e.g., see Acemoglu 2009) to model the dynamic evolution of key variables such as physical capital, human capital, capital-labor ratio, technology, income, investment, and household assets.

These few examples highlight how pervasive and flexible the law of motion is in economic theory. Despite differences in application, what unifies these contributions is the idea that economic phenomena are governed by intertemporal equations that specify how today's quantities transition into tomorrow's, which is precisely the structural intuition embedded in the framework developed in this paper.

Law of conservation. Conservation laws are ubiquitous in economics and finance, often underpinning key theoretical identities and practical modeling frameworks. In his seminal paper, Mirowski (1984b) highlights the parallels between thermodynamic conservation principles and economic accounting systems, studying the deep analogies between economic theory and thermodynamics. In a related analysis, Mirowski (1991) shows how the mathematization of neoclassical economics was shaped by metaphors from 19th-century physics − particularly the concept of equilibrium − thereby embedding conservation principles at the core of economic modeling. In a similar vein, Dawson (Dawson 2021, p. C429) describes the first law of thermodynamics as a "central thermodynamic accounting principle" (p. C440), introduces the notion of "energy accounting" (p. C440), and explicitly likens the first law of thermodynamics to Fisher's equation of exchange: "both are essentially accounting identities that involve adding up and comparing amounts of money or energy, respectively" (p. C443).

The concept of conservation in economics appears to trace back to Samuelson (1970), who explicitly introduced the idea of a conservation law in economics. According to Dawson (2021), earlier economists such as Jevons and Walras used the metaphor like the lever or balance to describe equilibrium conditions driven by economic forces. Even Irving Fisher illustrated his equation of exchange through a mechanical balance (see Morgan 1997, Fig, 1. and Dawson 2021, Fig. 6. See also Fisher's hydraulic model of an economic system in Morgan 1997, Fig. 2).[17]

The language and the analogies from mechanics remain pervasive to this day. Turk (2006) reminds that Arrow and Hahn (1971) used the expression "balance of forces" and "equal weight" to characterize a general equilibrium in economics. The MIT Dictionary of Modern Economics, as Turk (Turk 2006, p. 209) observes, defines equilibrium as a term borrowed from physics. Similarly, in his text on macroeconomics, Mankiw (2010) defines equilibrium as "a state of balance between opposing forces" (p. 557).

Over time, conservation laws in economics and finance have been the subject of extensive theoretical explorations. Numerous variants have been proposed and ana-

[17] In (Magni 2020, Fig. 2.2), a similar balance scale illustrates the law of conservation as it applies to an economic entity's capital, income, and cash flow.

lyzed by both economists and mathematicians (e.g., Sato and Ramachandran 1990; De León and De Diego 1998; Sato and Kim 2002; Samuelson 2004). A particularly powerful illustration of this perspective is found in the commencement address to the Department of Economics at the University of California, Berkeley, held by Nobel Laureate Thomas Schelling (Schelling 2016). Seeking to identify fundamental truths, Schelling draws a direct parallel between economics and physics, asserting that many foundational economic propositions functions as conservation principles:

> accounting propositions …hold in the same way that the laws of conservation do −conservation of energy, mass, or momentum. …In the physical sciences many of these accounting identities are dignified with the title, “law” (p. 79)

Schelling goes even further, arguing that the core truths of economics are accounting identities. In this view, accounting serves as the foundation of economic reasoning just as conservation laws underpin physics:

> my program was just to take inventory of how many things I knew in economics that were true, important, and not obvious, and to see whether they added up to five …These are what are sometimes called accounting identities …So if I am allowed five candidates, they are all accounting identities (pp.77-79)

Notably, Schelling also references a consolidated conservation principle in economics through the well-known metaphor “there is no free lunch”. This is a formulation equivalent to the *Law of One Price*, itself equivalent to the *no-arbitrage condition* (e.g., Brealey et al. 2020, p. 13, footnote 10). In financial markets, the Law of One Price manifests in terms of NPV as a zero-NPV condition: in a competitive market, financial transactions are zero-NPV opportunities that neither add nor destroy value (e.g., see Berk and DeMarzo 2017).

This conservation law is widely recognzied as the cornerstone of modern corporate finance, as emphasized by Berk and DeMarzo (2017), whose influential textbook is fundamentally built on this principle (see Section 5 of this paper). Virtually every textbook highlights the centrality of Modigliani-Miller Theorems, which are applications of the Law of One Price. The term *law of conservation of value* is often used synonymously (e.g., Brealey et al. 2020, p. 454), and the principle of value additivity is frequently presented as a direct expression of it (e.g., Brealey et al. 2020, p. 910). In their final chapters, these authors identify “value additivity and the law of conservation of value” as one of the seven most important ideas in corporate finance, alongside “capital structure theory”, also being grounded in the conservation principle as expressed by the Modigliani- Miller Theorems.[18]

[18] Schelling rightly notes that “free lunches” do occur in real economies − not through violations of the conservation principles, but through value creation. These arise when economic agents unlock trapped value or generate new wealth via “Pareto improvements, or the gains from trade” (Schelling 2016, p. 79). Positive-NPV projects may illustrate this idea: firms create value through innovation (e.g., new products or services), operational efficiency (e.g., technology or process upgrades), market expansion, removal of inefficiencies (e.g., cost-savings investments, better logistics), or strategic acquisitions. These actions do not violate accounting identities but work within their constraints to improve outcomes. A firm undertaking a positive-NPV project generates new net wealth through more effective use of resources. Admittedly, while positive-NPV projects always benefit shareholders (i.e., reflect private value creation), they may not be Pareto-improving for society if they generate negative externalities. However, if externalities are

Appendix B. The CAD Inc. project.

In this Appendix, we outline the CAD Inc. project, originally presented in Baschieri and Magni (2023), and suitably adapted for the purposes of this paper. For details, we refer the reader to that publication.[19] The Microsoft Excel file for this project is available at https://doi.org/10.5281/zenodo.17675701.

The project involves 25 input variables. For advanced applications, a case study utilizing 2,303 inputs is presented here: https://doi.org/10.3390/jrfm18110613.

A legend of all the symbols used in this Appendix is provided in Figure 5. All figures are either drawn or adapted from Baschieri and Magni (2023) or extracted as screenshots from this paper's accompanying Excel file.

DESCRIPTION OF THE PROJECT

CAD Inc. is evaluating a five-year project involving the production and sale of a manufactured good.

1. Investment and Depreciation

- Initial investment in fixed assets (property, plant and equipment): USD 20,000, fully incurred at time $t = 0$.
- Fixed assets are depreciated on a straight-line basis over five years.

2. Sales and Production

The unit sale price is fixed at USD 10 for each year. Unit manufacturing costs are composed of:

- Year 1 sales forecast: 6,000 units, with an annual growth rate of 10%
- Unit selling price: USD 10 (constant over the five years)
- USD manufacturing costs:
  - USD 2.50 for materials;
  - USD 4.00 for direct and indirect labor.
- No finished goods inventory: production equals sales each year.
- Inventory policy: at year-end, materials inventory must cover 25% of the following years' expected material consumption.

3. SG&A Expenses

- Non-manufacturing materials: 15% of annual sales revenue;
- Incremental salaries (non-manufacturing): fixed at USD 6,000 per year.

4. Working Capital Assumptions

- Receivables: average collection period is 30 days.

internalized – e.g., through carbon pricing – and multiple-accounting frameworks are used, then positive-NPV aligns with the Pareto improvements mentioned by Schelling.

[19] The application of the split-screen approach to spreadsheet enables modelers to both avoid financial functions usually employed in financial modeling (e.g., see Avon 2021; Benninga 2014; Benninga and Mofkadi 2018) and establish a 'spreadsheet engineering' discipline that reduces errors (Thorne 2009; Powell et al. 2008).

	Balance Sheet	C	Income Statement	I	Cash Flow Statement	F
Operating Area	Accounts Receivable	AR	Sales	S	Cash receipts from customers	F^{ar}
	Inventory	Inv	Income from inventory (Δ Inventory)	ΔInv	Cash flow from inventory (= 0)	0
	Accounts Payable (manufacturing)	AP^m	Cost of Purchases (manufacturing)	COP^m	Payments to suppliers (manufacturing)	$F^{ap,m}$
	Accounts Payable (nonmanufacturing)	AP^{nm}	Cost of Purchases (nonmanufacturing)	COP^{nm}	Payments to suppliers (nonmanufacturing)	$F^{ap,nm}$
	S&W Payable (manufacturing)	SWP^m	Labor costs (manufacturing)	LC^m	Payments to employees (manufacturing)	$F^{swp,m}$
	S&W Payable (nonmanufacturing)	SWP^{nm}	Labor costs (nonmanufacturing)	LC^{nm}	Payments to employees (nonmanufacturing)	$F^{swp,nm}$
	(Net) Fixed Assets	NFA	Depreciation	Dep	Asset disposal (net of capital expenditures)	F^{nfa}
	Taxes Payable	TP	Taxes	T	Payments for income taxes	F^{tp}
Non-operating area	Liquid Assets	C^l	Interest income	I^l	Cash flow from liquid assets (net of deposits	F^l
Debt area	Debt	C^d	Interest expenses	I^d	Cash flow to debt	F^d
Equity Area	Equity	C^e	Net Income	I^e	Cash flow to equity	F^e

Fig. 5 Legend of the symbols used in Appendix B: *m* represents manufacturing activities related to production processes, whereas *nm* represents non-manufacturing activities, including administrative and general functions

- Payables: all materials (both manufacturing and non-manufacturing) are paid after 90 days, except in Year 0 and Year 5, when paid within the same year.
- Wages and salaries: paid monthly.
- Tax rate: $\tau = 30\%$, with taxes paid in the year they are incurred.

5. Financing Structure

The initial capital expenditure of USD 20,000 is financed as follows:

- Equity: USD 8,000
- Bank loans: USD 12,000 at an interest rate $i^d = 2\%$, to be repaid through five equal annual principal repayments.

The Year 0 investment in materials is financed using cash withdrawals. The interest rate on liquid assets is $i^l = 2.5\%$.

6. Dividend/Reinvestment Policy

CAD Inc. distributes $\alpha = 20\%$ of its net income to shareholders annualy (Years 1-4). The remaining cash is invested in liquid assets.

7. Costs of capital

- On operating assets: $r^o = 15\%$ (pre-tax WACC),
- On liquid assets: $r^l = 1\%$
- On debt: $r^d = 1.5\%$.

8. Valuation method

Compressed APV: the pre-tax WACC r^o is equal to the unlevered cost of capital k^u, which also serves as the discount rate for tax shields: $r^o = k^u = r^\tau$.

FINANCIAL PLANNING

The analyst begins by extracting the 25 inputs from the project description and entering them into a spreadsheet, each associated with an assumption (i.e., an estimated input value), as illustrated in Figure 6 (see the "Assumptions" worksheet in the Excel file).

Subsequent to the formalization of inputs, the analyst must preliminarily process some of the input data, transforming it into a monetary amount representing either a capital component, an income component, or a cash flow component, and assign it to the appropriate area.

	INPUTS			ASSUMPTIONS	UoM
01		n	Project length	5	y
02	**Fixed Assets**	NFA_0	Investment	20,000	$
03			Depreciation	*straight-line*	
04	**Sales**	S_0	Sales projection of the first period	6,000	u
05			Growth rate (%/year)	10%	% / y
06			Unit Price	10	$ / u
07	**Manufacturing Costs (COGS)**	COP^m	Unit Cost of Purchases	2.5	$ / u
08		LC^m	Unit Cost of Labor	4.0	$ / u
09			Raw Materials Inventory (% of next-period consumption)	25%	%
10	**NonManufacturing Costs (SGA)**		Cost of purchases (SGA) (% of Sales)	15%	%
11		LC^{nm}	Labor costs (SGA)	6,000	$ / y
12	**Operating cycle**		DSO (Days Sales Outstanding)	30	dd
13			DPO (Days Payable Outstanding)	90	dd
14	**Pay period**		Salaries and Wages will be paid	*monthly*	
15	**Taxes**	τ	Tax Rate	30%	%
16			Taxes are paid	*yearly*	
17	**Equity financing**	C_0^e	Capital Payment (period 0)	8,000	$
18	**Debt financing**	C_0^l	Loan amount	12,000	$
19		i^d	Loan Interest Rate	2.00%	%
20			Loan reimbursement periods	5	y
21	**Liquid Assets**	i^l	Interest rate on liquid assets	2.50%	%
22	**Payout policy**	α	Payout ratio (% Net Income)	20%	%
23	**Market data**	r^o	Cost of operating assets	15.00%	%
24		r^l	Cost of non-operating assets	1.00%	%
25		r^d	Cost of debt	1.50%	%
	Valuation method		Compressed APV (i.e., $r^o = r^\tau = k^u$)		

Fig. 6 Inputs and Assumptions are formalized on a spreadsheet (UoM = Unit of Measure). They are numbered consecutively (first column), for easy reference

For example, assumptions 04, 05, 06 allow the analyst to calculate sales (an income component), for each period:

$$\begin{aligned}\text{Units sold in } t &= (1.1)\cdot\text{Units sold in } t-1\\ S_t &= \text{Units sold in } t\cdot\text{unit sale price}\end{aligned}$$

Assumptions 07 and 09 enable the calculation of material used in production and raw material inventory:

$$\begin{aligned}\text{Material used in } t &= \text{Units sold in } t\cdot\text{unit cost}\\ \text{Inv}_t &= 25\%\cdot\text{Material used in } t+1\end{aligned}$$

Assumption 12 allows the computation of accounts receivable:

$$\text{AR}_t = S_t\cdot\frac{30}{365}.$$

Assumption 18 enables the computation of debt principal repayment, and thus the remaining the principal outstanding:

$$\text{Principal Repayments} = C_t^d - C_{t-1}^d = \frac{C_0^d}{5} = \frac{12{,}000}{5} = 2{,}400.$$

Fig. 7 Split Screen Matrix ($t = 0, 1, \ldots, n$). The yellow cells contain values that can be computed directly from the assumptions via preliminary processing

$\mathbf{C}_t$	$\mathbf{C}_{t-1}$	$\mathbf{I}_t$	$-\mathbf{F}_t$
$+\mathrm{AR}_t$	$+\mathrm{AR}_{t-1}$	$+\mathrm{S}_t$	$-F_t^{ar}$
$+\mathrm{Inv}_t$	$+\mathrm{Inv}_{t-1}$	$+\Delta\mathrm{Inv}_t$	-0
$-\mathrm{AP}_t^m$	$-\mathrm{AP}_{t-1}^m$	$-\mathrm{COP}_t^m$	$+F_t^{ap,m}$
$-\mathrm{AP}_t^{nm}$	$-\mathrm{AP}_{t-1}^{nm}$	$-\mathrm{COP}_t^{nm}$	$+F_t^{ap,nm}$
$-\mathrm{SWP}_t^m$	$-\mathrm{SWP}_{t-1}^m$	$-\mathrm{LC}_t^m$	$+F_t^{swp,m}$
$-\mathrm{SWP}_t^{nm}$	$-\mathrm{SWP}_{t-1}^{nm}$	$-\mathrm{LC}_t^{nm}$	$+F_t^{swp,nm}$
$+\mathrm{NFA}_t$	$+\mathrm{NFA}_{t-1}$	$-\mathrm{Dep}_t$	$-F_t^{nfa}$
$-\mathrm{TP}_t$	$-\mathrm{TP}_{t-1}$	$-\mathrm{T}_t$	$+F_t^{tp}$
$+C_t^l$	$+C_{t-1}^l$	$+I_t^l$	$-F_t^l$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

Proceeding this way, the analyst can preliminarily compute and classify the following items for each year, grouping them into the three fundamental components (capital, income, cash flow):

- **Capital**: Accounts receivable, Accounts payable (manufacturing and non-manufacturing), Taxes payable, Debt, Raw Materials Inventory
- **Income**: Depreciation, Sales, Cost of purchases (manufacturing and non-manufacturing), Labor costs (manufacturing and non-manufacturing)
- **Cash Flow**: Net cash flow from fixed assets (asset disposals minus capital expenditures), Payments to employees (manufacturing and non-manufacturing)

These values are highlighted in yellow in Figure 7 (see the "DataProcess" worksheet in the Excel file).

The analyst must then complete all remaining cells. Since there are $n + 1$ SSMs (one for each year from 0 to n) this results in a strip of $n + 1$ SSMs to be completed). For spreadsheet use, the $n + 1$ matrices can be merged into a single *split-screen strip* (see Figure 8).

To build the split-screen strip, the analyst must first fill in the cells corresponding to the 33 accounting and financial magnitudes for period 0. These values are obtained in the following ways:

- Cells whose values are copied and pasted from the preliminary data processing (see list mentioned above)

0	$+\mathrm{S}_0$	$-F_0^{ar}$	$+\mathrm{AR}_0$	$+\mathrm{S}_1$	$-F_1^{ar}$	$+\mathrm{AR}_1$		$+\mathrm{S}_n$	$-F_n^{ar}$	0
0	$+\Delta\mathrm{Inv}_0$	$+0$	$+\mathrm{Inv}_0$	$+\Delta\mathrm{Inv}_1$	$+0$	$+\mathrm{Inv}_1$		$+\Delta\mathrm{Inv}_n$	$+0$	0
0	$-\mathrm{COP}_0^m$	$+F_0^{ap,m}$	$-\mathrm{AP}_0^m$	$-\mathrm{COP}_1^m$	$+F_1^{ap,m}$	$-\mathrm{AP}_1^m$		$-\mathrm{COP}_n^m$	$+F_n^{ap,m}$	0
0	$-\mathrm{COP}_0^{nm}$	$+F_0^{ap,nm}$	$-\mathrm{AP}_0^{nm}$	$-\mathrm{COP}_1^{nm}$	$+F_1^{ap,nm}$	$-\mathrm{AP}_1^{nm}$		$-\mathrm{COP}_n^{nm}$	$+F_n^{ap,nm}$	0
0	$-\mathrm{LC}_0^m$	$+F_0^{swp,m}$	$-\mathrm{SWP}_0^m$	$-\mathrm{LC}_1^m$	$+F_1^{swp,m}$	$-\mathrm{SWP}_1^m$		$-\mathrm{LC}_n^m$	$+F_n^{swp,m}$	0
0	$-\mathrm{LC}_0^{nm}$	$+F_0^{swp,nm}$	$-\mathrm{SWP}_0^{nm}$	$-\mathrm{LC}_1^{nm}$	$+F_1^{swp,nm}$	$-\mathrm{SWP}_1^{nm}$	……	$-\mathrm{LC}_n^{nm}$	$+F_n^{swp,nm}$	0
0	$-\mathrm{Dep}_0$	$-F_0^{nfa}$	$+\mathrm{NFA}_0$	$-\mathrm{Dep}_1$	$-F_1^{nfa}$	$+\mathrm{NFA}_1$		$-\mathrm{Dep}_n$	$-F_n^{nfa}$	0
0	$-\mathrm{T}_0$	$+F_0^{tp}$	$-\mathrm{TP}_0$	$-\mathrm{T}_1$	$+F_1^{tp}$	$-\mathrm{TP}_1$		$-\mathrm{T}_n$	$+F_n^{tp}$	0
0	$+I_0^l$	$-F_0^l$	$+C_0^l$	$+I_1^l$	$-F_1^l$	$+C_1^l$		$+I_n^l$	$-F_n^l$	0
0	$+I_0^d$	$-F_0^d$	$+C_0^d$	$+I_1^d$	$-F_1^d$	$+C_1^d$		$+I_n^d$	$-F_n^d$	0
0	$+I_0^e$	$-F_0^e$	$+C_0^e$	$+I_1^e$	$-F_1^e$	$+C_1^e$		$+I_n^e$	$-F_n^e$	0

Fig. 8 Split-screen strip ($n = 5$). Note that capital starts at zero in $t = -1$ and returns to zero at $t = n$. The convention for the vertical bar is changed as follows: for each row, the sum of the three elements before the bar (capital at time $t-1$, income at time t, and cash flow at time t) equals the element after the bar (capital at time t)

- Cells whose value are computed using the law of motion (income from inventory, cash recepits from customers, payments to suppliers, tax payments, cash flow to debt, salaries and wages payable, net fixed assets, liquid assets, equity)
- Cells whose values are calculated using the law of conservation (net income, cash flow from liquid assets)[20]
- Cells whose values are calculated via simple accounting formulas (income taxes, interest income, interest expenses, cash flow to equity)

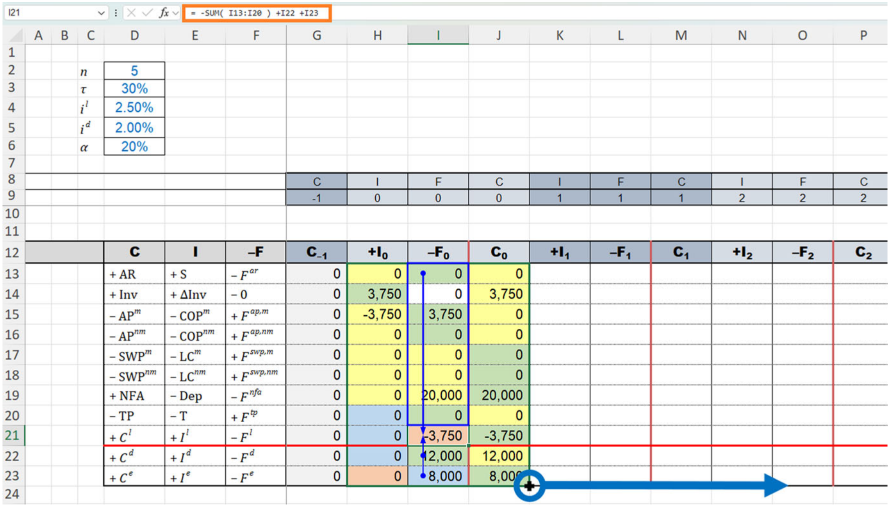
Figure 9 shows the populated cells for capital, income, and cash flow in Year 0. Using Excel's INDEX and MATCH functions, the analyst can extend the same formulas across periods 1 through 5 to complete the entire split-screen strip: this can be done by copying the formulas or dragging the fill handle to the right across all required columns. At this point, financial planning is completed (see Figure 10 and Figures 12-13) (see the "SplitScreenStrip" worksheet in the Excel file).

Once the split-screen strip is constructed, more compact or tailored representations can be derived from it. These serve as summarized statements that highlight specific accounting or financial magnitudes of interest to the analyst. For example, the analyst may frame the strip as a four-area strip, grouping the rows representing the operating area, as illustrated in Figure 11. This four-area strip can also be "transposed", so to speak, to produce a different layout: a three-panel layout that separates the three financial statements into distinct panels: the BS (top), the IS (middle), and the CFS (bottom) (see Figure 14) (see the "TransMatrix" worksheet in the Excel file). Further, the three financial statements can be simultaneously reclassified by nature, by function, or using any other preferred classification. See Baschieri and Magni (2023), Module 3, for further details and alternative framing approaches (see also the various configurations in the "SplitScreenStrip" worksheet in the Excel file).

FINANCIAL EVALUATION AND DECISION-MAKING

To evaluate the project, the analyst must construct a strip of the $n+1$ *benchmark matrices* by applying eq. (9) (via backward induction) to determine the market values of operating assets, non-operating assets, and debt. To determine the benchmark profits

20 Some items can be computed either by the law of motion or the law of conservation. For example, equity can be derived using either approach.



C	I	−F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
+ AR	+ S	$-F^{ar}$	0	0	0	0						
+ Inv	+ ΔInv	− 0	0	3,750	0	3,750						
$-AP^m$	$-COP^m$	$+F^{ap,m}$	0	-3,750	3,750	0						
$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0						
$-SWP^m$	$-LC^m$	$+F^{swp,m}$	0	0	0	0						
$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0						
+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000						
− TP	− T	$+F^{tp}$	0	0	0	0						
$+C^l$	$+I^l$	$-F^l$	0	0	-3,750	-3,750						
$+C^d$	$+I^d$	$-F^d$	0	0	12,000	12,000						
$+C^e$	$+I^e$	$-F^e$	0	0	8,000	8,000						

Fig. 9 The cells with values copied from preliminary data processing are highlighted in yellow. Green cells contain values calculated with the law of motion. Pink-orange cells contain values calculated using the law of conservation. Blue cells contain values calculated via specific formulas. This figure illustrates the calculation of the cash flow from liquid assets (with sign changed: $-F_0^l = -3{,}750$, cell I21) using the law of conservation. The financial plan is completed by dragging the fill handle right across all required columns

C	I	−F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	$+I_3$	$-F_3$	C_3	$+I_4$	$-F_4$	C_4	$+I_5$	$-F_5$	C_5
+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-55,068	4,932	66,000	-65,507	5,425	72,600	-72,058	5,967	79,860	-79,263	6,564	87,850	-94,414	0
+ Inv	+ ΔInv	− 0	0	3,750	0	3,750	375	0	4,125	413	0	4,538	454	0	4,991	499	0	5,491	-5,491	0	0
$-AP^m$	$-COP^m$	$+F^{ap,m}$	0	-3,750	3,750	0	-15,375	11,584	-3,791	-16,913	16,533	-4,170	-18,604	18,187	-4,587	-20,464	20,006	-5,046	-16,472	21,518	0
$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0	-9,000	6,781	-2,219	-9,900	9,678	-2,441	-10,890	10,646	-2,685	-11,979	11,710	-2,954	-13,178	16,132	0
$-SWP^m$	$-LC^m$	$+F^{swp,m}$	0	0	0	0	-24,000	24,000	0	-26,400	26,400	0	-29,040	29,040	0	-31,944	31,944	0	-35,140	35,140	0
$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0
+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000	-4,000	0	8,000	-4,000	0	4,000	-4,000	0	0
− TP	− T	$+F^{tp}$	0	0	0	0	-500	500	0	-899	899	0	-1,331	1,331	0	-1,809	1,809	0	-2,338	2,338	0
$+C^l$	$+I^l$	$-F^l$	0	0	-3,750	-3,750	-94	3,331	-513	-13	2,986	2,460	61	3,688	6,209	155	4,453	10,818	270	-11,089	0
$+C^d$	$+I^d$	$-F^d$	0	0	12,000	12,000	240	-2,640	9,600	192	-2,592	7,200	144	-2,544	4,800	96	-2,496	2,400	48	-2,448	0
$+C^e$	$+I^e$	$-F^e$	0	0	8,000	8,000	1,166	-233	8,933	2,097	-419	10,610	3,106	-621	13,095	4,222	-844	16,473	5,454	-21,927	0

Fig. 10 The split-screen strip completed for the five-period financial statements (BS, IS, CFS) using symbols and abbreviations (first three columns). While this format compactly presents the data, a clearer view of the accounting items is provided in Figures 12-13

C	I	−F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	$+I_3$	$-F_3$	C_3	$+I_4$	$-F_4$	C_4	$+I_5$	$-F_5$	C_5
$+C^o$	$+I^o$	$-F^o$	0	0	23,750	23,750	1,500	-6,204	19,046	2,301	-5,997	15,351	3,189	-6,854	11,686	4,163	-7,794	8,055	5,232	-13,287	0
$+C^l$	$+I^l$	$-F^l$	0	0	-3,750	-3,750	-94	3,331	-513	-13	2,986	2,460	61	3,688	6,209	155	4,453	10,818	270	-11,089	0
$+C^d$	$+I^d$	$-F^d$	0	0	12,000	12,000	240	-2,640	9,600	192	-2,592	7,200	144	-2,544	4,800	96	-2,496	2,400	48	-2,448	0
$+C^e$	$+I^e$	$-F^e$	0	0	8,000	8,000	1,166	-233	8,933	2,097	-419	10,610	3,106	-621	13,095	4,222	-844	16,473	5,454	-21,927	0

Fig. 11 The four-area split-screen strip

C_{-1}		$+I_0$		$-F_0$		C_0		$+I_1$		$-F_1$		C_1		$+I_2$		$-F_2$		C_2	
$+AR_{-1}$	0	$+S_0$	0	$-F^{ar}{}_0$	0	$+AR_0$	0	$+S_1$	60,000	$-F^{ar}{}_1$	-55,068	$+AR_1$	4,932	$+S_2$	66,000	$-F^{ar}{}_2$	-65,507	$+AR_2$	5,425
$+Inv_{-1}$	0	$+\Delta Inv_0$	3,750	− 0	0	$+Inv_0$	3,750	$+\Delta Inv_1$	375	− 0	0	$+Inv_1$	4,125	$+\Delta Inv_2$	413	− 0	0	$+Inv_2$	4,538
$-AP^m{}_{-1}$	0	$-COP^m{}_0$	-3,750	$+F^{ap,m}{}_0$	3,750	$-AP^m{}_0$	0	$-COP^m{}_1$	-15,375	$+F^{ap,m}{}_1$	11,584	$-AP^m{}_1$	-3,791	$-COP^m{}_2$	-16,913	$+F^{ap,m}{}_2$	16,533	$-AP^m{}_2$	-4,170
$-AP^{nm}{}_{-1}$	0	$-COP^{nm}{}_0$	0	$+F^{ap,nm}{}_0$	0	$-AP^{nm}{}_0$	0	$-COP^{nm}{}_1$	-9,000	$+F^{ap,nm}{}_1$	6,781	$-AP^{nm}{}_1$	-2,219	$-COP^{nm}{}_2$	-9,900	$+F^{ap,nm}{}_2$	9,678	$-AP^{nm}{}_2$	-2,441
$-SWP^m{}_{-1}$	0	$-LC^m{}_0$	0	$+F^{swp,m}{}_0$	0	$-SWP^m{}_0$	0	$-LC^m{}_1$	-24,000	$+F^{swp,m}{}_1$	24,000	$-SWP^m{}_1$	0	$-LC^m{}_2$	-26,400	$+F^{swp,m}{}_2$	26,400	$-SWP^m{}_2$	0
$-SWP^{nm}{}_{-1}$	0	$-LC^{nm}{}_0$	0	$+F^{swp,nm}{}_0$	0	$-SWP^{nm}{}_0$	0	$-LC^{nm}{}_1$	-6,000	$+F^{swp,nm}{}_1$	6,000	$-SWP^{nm}{}_1$	0	$-LC^{nm}{}_2$	-6,000	$+F^{swp,nm}{}_2$	6,000	$-SWP^{nm}{}_2$	0
$+NFA_{-1}$	0	$-Dep_0$	0	$-F^{nfa}{}_0$	20,000	$+NFA_0$	20,000	$-Dep_1$	-4,000	$-F^{nfa}{}_1$	0	$+NFA_1$	16,000	$-Dep_2$	-4,000	$-F^{nfa}{}_2$	0	$+NFA_2$	12,000
$-TP_1$	0	$-T_0$	0	$+F^{tp}{}_0$	0	$-TP_0$	0	$-T_1$	-500	$+F^{tp}{}_1$	500	$-TP_1$	0	$-T_2$	-899	$+F^{tp}{}_2$	899	$-TP_2$	0
$+C^l{}_{-1}$	0	$+I^l{}_0$	0	$-F^l{}_0$	-3,750	$+C^l{}_0$	-3,750	$+I^l{}_1$	-94	$-F^l{}_1$	3,331	$+C^l{}_1$	-513	$+I^l{}_2$	-13	$-F^l{}_2$	2,986	$+C^l{}_2$	2,460
$+C^d{}_{-1}$	0	$+I^d{}_0$	0	$-F^d{}_0$	12,000	$+C^d{}_0$	12,000	$+I^d{}_1$	240	$-F^d{}_1$	-2,640	$+C^d{}_1$	9,600	$+I^d{}_2$	192	$-F^d{}_2$	-2,592	$+C^d{}_2$	7,200
$+C^e{}_{-1}$	0	$+I^e{}_0$	0	$-F^e{}_0$	8,000	$+C^e{}_0$	8,000	$+I^e{}_1$	1,166	$-F^e{}_1$	-233	$+C^e{}_1$	8,933	$+I^e{}_2$	2,097	$-F^e{}_2$	-419	$+C^e{}_2$	10,610

Fig. 12 Split-screen strip with accounting labels showing five-period financial statements (BS, IS, CFS) with clearly organized symbols and abbreviations for each accounting item, aiding interpretation. (Part 1, from C_{-1} to C_2)

C_2		$+I_3$		$-F_3$		C_3		$+I_4$		$-F_4$	
$+AR_2$	5,425	$+S_3$	72,600	$-F^{ar}{}_3$	-72,058	$+AR_3$	5,967	$+S_4$	79,860	$-F^{ar}{}_4$	-79,263
$+Inv_2$	4,538	$+\Delta Inv_3$	454	-0	0	$+Inv_3$	4,991	$+\Delta Inv_4$	499	-0	0
$-AP^m{}_2$	-4,170	$-COP^m{}_3$	-18,604	$+F^{ap,m}{}_3$	18,187	$-AP^m{}_3$	-4,587	$-COP^m{}_4$	-20,464	$+F^{ap,m}{}_4$	20,006
$-AP^{nm}{}_2$	-2,441	$-COP^{nm}{}_3$	-10,890	$+F^{ap,nm}{}_3$	10,646	$-AP^{nm}{}_3$	-2,685	$-COP^{nm}{}_4$	-11,979	$+F^{ap,nm}{}_4$	11,710
$-SWP^m{}_2$	0	$-LC^m{}_3$	-29,040	$+F^{swp,m}{}_3$	29,040	$-SWP^m{}_3$	0	$-LC^m{}_4$	-31,944	$+F^{swp,m}{}_4$	31,944
$-SWP^{nm}{}_2$	0	$-LC^{nm}{}_3$	-6,000	$+F^{swp,nm}{}_3$	6,000	$-SWP^{nm}{}_3$	0	$-LC^{nm}{}_4$	-6,000	$+F^{swp,nm}{}_4$	6,000
$+NFA_2$	12,000	$-Dep_3$	-4,000	$-F^{nfa}{}_3$	0	$+NFA_3$	8,000	$-Dep_4$	-4,000	$-F^{nfa}{}_4$	0
$-TP_2$	0	$-T_3$	-1,331	$+F^{tp}{}_3$	1,331	$-TP_3$	0	$-T_4$	-1,809	$+F^{tp}{}_4$	1,809
$+C^l{}_2$	2,460	$+I^l{}_3$	61	$-F^l{}_3$	3,688	$+C^l{}_3$	6,209	$+I^l{}_4$	155	$-F^l{}_4$	4,453
$+C^d{}_2$	7,200	$+I^d{}_3$	144	$-F^d{}_3$	-2,544	$+C^d{}_3$	4,800	$+I^d{}_4$	96	$-F^d{}_4$	-2,496
$+C^e{}_2$	10,610	$+I^e{}_3$	3,106	$-F^e{}_3$	-621	$+C^e{}_3$	13,095	$+I^e{}_4$	4,222	$-F^e{}_4$	-844

C_4		$+I_5$		$-F_5$		C_5	
$+AR_4$	6,564	$+S_5$	87,850	$-F^{ar}{}_5$	-94,414	$+AR_5$	0
$+Inv_4$	5,491	$+\Delta Inv_5$	-5,491	-0	0	$+Inv_5$	0
$-AP^m{}_4$	-5,046	$-COP^m{}_5$	-16,472	$+F^{ap,m}{}_5$	21,518	$-AP^m{}_5$	0
$-AP^{nm}{}_4$	-2,954	$-COP^{nm}{}_5$	-13,178	$+F^{ap,nm}{}_5$	16,132	$-AP^{nm}{}_5$	0
$-SWP^m{}_4$	0	$-LC^m{}_5$	-35,140	$+F^{swp,m}{}_5$	35,140	$-SWP^m{}_5$	0
$-SWP^{nm}{}_4$	0	$-LC^{nm}{}_5$	-6,000	$+F^{swp,nm}{}_5$	6,000	$-SWP^{nm}{}_5$	0
$+NFA_4$	4,000	$-Dep_5$	-4,000	$-F^{nfa}{}_5$	0	$+NFA_5$	0
$-TP_4$	0	$-T_5$	-2,338	$+F^{tp}{}_5$	2,338	$-TP_5$	0
$+C^l{}_4$	10,818	$+I^l{}_5$	270	$-F^l{}_5$	-11,089	$+C^l{}_5$	0
$+C^d{}_4$	2,400	$+I^d{}_5$	48	$-F^d{}_5$	-2,448	$+C^d{}_5$	0
$+C^e{}_4$	16,473	$+I^e{}_5$	5,454	$-F^e{}_5$	-21,927	$+C^e{}_5$	0

Fig. 13 Split-screen strip with accounting labels showing five-period financial statements (BS, IS, CFS) with clearly organized symbols and abbreviations for each accounting item, aiding interpretation. (Part 2, from C_2 to C_5)

		C_0	C_1	C_2	C_3	C_4	C_5	ΣC_t
+ Operating Assets	$+C^o$	23,750	19,046	15,351	11,686	8,055	0	77,888
+ Liquid Assets	$+C^l$	-3,750	-513	2,460	6,209	10,818	0	15,224
Investments	$+C^{inv}$	**20,000**	**18,533**	**17,810**	**17,895**	**18,873**	**0**	**93,112**
+ Debt	$+C^d$	12,000	9,600	7,200	4,800	2,400	0	36,000
+ Equity	$+C^e$	8,000	8,933	10,610	13,095	16,473	0	57,112
Financings	$+C^{fin}$	**20,000**	**18,533**	**17,810**	**17,895**	**18,873**	**0**	**93,112**

		I_0	I_1	I_2	I_3	I_4	I_5	ΣI_t
+ Operating income	$+I^o$	0	1,500	2,301	3,189	4,163	5,232	16,385
+ Interest income	$+I^l$	0	-94	-13	61	155	270	381
Income from assets	$+I^{inv}$	**0**	**1,406**	**2,289**	**3,250**	**4,318**	**5,502**	**16,765**
+ Interest expenses	$+I^d$	0	240	192	144	96	48	720
+ Net Income	$+I^e$	0	1,166	2,097	3,106	4,222	5,454	16,045
Income to capital providers	$+I^{fin}$	**0**	**1,406**	**2,289**	**3,250**	**4,318**	**5,502**	**16,765**

		F_0	F_1	F_2	F_3	F_4	F_5	ΣF_t
+ Cash flow from operations (CCF)	$+F^o$	-23,750	6,204	5,997	6,854	7,794	13,287	16,385
+ Cash flow from non-operating assets	$+F^l$	3,750	-3,331	-2,986	-3,688	-4,453	11,089	381
Cash flow from assets	$+F^{inv}$	**-20,000**	**2,873**	**3,011**	**3,165**	**3,340**	**24,375**	**16,765**
+ Cash flow to debtholders	$+F^d$	-12,000	2,640	2,592	2,544	2,496	2,448	720
+ Cash flow to equity	$+F^e$	-8,000	233	419	621	844	21,927	16,045
Cash flow to capital providers	$+F^{fin}$	**-20,000**	**2,873**	**3,011**	**3,165**	**3,340**	**24,375**	**16,765**

Fig. 14 The three-panel layout of financial statements. The same components from the split-screen strip are reorganized to present the three financial statements separately

V	I^V	$-F^V$	V_{-1}	$+I^V{}_0$	$-F^V{}_0$	V_0	$+I^V{}_1$	$-F^V{}_1$	V_1	$+I^V{}_2$	$-F^V{}_2$	V_2	$+I^V{}_3$	$-F^V{}_3$	V_3	$+I^V{}_4$	$-F^V{}_4$	V_4	$+I^V{}_5$	$-F^V{}_5$	V_5
$+V^o$	$+I^{Vo}$	$-F^{Vo}$	0	0	25,498	25,498	3,825	-6,204	23,118	3,468	-5,997	20,589	3,088	-6,854	16,824	2,524	-7,794	11,554	1,733	-13,287	0
$+V^l$	$+I^{Vl}$	$-F^{Vl}$	0	0	-3,534	-3,534	-35	3,331	-238	-2	2,986	2,745	27	3,688	6,461	65	4,453	10,979	110	-11,089	0
$+V^d$	$+I^{Vd}$	$-F^{Vd}$	0	0	12,174	12,174	183	-2,640	9,716	146	-2,592	7,270	109	-2,544	4,835	73	-2,496	2,412	36	-2,448	0
$+V^e$	$+I^{Ve}$	$-F^{Ve}$	0	0	9,790	9,790	3,607	-233	13,164	3,320	-419	16,064	3,007	-621	18,449	2,516	-844	20,121	1,807	-21,927	0

Fig. 15 The benchmark split-screen strip

for operating assets, non-operating assets, and debt, one can employ the law of motion or, equivalently, the product $r_t^j \cdot V_{t-1}^j$ for $j = o, l, d$. Both the market values and the benchmark profit for equity are then derived from the law of conservation of value and profits, respectively.

The resulting benchmark matrix is shown in Figure 15 (split-screen layout) and Figure 16 (three-panel layout) (see also the "SplitScreenStrip" worksheet in the Excel file, rows 79-91). Subtracting the project matrix from the benchmark Matrix, a Differential Matrix is originated, presenting the Market Value Added (MVA) and the Economic Residul Income (ERI).

Net Present Value

The MVA is the first column of each triad in the Differential Matrix, obtained as $V_t^j - C_t^j$, $j = o, l, d, e$. For $t = 0$, it represents the NPVs of the four areas (cells J98-J101 in the "SplitScreenStrip" worksheet. See also cells J42-J48 in the "ValueCreation" worksheet):

Economic value		V_0	V_1	V_2	V_3	V_4	V_5	ΣV_t
Economic value of operating assets	$+V^o$	25,498	23,118	20,589	16,824	11,554	0	97,582
Economic value of non-operating assets	$+V^l$	-3,534	-238	2,745	6,461	10,979	0	16,413
	$+V^{\text{inv}}$	**21,964**	**22,880**	**23,334**	**23,285**	**22,532**	**0**	**113,995**
Economic value of debt	$+V^d$	12,174	9,716	7,270	4,835	2,412	0	36,408
Economic value of equity	$+V^e$	9,790	13,164	16,064	18,449	20,121	0	77,587
	$+V^{\text{fin}}$	**21,964**	**22,880**	**23,334**	**23,285**	**22,532**	**0**	**113,995**
Economic profit		I^V_0	I^V_1	I^V_2	I^V_3	I^V_4	I^V_5	ΣI^V_t
	$+I^{Vo}$	0	3,825	3,468	3,088	2,524	1,733	14,637
	$+I^{Vl}$	0	-35	-2	27	65	110	164
	$+I^{V\text{inv}}$	**0**	**3,789**	**3,465**	**3,116**	**2,588**	**1,843**	**14,801**
	$+I^{Vd}$	0	183	146	109	73	36	546
	$+I^{Ve}$	0	3,607	3,320	3,007	2,516	1,807	14,255
	$+I^{V\text{fin}}$	**0**	**3,789**	**3,465**	**3,116**	**2,588**	**1,843**	**14,801**
Benchmark's cash flows		F^V_0	F^V_1	F^V_2	F^V_3	F^V_4	F^V_5	ΣF^V_t
	$+F^{Vo}$	-25,498	6,204	5,997	6,854	7,794	13,287	14,637
	$+F^{Vl}$	3,534	-3,331	-2,986	-3,688	-4,453	11,089	164
	$+F^{V\text{inv}}$	**-21,964**	**2,873**	**3,011**	**3,165**	**3,340**	**24,375**	**14,801**
	$+F^{Vd}$	-12,174	2,640	2,592	2,544	2,496	2,448	546
	$+F^{Ve}$	-9,790	233	419	621	844	21,927	14,255
	$+F^{V\text{fin}}$	**-21,964**	**2,873**	**3,011**	**3,165**	**3,340**	**24,375**	**14,801**

Fig. 16 The benchmark three-panel layout

- Operating assets (core project): $\text{NPV}^o = 1,747.5$
- Non-operating assets (reinvestments): $\text{NPV}^l = 216.5$
- Debt: $\text{NPV}^d = 173.9$
- Equity: $\text{NPV}^e = 1,790.1$.

In total, the project NPV is 1,964:

$$\overbrace{1,747.5 + 216.5}^{\text{investment side}} = 1,964 = \overbrace{173.9 + 1,790.1}^{\text{financing side}}$$

The equity NPV is smaller than the project NPV by 173.9, which is the portion of the project's created value captured by debtholders.

We can further decompose value creation into contributions from the investment policy and the financial policy by first identifying the unlevered NPV and the tax benefits. To do this, we discount the tax shields at the unlevered cost of capital: $r^o = k^u = 15\%$.

The tax shields are given by $\tau I^D = (0, 100.1, 61.4, 24.8, -17.8, -66.7)$. Some of these values are negative, indicating periods when interest income exceeds interest expenses, resulting in an adverse tax effect. Overall, the NPV of the tax shields is positive:

$$\text{NPV}^\tau = \sum_{t=0}^{n} \frac{\tau I_t^D}{(1 + 15\%)^t} = 106.5.$$

The unlevered NPV can be obtained in two ways. First, by applying (23):

$$\text{NPV}^u = \text{NPV}^o - \text{NPV}^\tau = 1,747.5 - 106.5 = 1,641.$$

Alternatively, it can be computed by deriving the FCFs and discounting them at the unlevered cost of capital: $\text{NPV}^u = \sum_{t=0}^{n} \text{FCF}_t/(1+15\%)^t$.

Using eq. (24), we can then express the equity NPV as

$$\text{Equity NPV} = \underbrace{1,641}_{\text{value of investment policy}} + \underbrace{\overbrace{106.5}^{\text{tax benefits}} + \overbrace{216.5}^{\text{net reinvestment value}} - \overbrace{173.9}^{\text{debt NPV}}}_{\text{value of financial policy}} = 1,790.1.$$

Therefore, the shareholder value creation can be apportioned as follows:

- the investment policy contributes 1,790.1
- the financial policy contributes 149.1, consisting of:
  - Tax shields benefits: 100.2
  - Value from reinvestments: 231.0
  - Loss from borrowing costs: 173.9

(see the "ValueCreation" worksheet).

Residual Income

The economic residual income (ERI) allocates NPVs across the five periods and the four areas, preserving value additivity. This enables a twofold decomposition of the equity NPV: by period and by source. Specifically, the analyst calculates the four ERIs by subtracting the benchmark profit from the project income: $\text{ERI}_t^j = I_t^j - I_t^{Vj}$ with $I_t^{Vj} = r_t^j V_{t-1}^j$, $j = o, l, d,$. While r^o, r^l, r^d are inputs, the cost of equity is derived as

$$r_t^e = \frac{r^o V_{t-1}^o + r^l V_{t-1}^l - r^d V_{t-1}^d}{V_{t-1}^o + V_{t-1}^l - V_{t-1}^d}$$

so residual earnings can be computed as $\text{ERI}_t^e = I_t^e - r_t^e V_{t-1}^e$. Alternatively, they can be calculated indirectly applying the law of conservation:

$$\text{ERI}_t^e = \text{ERI}_t^o + \text{ERI}_t^l - \text{ERI}_t^d.$$

Figure 17 shows the residual incomes for all areas and for the project as a whole. As illustrated in this paper, the (undiscounted) sum of all residual incomes equals the NPV of the corresponding area (see last column). From the shareholder perspective, the equity NPV (1,790.1) is generated through two value-destroying periods and three value-creating periods. These period shares can in turn be apportioned to the contribution of operating assets, non-operating assets, and the debt (see the "ValueCreation" worksheet in the Excel file, rows 4-15).

Rates of return

As explained earlier in this paper, under the IARR approach, the rate of return is obtained as the ratio of total income to total capital (see eq. (30)). For example, to calculate the return on the operating assets (core project), one divides the total income generated by the operating assets (16,384.9), by the total capital invested in the operating assets (77,887.8) (see the totals in the last column of Figure 14). The

	$I_0-I^V_0$	$I_1-I^V_1$	$I_2-I^V_2$	$I_3-I^V_3$	$I_4-I^V_4$	$I_5-I^V_5$	Total ERI = NPV
o	0.0	-2,324.5	-1,166.3	100.4	1,639.0	3,498.9	**1,747.5**
l	0.0	-58.4	-10.4	34.0	90.6	160.7	**216.5**
d	0.0	57.4	46.3	34.9	23.5	11.8	**173.9**
e	0.0	-2,440.3	-1,223.0	99.5	1,706.2	3,647.7	**1,790.1**
inv	0.0	-2,382.9	-1,176.7	134.4	1,729.7	3,659.5	**1,964.0**
fin	0.0	-2,382.9	-1,176.7	134.4	1,729.7	3,659.5	**1,964.0**

Fig. 17 Economic Residual Income (ERI)

total capital		financial efficiency		NPV	
ΣC^o	77,887.8	$i^o-\rho^o$	2.24%	NPV^o	1,747.5
ΣC^l	15,224.0	$i^l-\rho^l$	1.42%	NPV^l	216.5
ΣC^{inv}	93,111.8	$i^{inv}-\rho^{inv}$	2.11%	NPV^{inv}	1,964.0
ΣC^d	36,000.0	$i^d-\rho^d$	0.48%	NPV^d	173.9
ΣC^e	57,111.8	$i^e-\rho^e$	3.13%	NPV^e	1,790.1
ΣC^{fin}	93,111.8	$i^{fin}-\rho^{fin}$	2.11%	NPV^{fin}	1,964.0

Fig. 18 Financial efficiency and NPV

same applies to the other areas. The average accounting rates of return in the four areas are

- Average Return On Investment: $i^o = \sum_0^n I_t^o / \sum_0^n C_t^o = 21.0\%$
- Average Return On Liquid Assets: $i^l = \sum_0^n I_t^l / \sum_0^n C_t^l = 2.5\%$
- Average Return On Debt: $i^d = \sum_0^n I_t^d / \sum_0^n C_t^d = 2.0\%$
- Average Return On Equity: $i^e = \sum_0^n I_t^e / \sum_0^n C_t^e = 28.1\%$

(see the "ValueCreation" worksheet, rows, 28-37). These rates are merely the book-value weighted averages of the various accounting rates of return, $i_t^j = I_t^j / C_{t-1}^j$, $j = o, l, d, e$. For example, for the operating assets,

$$i_1^o = 6.3\%, \quad i_2^o = 12.1\%, \quad i_3^o = 20.8\%, \quad i_4^o = 35.6\%, \quad i_5^o = 65\%.$$

The book values of the operating assets are

$$C_0^o = 23{,}750, \quad C_1^o = 19{,}046, \quad C_2^o = 15{,}351, \quad C_3^o = 11{,}686, \quad C_4^o = 8{,}055, \quad C_5^o = 0,$$

whence

$$21\% = \frac{6.3\% \cdot 23{,}750 + 12.1\% \cdot 19{,}046 + 20.8\% \cdot 15{,}351 + 35.6\% \cdot 11{,}686 + 65\% \cdot 8{,}055}{23{,}750 + 19{,}046 + 15{,}351 + 11{,}686 + 8{,}055}$$

(see the "AccDiagnostics" worksheet).

The overall project rate of return (average ROA) is $i = 18\%$. It is easy to verify that, unlike IRR (and AIRR), this rate of return equals the weighted average of its constituent assets' rates of return, both from an investment perspective and a financing perspective:

$$\overbrace{\frac{21\% \cdot 77{,}887.8 + 2.5\% \cdot 15{,}224}{77{,}887.8 + 15{,}224}}^{\text{investment side}} = 18\% = \overbrace{\frac{2\% \cdot 36{,}000 + 28.1\% \cdot 57,111.8}{36{,}000 + 57{,}111.8}}^{\text{financing side}}.$$

Financial efficiency
Dividing the total benchmark profits (last column of Figure 16) by the total invested capital in the various areas, one obtains the cutoff rates (average costs of capital) for value creation. Specifically, these are $\rho^o = 18.79\%$, $\rho^l = 1.08\%$, $\rho^d = 1.52\%$, $\rho^e = 24.96\%$. Subtracting ρ^j from i^j one gets a measure of how efficient the area is. The NPV of each area can then be decomposed into a scale effect (the total capital invested) and an efficiency effect (how efficiently funds are employed), as shown in eqs. (31)-(32): Figure 18 presents the results (see also the "ValueCreation" worksheet).

Model validation: accounting and financial consistency checks
Various diagnostic formulas for financial planning (accounting validation) and financial evaluation (financial validation) corroborate the internal consistency of the model (see the "AccDiagnostics" and the "FinDiagnostics" worksheets). All formulas are grounded on the law of motion and the law of conservation. A comprehensive list of these formulas is available in Magni (2025).

Acknowledgements The theoretical framework and proofs underpinning the results mentioned in the paper are developed in detail in Magni (2020, 2023, 2025). For an interactive overview of the Split Screen Matrix, see this video presentation.

Funding Open access funding provided by Università degli Studi di Modena e Reggio Emilia within the CRUI-CARE Agreement.

Declarations

Conflict of Interest The author has no relevant interests to disclose.

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